Fuzzy Logic

The binary logic of modern computers often falls short when describing the vagueness of the real world. Fuzzy logic offers more graceful alternatives

by Bart Kosko and Satoru Isaka

Computers do not reason as brains do. Computers "reason" when they manipulate precise facts that have been reduced to strings of zeros and ones and statements that are either true or false. The human brain can reason with vague assertions or claims that involve uncertainties or value judgments: "The air is cool," or "That speed is fast" or "She is young." Unlike computers, humans have common sense that enables them to reason in a world where things are only partially true.

Fuzzy logic is a branch of machine intelligence that helps computers paint gray, commonsense pictures of an uncertain world. Logicians in the 1920s first broached its key concept: everything is a matter of degree.

Fuzzy logic manipulates such vague concepts as "warm" or "still dirty" and so helps engineers to build air conditioners, washing machines and other devices that judge how fast they should operate or shift from one setting to another even when the criteria for making those changes are hard to define. When mathematicians lack specific algorithms that dictate how a system should respond to inputs, fuzzy logic can control or describe the system by using "commonsense" rules that refer to indefinite quantities. No known mathematical model can back up a truck-and-trailer rig from a parking lot to a loading dock when the vehicle starts from a random spot. Both humans and fuzzy systems can perform this nonlinear guidance task by using practical but imprecise rules such as "If the trailer turns a little to the left, then turn it a little to the right." Fuzzy systems often glean their rules from experts. When no expert gives the rules, adaptive fuzzy systems learn the rules by observing how people regulate real systems.

A recent wave of commercial fuzzy products, most of them from Japan, has popularized fuzzy logic. In 1980 the contracting firm of F. L. Smidth & Company in Copenhagen first used a fuzzy system to oversee the operation of a cement kiln. In 1988 Hitachi turned over control of a subway in Sendai, Japan, to a fuzzy system. Since then, Japanese companies have used fuzzy logic to direct hundreds of household appliances and electronics products. The Ministry of International Trade and Industry estimates that in 1992 Japan produced about $2 billion worth of fuzzy products. U.S. and European companies still lag far behind.

Applications for fuzzy logic extend beyond control systems. Recent theorems show that in principle fuzzy logic can be used to model any continuous system, be it based in engineering or physics or biology or economics. Investigators in many fields may find that fuzzy, commonsense models are more useful or accurate than are standard mathematical ones.

At the heart of the difference between classical and fuzzy logic is something Aristotle called the law of the excluded middle. In standard set theory, an object either does or does not belong to a set. There is no middle ground: the number five belongs fully to the set of odd numbers and not at all to the set of even numbers. In such bivalent sets, an object cannot belong to both a set and its complement set or to neither of the sets. This principle preserves the structure of logic and avoids the contradiction of an object that both is and is not a thing at the same time.

Sets that are fuzzy, or multivalent, break the law of the excluded middle—to some degree. Items belong only partially to a fuzzy set. They may also belong to more than one set. Even to just one individual, the air may feel cool, just right and warm to varying degrees. Whereas the boundaries of standard sets are exact, those of fuzzy sets are curved or taper off, and this curvature creates partial contradictions. The air can be 20 percent cool—and at the same time, 80 percent not cool.

Fuzzy degrees are not the same as probability percentages, a point that has eluded some critics of the field. Probabilities measure whether something will occur or not. Fuzziness measures the degree to which something occurs or some condition exists. The statement "There is a 30 percent chance the weather will be cool" conveys the probability of cool weather. But "The morning feels 30 percent cool" means that the air feels cool to some extent—and at the same time, just right and warm to varying extents.

The only constraint on fuzzy logic is that an object's degrees of membership in complementary groups must sum to unity. If the air seems 20 percent cool, it must also be 80 percent not cool. In this way, fuzzy logic just skirts the bivalent contradiction—that something is 100 percent cool and 100 percent not cool—that would destroy formal logic. The law of the excluded middle holds merely as a special case in fuzzy logic, namely when an object belongs 100 percent to one group.

The modern study of fuzzy logic and partial contradictions had its origins early in this century, when Bertrand Russell found the ancient Greek paradox at

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the core of modern set theory and logic. According to the old riddle, a Cretan asserts that all Cretans lie. So, is he lying? If he lies, then he tells the truth and does not lie. If he does not lie, then he tells the truth and so lies. Both cases lead to a contradiction because the statement is both true and false. Russell found the same paradox in set theory. The set of all sets is a set, and so it is a member of itself. Yet the set of all apples is not a member of itself because its members are apples and not sets. Perceiving the underlying contradiction, Russell then asked, "Is the set of all sets that are not members of themselves a member of itself?" If it is, it isn't; if it isn't, it is.

