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fsQCA: Dialog Between Jerry M. Mendel and Charles C. Ragin

by

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With email contributions from Prof. Ragin

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ABSTRACT

This report reprints extensive e-mail dialogs between the author and Prof. Charles Ragin about **fuzzy set Qualitative Comparative Analysis (fsQCA)**, organizing them by category and sub-category, so that others may also benefit from Prof. Ragin's insights and wisdom about fsQCA. Doing this should make fsQCA more accessible than it presently is, especially to people who want to understand the inner inner-workings of fsQCA.

The 2nd Edition of this report is due to a continuation of the e-mail dialogs between March 2011 and the end of December 2011. Revised or new items are printed in red.

I. Introduction

Beginning on October 8, 2009 (and continuing for more than two years), I began an extensive e-mail dialog with Prof. Charles C. Ragin, who is the inventor and developer of QCA (Qualitative Comparative Analysis) and fsQCA (Fuzzy Set Qualitative Comparative Analysis). He is also the author of two books about QCA and fsQCA [1] and [2] as well as the co-editor of and contributor to a short textbook about them [3].

QCA was originally developed by Prof. Ragin for crisp sets but was later extended by him to fuzzy sets, because he realized that categorizing social science causes and effects as black or white was not realistic. Fuzzy sets let him get around this. The former QCA is now referred to as "csQCA" and the latter QCA is now referred to as "fsQCA"[1].

There are many steps to fsQCA, but there was no single reference that explained them all. As my students and I tried to learn fsQCA (so that we could program it) and understand its mathematical nuances and how to apply it, I turned to Prof. Ragin for clarifications. Understanding all aspects of fsQCA turned out to be akin to peeling an onion. As I went from one step of fsQCA to the next, a new "ring" was uncovered that needed further explaining. Prof. Ragin was extremely patient and generous with his time and always responded to my enquiries in great detail.

In this report I have reprinted our e-mail dialogs, organizing them by category and sub-category, so that others may also benefit from Prof. Ragin's insights and wisdom about fsQCA. Doing this should make fsQCA more accessible than it presently is, especially to people who want to understand its inner-workings.

In my first e-mail to Prof. Ragin, I stated: "I am not a social scientist, but am a professor of electrical engineering at the Univ. of Southern California, who has been working in the fields of fuzzy logic and systems for more than 20 years. I recently came across your works and my students I have been reading your two extremely interesting books [1] *Fuzzy-Set Social Science* and [2] *Redesigning Social Inquiry Fuzzy Sets and Beyond*. We are very much interested in seeing if your fsQCA can be applied to some of the **linguistic summarization** problems that we are studying."

A word about notation: A and a denote a set and its complement, respectively; and, $Abc + Cd$ is short for (A and b and c) or (C and d).

Finally, not every aspect of this on-going dialog may be crystal clear, because it is a "dialog" and not a technical article. Readers are advised to extract as much useful material from this dialog as possible and to not bother with the rest.

Note: Revised or new items are printed in red. Most items are new and represent a continuing of the dialog between March 2011 and the end of December 2011.

[1] There is also a version of QCA called "Multi-Value QCA—mvQCA" [3] in which more than two discrete levels can be assigned to a variable.

II. Index of Topics

III. Questions and Answers

<u>Cases</u>

- **Negative Cases**

Q: On pp. 275-278 of your 2000 book you talk about "negative cases." Are they still used in fsQCA, and if so, are they generated automatically in fsQCA?

A: I'm not sure about the context of this question. In social science parlance, a "negative case" is any case that did not display the outcome. So it's a candidate for the outcome that nevertheless failed to produce/display it. With csQCA, this is straightforward; any case with a 0 in the outcome column is a negative case. In fsQCA, of course, there are degrees to which each case is a negative case, based on the fuzzy coding of the outcome.

- **Negative Cases: Re-specifying the Causal Conditions**

Q: On page 286 of your 2000 book[2], you state: "One also should look at the 'negative cases,' especially the ones that have a high score in the necessary conditions but non-membership (0 scores) in the desired outcome." Since necessary conditions will be appended to all sufficient conditions [this has changed: see fsQCA: Necessary Conditions], what does this mean?

