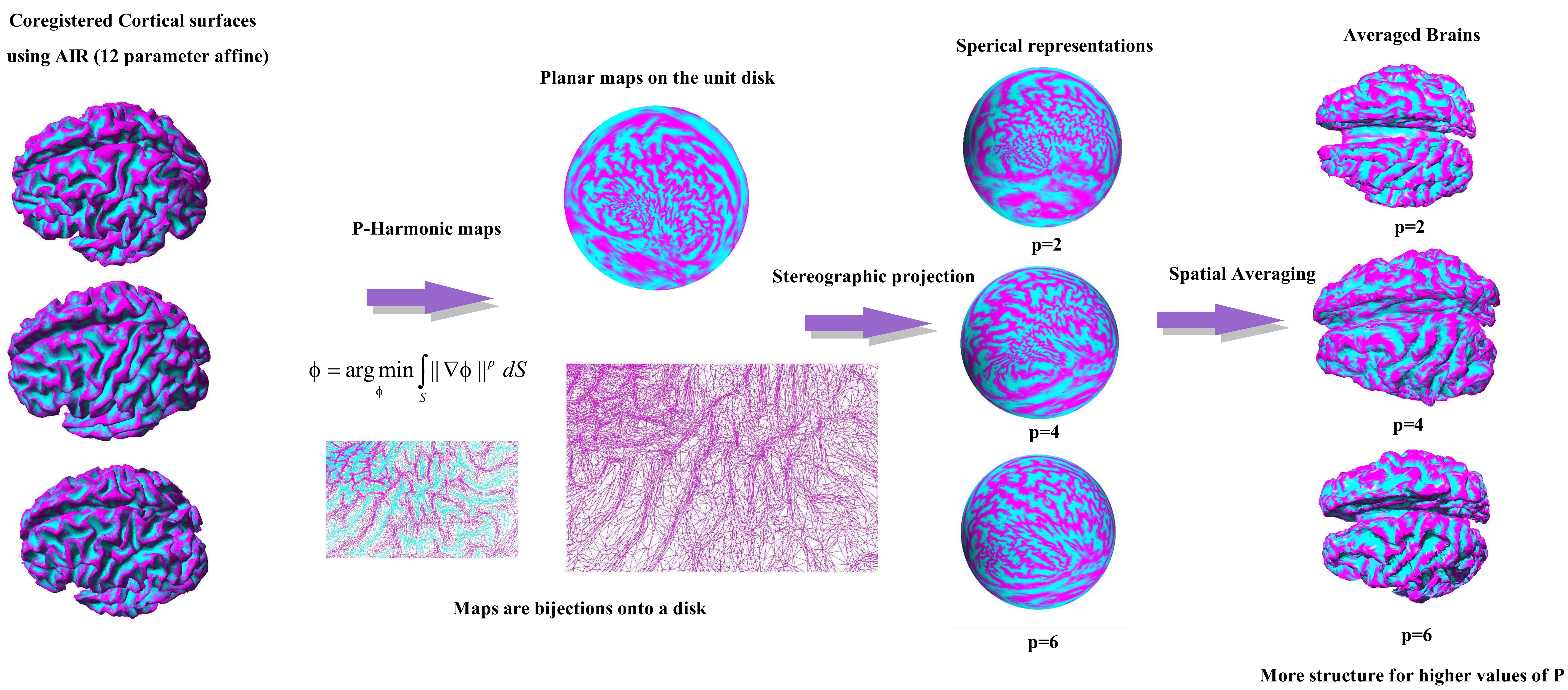
CORTICAL SURFACE PARAMETERIZATION BY P-HARMONIC ENERGY MINIMIZATION

Anand Joshi[1], David Shattuck[2], Paul Thompson[2], Richard Leahy[1],

[1]University of Southern California, Signal and Image Processing Institute, Los Angeles, USA

[2]University of California, Los Angeles, USA



$$\phi = \arg\min_{\phi} \int_S \| \nabla\phi \|^p \, dS$$

INTRODUCTION

Cortical surface parameterization has several applications in visualization and analysis of the brain surface. Here we propose a scheme for parameterizing the surface of the cerebral cortex. The parameterization is formulated as the minimization of an energy functional in the p^{th} norm. A numerical method for obtaining the solution is also presented. Brain surfaces from multiple subjects are brought into common parameter space using the scheme. 3D spatial averages of the cortical surfaces are generated by using the correspondences induced by common parameter space.