Faced with such a conundrum, classical logic surrenders. But fuzzy logic says that the answer is half true and half false, a 50-50 divide. Fifty percent of the Cretan's statements are true, and 50 percent are false. The Cretan lies 50 percent of the time and does not lie the other half. When membership is less than total, a bivalent system might simplify the problem by rounding it down to zero or up to 100 percent. Yet 50 percent does not round up or down.

In the 1920s, independent of Russell, the Polish logician Jan Łukasiewicz worked out the principles of multivalued logic, in which statements can take on fractional truth values between the ones and zeros of binary logic. In a 1937 article in *Philosophy of Science,* quantum philosopher Max Black applied multivalued logic to lists, or sets of objects, and in so doing drew the first fuzzy set curves. Following Russell's lead, Black called the sets "vague."

Almost 30 years later Lotfi A. Zadeh, then chair of the electrical engineering department at the University of California at Berkeley, published "Fuzzy Sets," a landmark paper that gave the field its name. Zadeh applied Łukasiewicz's logic to every object in a set and worked out a complete algebra for fuzzy sets. Even so, fuzzy sets were not put to use until the mid-1970s, when Ebrahim H. Mamdani of Queen Mary College in London designed a fuzzy controller for a steam engine. Since then, the term "fuzzy logic" has come to mean any mathematical or computer system that reasons with fuzzy sets.

Fuzzy logic is based on rules of the form "if...then" that convert inputs to outputs—one fuzzy set into another. The controller of a car's air conditioner might include rules such as "If the temperature is cool, then set the motor speed on slow" and "If the temperature is just right, then set the motor speed on medium." The temperatures (cool, just right) and the motor speeds (slow, medium) name fuzzy sets rather than specific values.

To build a fuzzy system, an engineer might begin with a set of fuzzy rules from an expert. An engineer might define the degrees of membership in various fuzzy input and output sets with sets of curves. The relation between the input and output sets could then be plotted. Given the rule "If the air feels cool, then set the motor to slow," the inputs (temperature) would be listed along
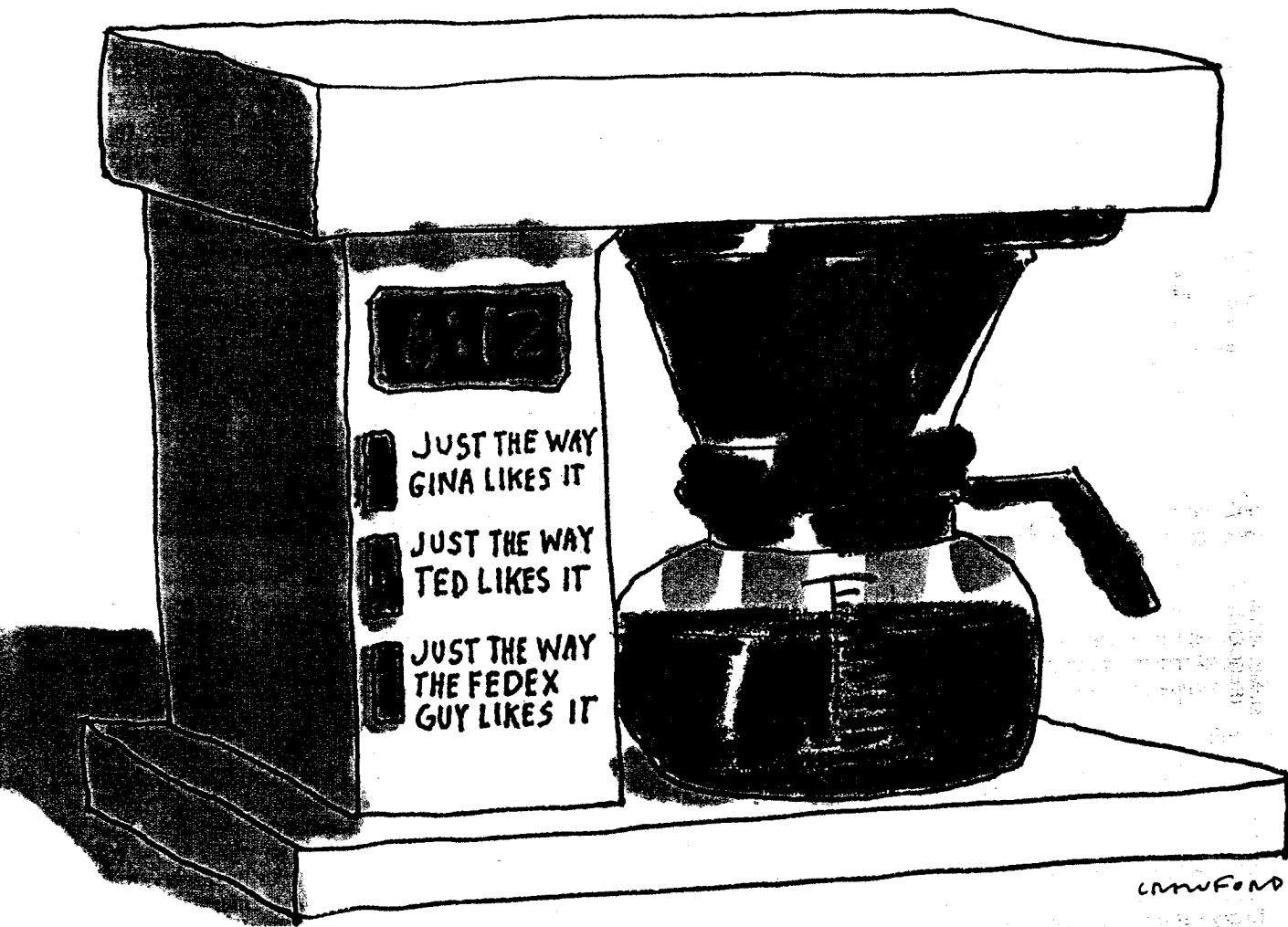
BUILDING MACHINES THAT UNDERSTAND the vagueness of the real world is the goal of those working on fuzzy logic. Fuzzy logic can model and control nuances overlooked by the binary logic of conventional computers.