A: This section addresses possibilities for re-specifying the causal conditions included in the analysis, so that a higher coverage score might be obtained. It is straightforward that it is a good idea to look at the "uncovered" positive cases (high *y*, low *x*). It is not so obvious that it is also useful to look at cases that met the necessary conditions but still failed to display the outcome. It could be, for example, that the positive cases all had an absence of severe government repression and this would be relatively invisible because researchers tend to look for conditions that are "present." However, by looking at the cases that met the necessary conditions (i.e., they are good candidates for the outcome) but did not experience/display the outcome, it would be quickly apparent that severe government repression was "present." This condition could then be added as a causal condition (the absence of severe government repression) in a subsequent analysis of the conditions leading to IMF protest.

- **Number of Cases (see, Causal Conditions/Combinations: Number of Causal Conditions)**

Q1: I have read some works about the maximum number of causal conditions that one can use, and that the number depends upon the number of cases. The phrase "number of cases" is ambiguous (to me) because it can be interpreted to mean (1) the entire set of cases, or the (2) set of cases that are associated with a desired outcome. The second interpretation is further confounded in fsQCA by the fact that each case is associated with a desired outcome to a degree. So an interpretation of "set of cases that are associated with a desired outcome" could/should be "set of cases that are associated with a desired outcome to degree > 0.5."

[2] This is at the end of a section, entitled "Example 1: IMF Protest/Discussion of Results," and is about Table 10.5, "Summary of fuzzy-set analysis of 'IMF protest'."

This turns out to be extremely important to us in our trying to apply fsQCA to oil field data. For 60 oil wells [in an undisclosed oil field], when the desired outcome is cumulative high production rate, only three (!) are in this set to degree > 0.5, whereas when the desired outcome is cumulative low production rate, more than 50 wells are in this set [this is a very difficult field to extract oil from]. We have been using six causal conditions, and it has occurred to me that this is okay for low production rate, but is not for high production rate.

Have you encountered such a situation before? I ask because I have not seen it discussed in any of the books I have read.

A1: When social scientists use the phrase "number of cases" they usually mean the number of independent observations, which in your research would be seen as the 60 oil wells.

I actually think that trying to apply a formula for the maximum number of causal conditions that can be applied to a given number of cases is misguided. This thinking has its source in the degrees of freedom literature in statistics, where inference is impossible if there are too many predictors. In statistics, the idea is that the ratio of cases to explanatory variables should be substantial.

More important than the number of cases is their diversity. If you have 60 cases that all look pretty much the same, there is no analytic leverage; however, there is a great deal of leverage with 10 cases that differ substantially from each other (with respect to their memberships in the relevant causal conditions).

The number of cases with the outcome is relevant to the assessment of necessary conditions—at least this is what I argued in *Fuzzy-Set Social Science*. Is a case without the outcome relevant to the evaluation of a necessary conditions argument? Not directly, because the assessment is of necessity without sufficiency. So this line of thought adds some complications to the "number of cases" issue.

In general, the interface between set theoretic reasoning and probabilistic reasoning is messy.

If only 3 of 60 cases have greater than 0.5 membership then this does present analytic challenges: (1) You will find few if any sufficient conditions (or combinations). In some ways this is not bad. If an outcome is very rare, then more than likely there are several necessary conditions for its occurrence and these conditions are only rarely met. (2) In the analysis of sufficiency, much of the action in the calculation of consistency and coverage will be dictated by patterns among cases with less than 0.5 membership in the outcome. This may or may not be okay. It's like looking for clues to what might be found if you had more cases with greater than 0.5 membership in the outcome.

Q2: We observed that if N_1 cases (where N_1 is small) have membership > 0.5 in the desired outcome, and consistency is computed for these cases (they all survive the frequency threshold test), we are led to a certain set of sufficient conditions; but, when more cases are included for which their memberships in the desired outcome is very small, these new cases can obliterate the prior sufficient conditions, and, in fact, we actually have a situation where now fsQCA leads to no sufficient conditions. Using a new recursive formula [that we have developed] for consistency, it is possible to establish precise conditions for when can happen (or not happen). And, those conditions demonstrate that it can indeed happen. This may not have been observed for small N, but now that fsQCA is being used for large N, it can happen, and this suggests that one has to be "careful" when applying fsQCA to the large-N situation.

