A Framework for Registration, Statistical Characterization and Analysis for Surface Constrained Functional Imaging Data

Anand A. Joshi[1], David W. Shattuck[2], Paul M. Thompson[2] and Richard M. Leahy[1]

[1]Signal and Image Processing Institute, University of Southern California, Los Angeles, USA
[2]Laboratory of Neuro Imaging, University of California, Los Angeles, USA

Introduction

We present a framework for registering and analyzing functional neuroimaging data constrained to the cortical surface of the brain. We assume as input a set of labeled data points that lie on a set of parameterized topologically spherical surfaces that represent the cortical surfaces of multiple subjects. To perform analysis across subjects, we first co-register the coordinates from each surface to a cortical atlas or canonical template using labeled sulcal maps as constraints. We then present a method for performing statistical analysis of points on this atlas surface which replaces the Gaussian function by the heat kernel. We demonstrate the utility of this framework in the development of a maximum likelihood classifier for parcellation of somatosensory cortex in the atlas based on current dipole fits to MEG data, simulated to represent a somatotopic mapping of S1 sensory areas in multiple subjects.

Surface extraction and parameterization

We first extract a mask for the cortical surface from an MRI volume using the Brainsuite software [1]. The topology of the mask is corrected automatically using a graph-based approach and tessellated to produce a genus zero surface. We then use a *p*-harmonic functional minimization scheme [2] to map the each cortical hemisphere onto the unit square. The result is a bijective mapping between each hemisphere and the unit square in which the interhemispheric fissure is constrained to map to the boundary of the unit square. This allows us to calculate partial derivatives across the boundary and explicitly model continuity between the two cortical hemispheres.

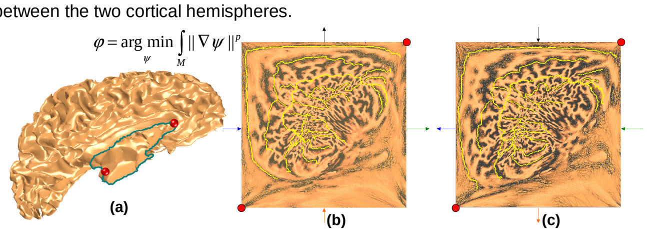

Figure 1: (a) Cortical hemisphere, (b),(c) *p*-harmonic maps of left and right hemispheres. The arrows indicate connectivity of the two hemispheres.

TPS warping in the intrinsic geometry

Having parameterized the cortical surfaces of the subject and atlas, we align the coordinate systems between the subject and atlas such that a set of hand-labelled sulci are brought into register. The alignment uses a set of interactively labelled sulci, sampled uniformly along their lengths, as a set of point constraints. We use the thin-plate spline bending energy on the atlas surface as a regularizing function for the deformation field. We minimize the bending energy with respect to the intrinsic geometry of the 3D surface rather than the parameter space itself, using a covariant PDE approach [3]. To do this, we solve the biharmonic equation using covariant derivatives to obtain a thin-plate spline warp from subject to atlas coordinates.

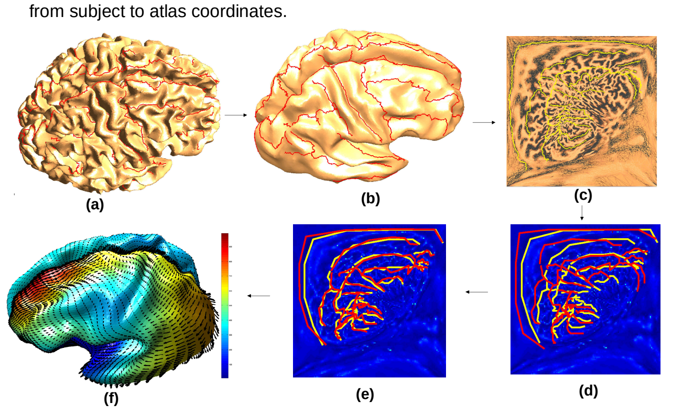

Figure 1: (a) sulcal curves marked on the cortical surface, (b) smoothed cortex with sulcal curves, (c) flattened hemisphere, (d) maps for one hemisphere showing sulcal curves for two subjects before re-parameterization/alignment, (e) alignment of the sulcal curves after thin plate spline based warping, (f) mapping of the deformation field that aligns one cortex to the other mapped back onto a smoothed representation of the cortex (color indicates magnitude of deformation and arrows the direction).

$$E_b = \int \left(\frac{\partial^2\phi}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2\phi}{\partial x\partial y}\right)^2 + \left(\frac{\partial^2\phi}{\partial y^2}\right)^2 dxdy$$ **Thin Plate Bending Energy in Euclidean space**

$$E_b = \int \left(\left(\psi^j{}_{,11}\right)^2 + \left(\sqrt{2}\psi^j{}_{,12}\right)^2 + \left(\psi^j{}_{,22}\right)^2\right) g\,dudv$$ **Bending Energy in non-Euclidean space**

Calculations of covariant derivatives

$$\phi^{\beta}{}_{,\sigma} = \frac{\partial\phi^{\beta}}{\partial u^{\sigma}} + \phi^{\kappa}\Gamma_{\kappa\sigma}{}^{\beta} \quad \text{where} \quad \sigma,\beta,\kappa \in \{1,2\}$$

$$\phi^{\zeta}{}_{,\beta\sigma} = \frac{\partial\phi^{\zeta}{}_{,\beta}}{\partial u^{\sigma}} - \phi^{\zeta}{}_{,\mu}\Gamma_{\beta\sigma} + \phi_{\beta}{}^{\nu}\Gamma_{\nu\sigma}{}^{\zeta} \quad \text{where} \quad \sigma,\beta,\zeta,\mu,\kappa \in \{1,2\}$$

Corresponding Author: Anand A. Joshi (ajoshi@sipi.usc.edu)

Pattern classification in the intrinsic geometry

We present a scheme for pattern classification scheme that considers the intrinsic geometry of the cortical surface of the cortical surface. Our approach uses the heat kernel to replace the Gaussian distribution so that a probability density function on the surface can be defined by analogy to heat propagation on a surface.