one axis of a graph and the outputs (motor speed) along a second axis. The product of these fuzzy sets forms a fuzzy patch, an area that represents the set of all associations that the rule forms between those inputs and outputs.

The size of the patch reflects the rule's vagueness or uncertainty. The more precise the fuzzy set, the smaller it becomes. If "cool" is precisely 68 degrees Fahrenheit, the fuzzy set collapses to a spike. If both the cool and slow fuzzy sets are spikes, the rule patch is a point.

The rules of a fuzzy system define a set of overlapping patches that relate a full range of inputs to a full range of outputs. In that sense, the fuzzy system approximates some mathematical function or equation of cause and effect. These functions might be laws that tell a microprocessor how to adjust the power of an air conditioner or the speed of a washing machine in response to some fresh measurement.

Fuzzy systems can approximate any continuous math function. One of us (Kosko) proved this uniform convergence theorem by showing that enough small fuzzy patches can sufficiently cover the graph of any function or input/output relation. The theorem also shows that we can pick in advance the maximum error of the approximation and be sure there is a finite number of fuzzy rules that achieve it. A fuzzy system rea-

sons, or infers, based on its rule patches. Two or more rules convert any incoming number into some result because the patches overlap. When data trigger the rules, overlapping patches fire in parallel—but only to some degree.

Imagine a fuzzy air conditioner that relies on five rules and thus five patches to match temperatures to motor speeds. The temperature sets (cold, cool, just right, warm and hot) cover all the possible fuzzy inputs. The motor speed sets (very slow, slow, medium, fast and very fast) describe all the fuzzy outputs. A temperature of, say, 68 degrees F might be 20 percent cool (80 percent not cool) and 70 percent just right (30 percent not just right). At the same time, the air is also 0 percent cold, warm and hot. The "if cool" and "if just right" rules would fire and invoke both the slow and medium motor speeds.

The two rules contribute proportionally to the final motor speed. Because the temperature was 20 percent cool, the curve describing the slow engine speed must shrink to 20 percent of its height. The "medium" curve must shrink to 70 percent. Summing those two reduced curves produces the final curve for the fuzzy output set.

In its fuzzy form, such an output curve does not assist controllers that act on binary instructions. So the final step is a process of defuzzification, in

which the fuzzy output curve is turned into a single numerical value. The most common technique is to compute the center of mass, or centroid, of the area under the curve. In this instance, the centroid of the fuzzy output curve might correspond to a motor speed of 47 revolutions per minute. Thus, beginning with a quantitative temperature input, the electronic controller can reason from fuzzy temperature and motor speed sets and arrive at an appropriate and precise speed output.