When we focused first on the subset of N_1 high production-rate wells, we obtained some sufficient conditions; however, when we focused on all of the wells ($N_2 = 60$), N_2 - N_1 of which had very small membership values in "high production rate wells," no sufficient conditions resulted.

Naturally, one could take different points of view about this, depending upon one's objectives. If, e.g., my objective was to establish causal connections to high production rate for all 60 wells, then knowing there are none is useful. If, on the other hand, I want to know the causal connections to high production rate wells (notice the subtle distinction between "high production rate for all 60 wells" and "high production rate wells"), then I think that fsQCA should only be applied to the subset of such wells.

I think one analogous situation for your breakdown in democracy example is "world-wide breakdown in democracy" versus "breakdown in democracy in the 18 European countries." But, I think there could also be a distinction between "likely breakdown in democracy" and "breakdown in democracy." For the former, one would have to set a threshold on membership of the outcome (e.g., 0.5) and only focus on those countries in this subset of the 18 countries.

So, the choice of the cases seems to be extremely important for fsQCA. While this may seem obvious to you, it has not been so obvious to us.

[Perhaps a better (more correct) way to handle "likely breakdown in democracy" is to derive a membership function for it that is different from the membership function that is used for "breakdown of democracy." Such a membership function would also be S-shaped, but it would be much more compressed toward the right.]

A2: About N_1 and N_2 and the assessment of sufficiency: An argument of sufficiency is technically based on a process of selecting cases with high scores on a causal condition or combination and then assessing the degree to which they agree in displaying the outcome. In fuzzy *XY* plot terms, you want to establish that there are few, if any, cases in lower right triangle (or, less restrictively, in the lower right quadrant).

When you select cases with > 0.50 membership value in the outcome, you are, in effect, making it very difficult to contradict sufficiency. Again, in fuzzy *XY* plot terms, you have drastically reduced the relevant contradictory space (0.75 of the area below $Y = 0.50$ contradicts sufficiency while only 0.25 of the area above $Y = 0.50$ contradicts sufficiency).

So it is not surprising that when you restrict the analysis to $Y > 0.50$, you find consistency but then this consistency erodes when you include cases that are $Y \leq 0.50$.

Here's another take, a more positive one, on what you are doing. When you select on the outcome (as in $Y > 0.5$), and then conduct a truth table analysis, you are in effect asking what clumps of characteristics are shared by the positive cases. This is a perfectly reasonable question to ask, and it is directly relevant to the assessment of shared conditions (and "substitutable" conditions). But this analysis does not establish sufficiency, per se, because there can still be more or less identical cases lacking the outcome.

I conduct this type of analysis from time to time because it is informative and descriptively powerful, but I still avoid claims of sufficiency. The latter requires looking at all relevant instances (e.g., those with $X = 1$ and $Y = 0$).

Causal Conditions/Combinations

• **Adjective Terms**

Q: In your works, do you encounter conditions that don't have a natural “yes” or “no” connotation (e.g., *literate*, *urban*, *unstable* do have such a connotation)? If so, how do you handle them? I ask because in our work we frequently (more likely than not) have conditions that don't have a natural “yes” or “no” connotation (e.g., pressure, volume, temperature, etc.).

A: My general position is that it is not possible to assess membership in abstract nouns, like “pressure.” There's got to be an adjective in the mix, like “high pressure.” Of course, you may not know if “high” is the operative adjective.

• **Appearance of Both a Causal Condition and its Complement in Different Parsimonious Terms**

Q: Can it happen that a causal condition appears as part of one of the parsimonious terms and its complement appears as part of another one of the parsimonious terms?

A: Yes. I think it’s interesting when it happens. It’s like the solution starts with a branching, X versus $\sim X$. (Terms without either X or $\sim X$ are on both branches.)