MATHEMTICAL FORMULATION

The parameterization of an arbitrary surface, tessellated using a set of triangles can be viewed as an assignment of complex numbers to each vertex. We do this assignment by minimizing p-harmonic energy.
The p-harmonic energy functional is defined as

This energy functional is minimized to obtain the p-harmonic mapping of the surface. The equation is discretized using finite elements. It results in the norm minimization problem

$$\begin{aligned} E_s &= \int_S \| \nabla\phi \|^p \, dS \\ &= \int_S \| \nabla\alpha \|^p \, dS + \int_S \| \nabla\beta \|^p \, dS \end{aligned}$$

We assume alpha and beta are piecewise linear.

$$\alpha(x,y) = \alpha_0 + \alpha_1 x + \alpha_2 y$$

For ith triangle, we write these equations in matrix format

$$\begin{bmatrix} 1 & x_1^i & y_1^i \\ 1 & x_2^i & y_2^i \\ 1 & x_3^i & y_3^i \end{bmatrix} \begin{bmatrix} a_0^i \\ a_1^i \\ a_2^i \end{bmatrix} = \begin{bmatrix} \alpha(x_1,y_1) \\ \alpha(x_1,y_1) \\ \alpha(x_3,y_3) \end{bmatrix}$$

$$\overrightarrow{\nabla\alpha^i} = \frac{1}{|D^i|} \underbrace{\begin{bmatrix} y_2^i - y_1^i & y_3^i - y_1^i & y_1^i - y_2^i \\ x_1^i - x_2^i & x_1^i - x_3^i & x_2^i - x_1^i \end{bmatrix}}_{B^i} \underbrace{\begin{bmatrix} \alpha(x_1,y_1) \\ \alpha(x_2,y_2) \\ \alpha(x_3,y_3) \end{bmatrix}}_{\Gamma^i} = \frac{1}{2A^i} B^i \Gamma^i$$

$$\min_{\alpha} \int_S \| \nabla\alpha \|^p \, dS = \sum_i \| \nabla\alpha^i \|^p \, A^i \qquad \text{for piecewise linear functions } \alpha$$

$$\min_{\alpha} \int_S \| \nabla\alpha \|^p \, dS = \min_{\Gamma} \sum \| \frac{(A^i)^{\frac{1}{p}-1}}{2} B^i \Gamma^i \|^p$$

Assembling this system of equations for all traingles, we get

$$\min_{\alpha} \int_S \| \nabla\alpha \|^p \, dS = \min_{\Gamma} \sum_i \| M^i \Gamma^i \|^p = \min_{\Gamma_c} \| M_f \Gamma_f + M_c \Gamma_c \|^p$$

Here corpus callosum is constrained to lie on the circumcircle of the unit disk. The minimization of the p-harmonic functional results in a map of a brain hemisphere inside the disk. The map is a bijection.
The minimization for β is done similarly. This norm minimization is done by conjugate gradient method.
Because of the convexity for $p > 1$, there is a global minima and the solution is unique.

METHODOLOGY

We use our BrainSuite software package (Shattuck and Leahy, 2002) to generate cortical surfaces. Prior to tessellation, the topological defects are identified and removed using a graph-based approach. The resulting mask is then tessellated to produce a genus-zero surface.

Our spherical mapping is computed from the genus-zero surface in two stages. First, each brain hemisphere is mapped onto a unit disk. This is done by constraining a closed contour, representing the intersection of a plane through the inter-hemispheric fissure with the corpus callosum, to lie on the unit circle. With this constraint we solve for the p-harmonic map, as described, for each hemisphere in turn.

As a preliminary evaluation of this method, we have investigated the average behavior of these maps for several subjects as a function of P. We observe that as P increases, distortion decreases and the maps are more uniform. There is indeed improved correspondence between brains so that the spacial average reveals a more realistic representation of common cortical features.

[1] M. K. Hurdal, et al. Coordinate systems for conformal cerebellar at maps. *NeuroImage*, volume 11, page S467, 2000.
[2] S. Haker, et al. Conformal surface parameterization for texture mapping, *IEEE Transactions on Visualization and Computer Graphics,* 6(2):181-189, April-June 2000.
[3] M. Eck, et al. Multiresolution analysis of arbitrary meshes. In *Computer Graphics (Proceedings of SIGGRAPH 95),* August 1995.

Correspondence: Anand Joshi (ajoshi@usc.edu)