All fuzzy systems reason with this fire-and-sum technique—or something close to it. As systems become more complex, the antecedents of the rules may include any number of terms conjoined by "and" or disjoined by "or." An advanced fuzzy air conditioner might use a rule that says, "If the air is cool and the humidity is high, then set the motor to medium."

Fuzzy products use both microprocessors that run fuzzy inference algorithms and sensors that measure changing input conditions. Fuzzy chips are microprocessors designed to store and process fuzzy rules. In 1985 Masaki Togai and Hiroyuki Watanabe, then working at AT&T Bell Laboratories, built the first digital fuzzy chip. It processed 16 simple rules in 12.5 microseconds, a rate of 0.08 million fuzzy logical inferences per second. Togai InfraLogic, Inc., now offers chips based on Fuzzy Computational Acceleration hardware that processes up to two million rules per second. Most microprocessor firms currently have fuzzy chip research projects. Fuzzy products largely rely on standard microprocessors that engineers have programmed with a few lines of fuzzy inference code. Although the market for dedicated fuzzy chips is still tiny, the value of microprocessors that include fuzzy logic already exceeds $1 billion.

The most famous fuzzy application is the subway car controller used in Sendai, which has outperformed both human operators and conventional automated controllers. Conventional controllers start or stop a train by reacting to position markers that show how far the vehicle is from a station. Because the controllers are rigidly programmed, the ride may be jerky: the automated controller will apply the same brake pressure when a train is, say, 100 meters from a station, even if the train is going uphill or downhill.

In the mid-1980s engineers from Hitachi used fuzzy rules to accelerate, slow and brake the subway trains more smoothly than could a deft human operator. The rules encompassed a broad