• **Correlated Causal Conditions**

Q1: How important is correlation between the causal conditions in fsQCA? If, e.g. two causal conditions are correlated will fsQCA always discover this? For example, if C_1 and C_2 are positively correlated, then will a causal combination that contains both of them always contain, e.g., low C_1 and low C_2, or high-C_1 and high C_2, but not low C_1 and high C_2, or high C_1 and Low C_2? If the latter can occur, then this would seem to contradict the correlation between C_1 and C_2 and would cast a “shadow” on the credibility of fsQCA. What are your thoughts about this?

A1: More important than correlated conditions are conditions that have a set theoretic relation (e.g., $C_2 <= C_1$). If two conditions are strongly correlated, then there may be few cases of $c_1 C_2$ and few cases of C_1c_2. In all likelihood, rows with these combinations will be low frequency and many will be empty. These rows become potential counterfactuals, so their fate in the solution will be shaped in part by how CA is set up. My guess is that there will be recipes with C_1 and recipes with C_2, but relatively few with both (assuming it is the presence of C_1 and C_2 that it is linked to Y). Another way to approach correlated conditions is to combine them into single conditions, using min or max, depending on which logic makes better sense.

Q2: I wonder whether this can be approached theoretically? A starting point might be to quantify $C_2 <= C_1$. Any thoughts on how to do this?

A2: Not sure I understand the question. When there are superset subset relations among causal conditions, it is important to understand why. If C_1 is a subset of C_2, there are no cases of C_1c_2. Is this combination even possible? If not, then why not? This is the first question. If it is an impossible combination, then you have to decide what to do with the remainder combinations that include C_1c_2. You may, for example, want to keep them false in the analysis of both Y and

~*Y*. Another way to think about this is that you have a multi-value set with three categories c_1c_2, c_1C_2 and C_1C_2.

• Disappearance of Causal Condition after Counterfactual Analysis

Q: If after CA one or more causal conditions no longer appear in the intermediate solutions, couldn't one start the entire fsQCA all over excluding those causal conditions? Have you ever tried this, and if so will one obtain the same solution as obtained from the first fsQCA?

A: Usually yes, but not always. Sometimes the solution is different. This can occur for a variety of reasons because the initial truth table spreadsheet is now different, with different row frequencies, consistencies, etc.

• Irrelevant/Nonsensical Causal Conditions

Q1: If one assumes a causal condition that is totally irrelevant will fsQCA discard it? If so, do you know at which step in fsQCA this occurs and why?

A1: Not quite sure what the context for this question is. In general, fsQCA is not as good as say regression analysis at "weeding out" causal conditions. The inclusion of an irrelevant condition in a truth table analysis impacts the row frequencies and the consistency scores (and later on, the counterfactual analysis), so there is no guarantee that it will be excluded from the solution. In general, for a condition to be excluded from the solution, it must appear to be irrelevant (eliminate-able) in all contexts in which it appears.

I think of the specified causal conditions as an expression of how the investigator wishes to structure the analysis—as an expression of his/hers interests via an analytic frame.

Q2: It seems that fsQCA is very much dependent on the choice of the "candidate" causal conditions. For example, in your breakdown of democracy example, if I included a nonsensical candidate causal condition such as "H = consumes sausages," some countries would score very high and others might score low. Such a causal condition could be eliminated (by absorption) during counterfactual analysis by assuming that the desired outcome could also have occurred if h occurred. Otherwise, it seems that intermediate solutions could include this nonsensical causal condition.

A2: Yes, the intermediate and complex solutions are especially vulnerable to these kinds of "errors."

Q3: Have you ever had criticisms of fsQCA that focus on this? if so, how have you responded to them?

A3: In fact, I haven't had much criticism along these lines. In quantitative social science, it is generally recognized that results are profoundly specification dependent, even using techniques such as regression that are more ruthless when it comes to irrelevant variables.

Think of the specification (of causal conditions) as a lens. Using a different lens gives a different view. Most QCA researchers will try different specifications until they find the lens that helps them see the best.

In the end, all research is theory and knowledge dependent. QCA is more up front about this dependence than most conventional techniques.

• Methodology for Choosing Causal Conditions

Q: At the very start of an fsQCA one begins with a set of potential causal conditions. Is there a methodology for choosing the subset from that set that is actually used in fsQCA?