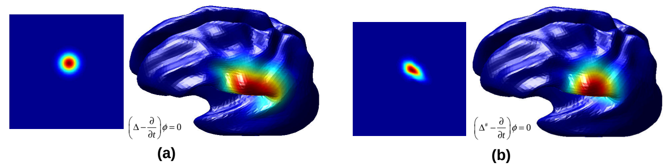

Figure 3: (a) The heat kernel computed using Laplacian in the (u,v) parameter space, (b) The heat kernel computed using the Laplace-Beltrami operator on the cortical surface

Heat equation in Euclidian space	Heat equation on the cortical surface
$\left(\Delta - \frac{\partial}{\partial t}\right)\phi = 0$	$\left(\Delta^{\#} - \frac{\partial}{\partial t}\right)\phi = 0 \quad i,j \in \{1,2\}$
$\Delta = \frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2}$ Laplacian operator	$\Delta^{\#} = \frac{1}{\sqrt{g}}\frac{\partial}{\partial u^i}\sqrt{g}g^{ij}\frac{\partial}{\partial u^j}$ Laplace-Beltrami operator

Just as we can characterize an isotropic Gaussian distribution in the Euclidean plane through its mean and standard deviation, so we can characterize distributions on the surface through mean and variance-like parameters that characterize the ‘center’ of the heat kernel and the ‘time' at which it is observed. For isotropic distributions the corresponding heat kernel $K_t(m,x)$ on a Riemannian manifold can be completely specified by two parameters: m, the location of the initial impulse, and the time t. Parameters m and t play the role of the mean and variance in the Gaussian case. Thus the probability of finding a sample at x is modeled as $p(x\,|\,m,t) = K_t(m,x)$. To use this scheme for classification of two clusters of points, we first compute ML estimates of the parameters (m_1,t_1) and (m_2,t_2) for the two clusters. We illustrate the technique presented above for classification of point localizations of S1 somatosensory regions.

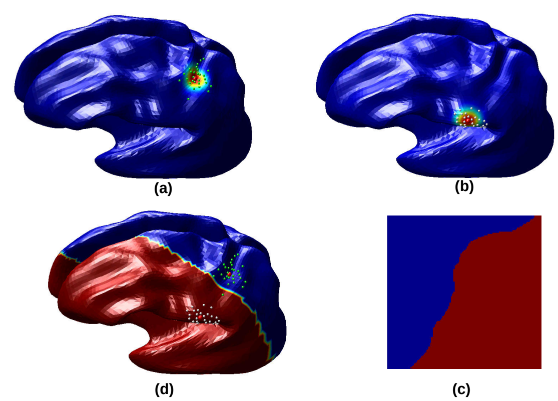

Figure 4: (a)(b) The figures show heat kernels estimated to fit the two datasets for MEG Somatosensory data (c) shows the ML classifier for left hemisphere plotted in the parameter space (d) The classifier on the cortical surface. Red and blue regions show the two decision regions

Conclusion

We have presented a unified framework for analyzing cortically constrained functional data from multiple subjects where the analysis is performed in the intrinsic geometry of the surface. This allows us, for example, to compute the mean with respect to a cluster of points, such that the mean also lies in the surface. We have illustrated this framework by applying the analysis to produce functional parcellation of somatosensory cortex based on (simulated) MEG source localizations across multiple subjects. The method is currently limited to isotropic distributions and to point-wise analysis, but the idea of using the intrinsic heat equation, and kernels of covariant differential operators in place of the Gaussian distribution generalizes to the development of multivariate statistical analysis tools for data constrained to Riemannian manifolds.

The metric Tensor and Christoffel Symbols

$$g_{11} = \left\|\frac{\partial\mathbf{x}}{\partial u}\right\|^2, g_{22} = \left\|\frac{\partial\mathbf{x}}{\partial v}\right\|^2, g_{12} = \left\langle\frac{\partial\mathbf{x}}{\partial u}, \frac{\partial\mathbf{x}}{\partial v}\right\rangle^2, g = g_{11}g_{22} - (g_{12})^2$$

$$g^{11} = \frac{g_{22}}{g},\ g^{12} = g^{21} = -\frac{g_{12}}{g},\ g^{22} = \frac{g_{11}}{g}$$

$$\Gamma_{i,j}{}^{m} = \frac{1}{2}g^{km}\left(\frac{\partial g_{ik}}{\partial u^j} + \frac{\partial g_{jk}}{\partial u^i} - \frac{\partial g_{ij}}{\partial u^k}\right) \quad i,j,k,m \in \{1,2\}$$

References

[1] Shattuck DW, Leahy RM (2002). *Brain Suite: A Cortical Surface Identification Tool*, Med. Image Anal., 6(2): 129-142.
[2] Joshi AA, Leahy RM, Thompson PM., Shattuck DW (2004). *Cortical Surface Parameterization by P-Harmonic Energy Minimization*, IEEE Int. Symp. Biomed. Imag. ISBI 2004: 428-431.
[3] Thompson PM et al. (2004). *Mapping Cortical Change in Alzheimer's Disease, Brain Development, and Schizophrenia*, NeuroImage 23(1):S2-18.

This work is supported by NIBIB under Grant No: R01 EB002010 and NCRR under Grant No: P41RR013642