NONFUZZY SET

FUZZY SET

FUZZY SET AND ITS COMPLEMENT
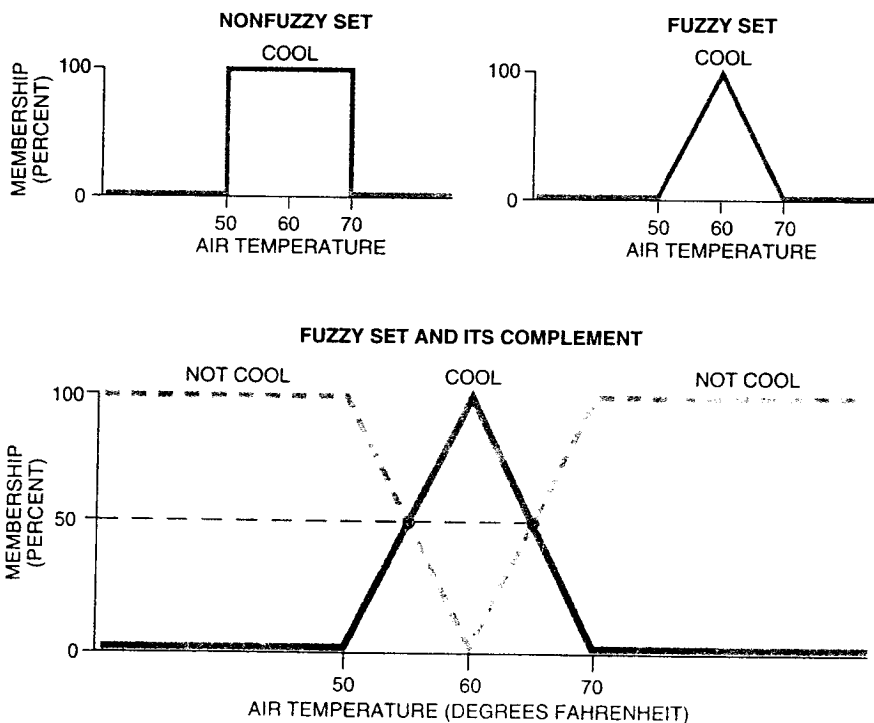
SET THEORY underlies the difference between standard and fuzzy logic. In standard logic, objects belong to a set fully or not at all (*top left*). Objects belong to a fuzzy set only to some extent (*top right*) and to the set's complement to some extent. Those partial memberships must sum to unity (*bottom*). If 55 degrees is 50 percent "cool," it is also 50 percent "not cool."
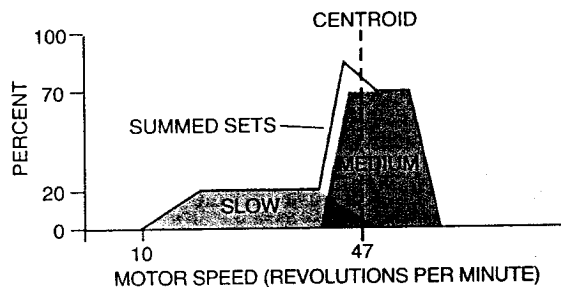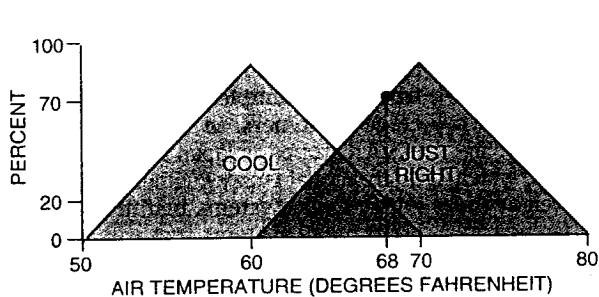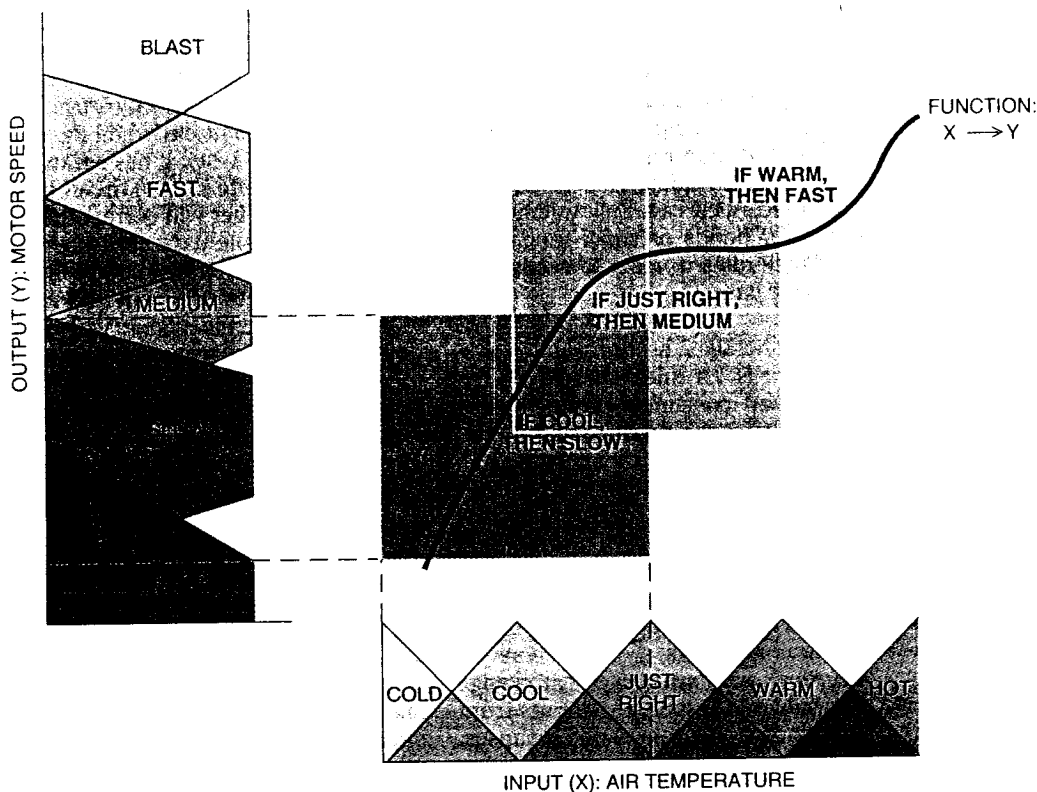
APPLICATION OF FUZZY LOGIC to the control of an air conditioner shows how manipulating vague sets can yield precise instructions. The air conditioner measures air temperature and then calculates the appropriate motor speed. The system uses rules that associate fuzzy sets of temperatures, such as "cool," to fuzzy sets of motor outputs, such as "slow." Each rule forms a fuzzy patch. A chain of patches can approximate a performance curve or other function (*top*). If a temperature of 68 degrees Fahrenheit is 20 percent "cool" and 70 percent "just right" (*bottom left*), two rules fire, and the system tries to run its motor at a speed that is 20 percent "slow" and 70 percent "medium" (*bottom right*). The system arrives at an exact motor speed by finding the center of mass, or centroid, for the sum of the motor output curves.

range of variables about the ongoing performance of the train, such as how frequently and by how much its speed changed and how close the actual speed was to the maximum speed. In simulated tests the fuzzy controller beat an automated version on measures of riders' comfort, shortened riding times and even achieved a 10 percent reduction in the train's energy consumption. Today the fuzzy system runs the Sendai subway during peak hours and runs some Tokyo trains as well. Humans operate the subway during nonpeak hours to keep up their skills.