A: In general, the answer is no. Usually it's trial and error. However, when you run analysis with *A*, *B*, *C*, *D* and *D/d* does not appear in the solution, it's a good idea to rerun the analysis with just *A*, *B*, *C*.

Sometimes I use the *subset/superset procedure* in the software to narrow the list of conditions. I've attached a brief description (though not with this specific application in mind). See Appendix B (prepared by Ragin).

• More Than One-Term Causal Conditions

Q1: In our applications we tend to use more than one-term causal conditions, e.g., *low acceleration*, *moderate acceleration* and *high acceleration*. Peer Fiss told us that the way he handles this is to treat each of these as a separate causal condition. Doing this increases the dimensionality of fsQCA very rapidly. Is this the way you do this?

A1: Often. I mentioned before my preference for calibrating both ends of a range (e.g., calibrating membership in *high income* and *low income* separately) and then interpreting *not-high* intersected with *not-low* as *high membership* in middle.

Q2: Can you pick and choose which of the terms you want to include in fsQCA? If so, must you consider all possible combinations of them in a multitude of fsQCAs?

A2: Usually, I just compute the multiple calibrations and then study them in *XY*-plots, with the outcome. This usually gives strong clues about which calibration(s) is (are) most useful. So yes I pick and chose using a combination of substantive knowledge and data exploration.

• Multiple Causal Conditions

Q: Suppose I have a problem where, e.g., there are 5 potential causes [causal conditions[3]]. I could examine them 1, 2, ..., or 5 at a time. If I examine them 5 at a time will the Quine reduction include the same rules that I would have obtained from the 1, 2, 3, or 4 cause studies; or do I perform all of the studies and then combine the (seemingly) uncommon rules across them, and then apply Quine reduction to the combined rules?

A: I usually stay focused on the 5-condition analysis, although in my 2000 book, I did all combinations, as in your example. Very often, the solutions resulting from the 5 condition analysis will reveal that one or more of the original 5 conditions does not appear in any of the causal recipes generated by the truth table analysis (especially if the focus is on the parsimonious solution). This finding would permit moving to the simpler specification (e.g., a four condition analysis).

In essence, if there are 5 causal conditions and you believe that the world works via causal

[3] A *causal condition* is analogous to an antecedent in a rule.

recipes (multiple intersections of conditions for a given outcome), then to work with only 3 or 4 conditions (a subset of the known/thought-to-be-relevant conditions) is a **misspecification** of the analysis and should give "incorrect" results (if the five conditions are indeed all relevant).

In practice, researchers are always experimenting with different sets (and subsets) of conditions to see which specification yields the most interpretable results.

- **Name of Causal Combination after Sufficiency Test**

Q: What name do you use for the causal combinations that make it through the sufficiency test, just before QM, e.g., "Primitive Boolean expressions"?

A: Yes, or simply "truth table rows."

- **Number of Causal Conditions**

Q1: You advocate limiting fsQCA to 3-8 causal conditions. Suppose one had more than 8 potential causal conditions. Would the results of using fsQCA for so many causal conditions be "reliable?"

A1: Reliability is not the problem; it's more a matter of the researcher's ability to interpret the results. Of course, the other issue is that the greater the number of causal conditions, the greater the number of configurations lacking empirical cases.

Q2: My interest in the number of causal conditions stems from Marx's 2005 article. He concludes that: "a QCA-application is restricted by the proportion of variables on cases and by an upper limit of variables which can be used in an analysis [this limit occurs when contradictions do not occur]. If both restrictions are not taken into account QCA cannot make a distinction between random and real data." For 10 cases [Marx (2005, Table 5)], the number of variables has to be limited to 4; for 15 cases, the number of variables has to be limited to 5; for 25 cases, the number of variables has to be limited to 6; for 30 cases, the number of variables has to be limited to 7; and, for 45 cases, the number of variables has to be limited to 8." He also states [Marx (2005, p. 18)]: " … for 50 cases the upper-limit of variables is 8." So, what do you make of this?

A2: [Marx] leaves out the fact that we use substantive knowledge to address the problem of limited diversity. The problem of "too few" cases (which really means cases that are limited in their diversity) is partially ameliorated via "easy" counterfactuals. [Marx] does not address this key feature of the approach. [Marx performed his study for csQCA and not fsQCA.]