Companies in Japan and Korea are building an array of fuzzy consumer goods that offer more precise control than do conventional ones. Fuzzy wash-ing machines adjust the wash cycle to every set of clothes, changing strategies as the clothes become clean. A fuzzy washing machine gives a finer wash than a "dumb" machine with fixed commands. In the simplest of these machines, an optical sensor measures the murk or clarity of the wash water, and the controller estimates how long it would take a stain to dissolve or saturate in the wash water. Some machines use a load sensor to trigger changes in the agitation rate or water temperature. Others shoot bubbles into the wash to help dissolve dirt and detergent. A washing machine may use as few as 10 fuzzy rules to determine a wide variety of washing strategies.

In cameras and camcorders, fuzzy logic links image data to various lens settings. One of the first fuzzy camcorders, the Canon hand-held H800, which was introduced in 1990, adjusts the autofocus based on 13 fuzzy rules. Sensors measure the clarity of images in six areas. The rules take up about a kilobyte of memory and convert the sensor data to new lens settings.

Matsushita relies on more rules to cancel the image jitter that a shaking hand causes in its small Panasonic camcorders. The fuzzy rules infer where the image will shift. The rules heed local and global changes in the image and then compensate for them. In contrast, camcorder controllers based on mathematical models can compensate for no more than a few types of image jitter.

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Systems with fuzzy controllers are often more energy efficient because they calculate more precisely how much power is needed to get a job done. Mitsubishi and Korea's Samsung report that their fuzzy vacuum cleaners achieve more than 40 percent energy savings over nonfuzzy designs. The fuzzy systems use infrared light-emitting diodes to measure changes in dust flow and so to judge if a floor is bare. A four-bit microprocessor measures the dust flow to calculate the appropriate suction power and other vacuum settings.

Automobiles also benefit from fuzzy logic. General Motors uses a fuzzy transmission in its Saturn. Nissan has patented a fuzzy antiskid braking system, fuzzy transmission system and fuzzy fuel injector. One set of fuzzy rules in an on-board microprocessor adjusts the fuel flow. Sensors measure the throttle setting, manifold pressure, radiator water temperature and the engine's revolutions per minute. A second set of fuzzy rules times the engine ignition based on the revolutions per minute, water temperature and oxygen concentration.

One of the most complex fuzzy systems is a model helicopter, designed by Michio Sugeno of the Tokyo Institute of Technology. Four elements of the craft—the elevator, aileron, throttle and rudder—respond to 13 fuzzy voice commands, such as "up," "land" and "hover." The fuzzy controller can make the craft hover in place, a difficult task even for human pilots.

A few fuzzy systems manage information rather than devices. With fuzzy logic rules, the Japanese conglomerate Omron oversees five medical data bases in a health management system for large firms. The fuzzy systems use 500 rules to diagnose the health of some 10,000 patients and to draw up personalized plans to help them prevent disease, stay fit and reduce stress. Other companies, including Hitachi and Yamaichi Securities, have built trading programs for bonds or stock funds that use fuzzy rules to react to changes in economic data.

The Achilles' heel of a fuzzy system is its rules. Almost all the fuzzy consumer products now on the market rely on rules supplied by an expert. Engineers then engage in a lengthy process of tuning those rules and the fuzzy sets. To automate this process, some engineers are building adaptive fuzzy systems that use neural networks or other statistical tools to refine or even form those initial rules.

Neural networks are collections of "neurons" and "synapses" that change their values in response to inputs from surrounding neurons and synapses. The neural net acts like a computer because it maps inputs to outputs. The neurons and synapses may be silicon components or equations in software that simulate their behavior. A neuron adds up all the incoming signals from other neurons and then emits its own response in the form of a number. Signals travel across the synapses, which have numerical values that weight the flow of neuronal signals. When new input data fire a network's neurons, the synaptic values can change slightly. A neural net "learns" when it changes the value of its synapses.

Depending on the available data, networks can learn patterns with or without supervision. A supervised net learns by trial and error, guided by a teacher. A human may point out when the network has erred—when it has emitted a response that differs from the desired output. The teacher will correct the responses to sample data until the network responds correctly to every input.

Supervised networks tune the rules of a fuzzy system as if they were synapses. The user provides the first set of rules, which the neural net refines by running through hundreds of thousands of inputs, slightly varying the fuzzy sets each time to see how well the system performs. The net tends to keep the changes that improve performance and to ignore the others.

A handful of products in Japan now use supervised neural learning to tune the fuzzy rules that control their operation. Among them are Sanyo's microwave oven and several companies' washing machines. Sharp employs this technique to modify the rules of its fuzzy refrigerator so that the device learns how often its hungry patron is likely to open the door and adjusts the cooling cycle accordingly. So far the neural net must learn "off-line" in the laboratory, from small samples of behavior by average customers. In time, researchers at such groups as Japan's Laboratory for International Fuzzy Engineering and the Fuzzy Logic Systems Institute hope to build fuzzy systems that will adapt to the needs of each consumer.

Supervised networks do have drawbacks. Tuning such systems can take hours or days of computer time be-

Fuzzy Sets and Paradoxes

Relations between sets show the paradox at the heart of fuzzy logic. In standard sets, an object either does or does not belong to a set: the glass of water is either empty or full. Fuzzy sets cover a continuum of partial sets, such as glasses only half full (*left*).

Two objects or two sets define a two-dimensional space (*right*). Standard set theory holds only at the corners of the square, where the values are either 1 or 0. The midpoint, the fuzziest point in the square, cannot be rounded off to a corner.
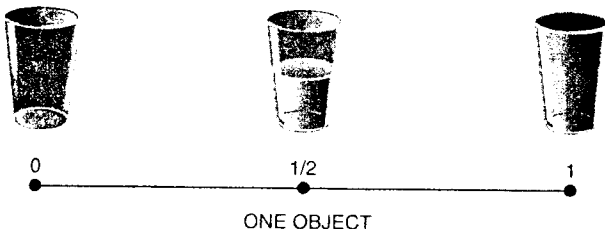
Fuzzy sets of two objects are points of the square, such as the point A. The union and intersection of a fuzzy set A and its complement A* lie at the corners of an interior square. In contrast, the union or intersection of a standard set and its complement results in the total set or the empty set at the square's corners, respectively.


ONE OBJECT

FIRST OBJECT
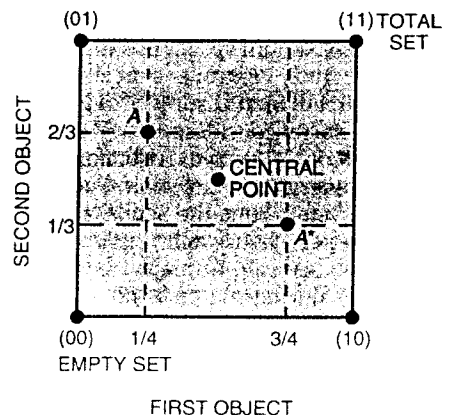
How Fuzzy Systems Learn Rules

Adaptive systems called neural networks can help fuzzy systems learn rules. A neural network accepts pairs of input and output data, such as temperatures and motor speeds for air conditioners, and groups them into a small number of prototypes, or classes. Within the network, each prototype acts as a quantization vector—a list of numbers—that stands for the synapses feeding into a neuron. When a new data point enters the network, it stimulates the neuron associated with the prototype that matches the data most closely. The values of the "winning" synapses adjust to reflect the data they are receiving. As the data cluster, so do the quantization vectors, which define rule patches. More data lead to more numerous and precise patches.