Q3: I am trying not to use statistical arguments; however, once someone has raised this as an issue and has even provided some guidelines it has to be addressed. Peer Fiss and I have met a few weeks ago and he very generously gave me an advance copy of his article in press. There he uses data from 205 firms and 8 causal conditions. So, this does not contradict Marx.

A3: A typical fsQCA analysis has 4-8 causal conditions, almost regardless of the number of cases. I think the real reason for this has more to do with our ability to decipher complexity than anything else.

Q4: In our engineering/computer-science applications of fsQCA I feel it is very important to provide potential users with some guidelines on how many causal conditions can be chosen. Since fsQCA is unknown to these communities and they are steeped in quantitative methods, and they are preconditioned about the impossibility of prediction with too many predictors (I know, I know, fsQCA is not doing prediction), they will find it impossible to believe that fsQCA has no limitations on the number of causal conditions.

A4: The limitation has more to do with the complexity factor I mentioned. Also, the stronger the substantive knowledge the less the pressure to have empirical instances of all logically possible combinations of causal conditions. I agree that the idea of "no limitation" is implausible, but the limitation varies with the strength and nature of the substantive knowledge that the researcher brings to the analysis.

• One-Term Causal Conditions

Q: Why do you prefer (not sure this is the right word) to use one-term causal conditions (linguistic variable), e.g. *literate*, rather than more than one-term causal conditions, e.g., *somewhat literate*, *moderately literate* and *highly literate*?

A: It's not really a preference. I actually prefer qualifiers and the analysis of multiple qualifiers. But my usual audience is beginner level, so I keep it simple.

• Used as Regressors

Q: Could fsQCA be used to establish a subset of causal conditions that might then be used as the regressors in a quantitative (not necessarily linear) variable analysis? I ask this because after I describe QCA to someone, they always seem to ask: "So, what can I do with the results from QCA?" (I am familiar with Box 1.4 on page 15 in your 2009 book [3][4].) In engineering we are often more interested in forecasting or classifying then we are in "describing."

A: As long as the regression equation is correctly specified, I see no real problem. Correct specification is not all that easy. For example, if your fsQCA results are $AB + CD \to Y$, the regression equation would have these two terms as interaction terms, AB and CD, (without the additive components), and it would also need to have a four-way interaction $ABCD$ to take care of the fact that cases of $ABCD$ should score 1, not 2, on the outcome. That is, cases that are doubly determined (from a logical point of view) shouldn't be doubly counted in a regression analysis.

[It seems, therefore that the fsQCA causal combinations could be used to establish the structure of a regression equation.]

[4] This Box is entitled "Five Types of Uses of QCA Techniques," and lists them as: (1) Summarizing data; (2) Checking coherence of data; (3) Checking hypotheses or existing theories; (4) Quick test of conjectures; and (5) Developing new theoretical arguments.

Counterfactual Analysis[5]

- **Believability of**

Q: The extreme situation for counterfactuals would be if there were no cases. Then the final results from QCA would be based entirely on one's substantive knowledge. I doubt anyone would take those results seriously. So, in your experience what is the "breakpoint," if you will, between this extreme situation and situations where counterfactuals are used, i.e. what percentage of the total number of causal combinations can be treated as counterfactuals with people still believing the results?

A: This is a great question. You are way ahead of my colleagues in social science. I like to think of it in terms of the width of the interval between the parsimonious and the complex solution. If the parsimonious solution is A and the complex solution is $AbCDe$, then the width is great, perhaps too great. It also depends on the number of initial conditions in the truth table, of course. This can be formalized and I think I've done it in my notes somewhere. Still, the short answer to your question is that *I don't have a breakpoint*. My guess also is that it depends in part on the nature of the research question.

There is a paper somewhere that talks about the ratio of observed combinations of logically possible combinations, but that's as far as it goes—no breakpoint.

- **Causes Not Necessarily Independent**

Q: In counterfactual analysis, a lot of expert knowledge is used to discard/accept possibilities. If causes are not necessarily independent (one of the strengths of QCA) can this analysis be done?