FIELD OF COMPETING NEURONS

SYNAPTIC VECTOR

RULE PATCH

QUANTIZATION VECTOR

DATA POINT

OUTPUT

INPUT
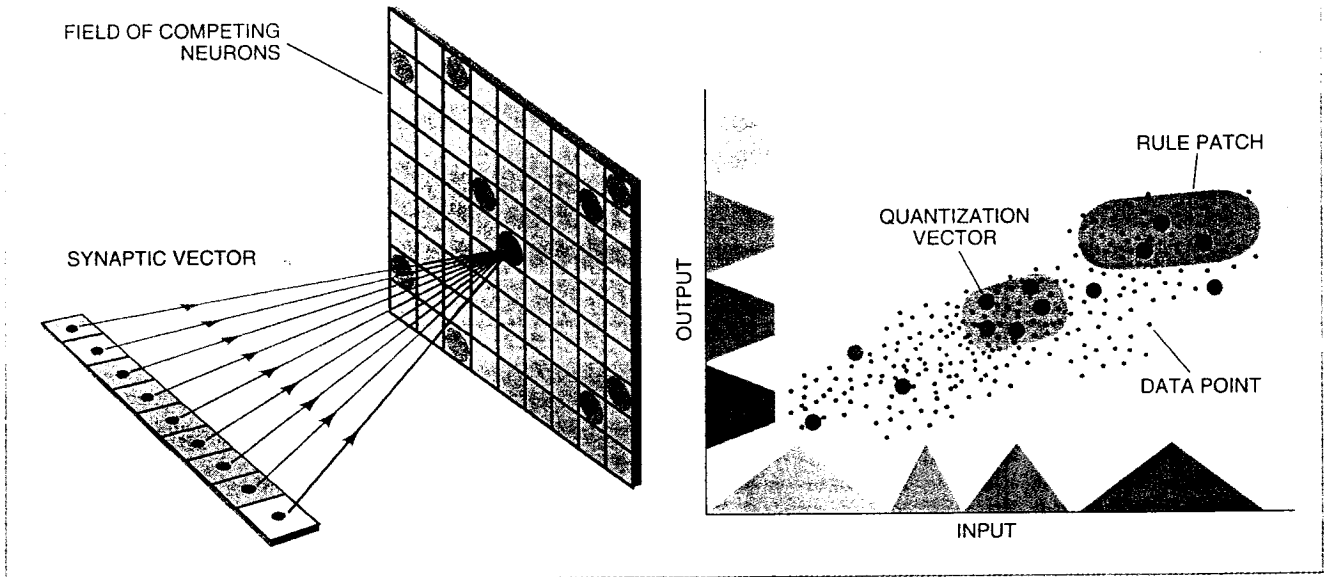
cause networks may converge on an inappropriate solution or rule or may fail to converge at all. Neural researchers have proposed hundreds of schemes to alleviate this problem, but none has removed it. Even after a lengthy tuning session, the final rules may not be much better than the first set.

Rather than relying on an expert to supply a training set of data and correct a network in the process of learning, unsupervised neural networks learn simply by observing an expert's decisions. In this way, an adaptive fuzzy system can learn to spot rule patterns in the incoming data. Broad rule patches form quickly, based on a few inputs. Those patches are refined over time.

Unsupervised neural networks blindly cluster data into groups, the members of which resemble one another. There may be no given right or wrong response or way to organize the data. The algorithms are simpler, and, at least in theory, the network need run through the data just once. (In some cases, when data are sparse, the neural net must cycle through them repeatedly.) Unsupervised learning is thus much faster than supervised learning. With numerical inputs and outputs supplied by an expert or a physical pro-

cess or even an algorithm, an unsupervised neural network can find the first set of rules for a fuzzy system. The quality of the rules depends on the quality of the data and therefore on the skills of the expert who generates the data. At this point, there are fewer unsupervised than supervised adaptive fuzzy systems. Because unsupervised networks are best used to create rules and supervised networks are better at refining them, hybrid adaptive fuzzy systems include both.

Most fuzzy systems have been control systems with few variables. That trend happened because most of the first fuzzy logic engineers were control theorists and because a control loop regulates most consumer products. The challenge for the next generation of fuzzy research will be tackling large-scale, nonlinear systems with many variables. These problems can arise when people try to supervise manufacturing plants or schedule airline flights or model the economy. No experts may be able to describe such systems. Common sense may falter or not apply. The neural nets that must learn the rules for modeling these hard problems may have little or no data to go on.

A further problem is that, like any other mathematical or computer mod-

el, fuzzy logic falls prey to the "curse of dimensionality": the number of fuzzy rules tends to grow exponentially as the number of system variables increases. Fuzzy systems must contend with a trade-off. Large rule patches mean the system is more manageable but also less precise. Even with that trade-off, fuzzy logic can often better model the vagueness of the world than can the black-and-white concepts of set theory. For that reason, fuzzy logic systems may well find their way into an ever growing number of computers, home appliances and theoretical models. The next century may be fuzzier than we think.

FURTHER READING

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