A: In the algorithm I have developed, a causal condition is not used in a blanket manner. It is more surgical. Let's say you believe that A, B, and C are linked to the outcome when present. You find a pattern: $ABc \to Y$. The "c" is perplexing, but you have no cases (empirically) of ABC (which would allow the simplification to $AB \to Y$. The counterfactual $ABC \to Y$ is used (surgically) to allow this simplification. Still "C" is not used in a blanket manner, but only when the context allows its use as a strategic "don't care." Of course, if we are not confident that a condition has this character (strongly linked to the outcome—either way), then its use to create strategically useful don't care combinations is not warranted.

- **Easy and Difficult**

Q1: Are all *difficult counterfactuals* always rejected by your fsQCA software?

A1: I hope so! My worry is the other way—that some are actually easy (if examined one at a time), and the software says they are difficult (according to the rules that are input by the user). This could occur in specific configurational contexts; the user's redefinition of a remainder as "easy" would have to be based on very good substantive knowledge about the specific context. Of course, it could operate the other way as well, where knowledge of a specific configurational context would reclassify an "easy" counterfactual to "difficult."

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[5] A counterfactual case [2, p. 9, footnote 4] is a substantively relevant combination of causal conditions that nevertheless does not exist empirically. Counterfactual analysis involves evaluating the outcome that such a case would exhibit if, in fact, it existed.

Q2: Once the answers have been obtained (and couldn't they contain some causal conditions and some complements of causal conditions?), then isn't the rest of CA mechanical, in which case I have not needed the notions of "easy" and "difficult" counterfactuals?

A2: Yes, everything follows from the answers because the answers define which remainders are easy counterfactuals. The idea of a difficult counterfactual is simply that you shouldn't remove an element from a complex solution if that element makes sense! The parsimonious solution sometimes does exactly that, and the purpose of CA is to put it back in! The parsimonious solution doesn't care which counterfactuals are easy and which are difficult (from the perspective of existing knowledge).

Q3: I realize that "easy" and "difficult" counterfactuals are useful for explaining things, but, partitioning counterfactuals into these subsets may cause an intellectual barrier to someone's understandings of counterfactuals, because they may believe that such a partitioning must be done prior to performing CA.

A3: No such partitioning is necessary; it follows from the answers to the questions posed in the intermediate solution dialogue box [in Ragin's fsQCA software].

Q4: I may be way off base, but if this partitioning of counterfactuals is needed prior to computation, then could you please explain why and what the thought process is?

A4: Again, no need to partition. The point is simply that the parsimonious solution can often be over inclusive with respect to remainders. In fact, it usually is, which is why I almost always favor the intermediate solutions.

• **Example**

Q1: Do you have an example you could send to me that works out all possible *counterfactuals*, and discusses the substantive knowledge that is used to accept or reject each?

A1: Try this reference: Alexander Hicks et al., "The Programmatic Emergence of the Social Security State," *American Sociological Review*, 60:329-49. It may do the trick. If not, let me know. [It did not do the trick.]

In general, social scientists have been very slow to catch on to the idea of counterfactual analysis and to the fact that some are easy and some are difficult. I published the ideas about five years ago because (almost) everyone was reporting parsimonious solutions and never checking to see which remainders had been incorporated and then assessing them. What I sketched then was how to do counterfactual analysis by hand, with a little algebra, comparing the parsimonious and complex solutions. Still, no one really did it, because it seemed all too technical. So I had it programmed into fsQCA. Finally, social scientists are starting to try it. Still, there are very few who (previously) checked all the remainders (one at a time), even though this was my original recommendation. It's just too demanding for most social scientists. They are accustomed to privileging parsimony, which I think is a mistake.

Q: We have another question about CA. It is hard for me to describe in words so let me ask it in two examples (these will probably seem very simple to you).

Complex solution: *BCdE*
Parsimonious solution: *B*+*E*
Assumptions: *b*, *C*, *D*, *E*

Intermediate solution #1: Focusing on Parsimonious solution term *B*, it is *BCE* —Correct?
Intermediate solution #2: Focusing on Parsimonious solution term *E* it is *CE* —Correct?

A: [Both] look correct to me; and, *BCE* + *CE* = *CE*.

• Intermediate Solutions [see, also Software: Intermediate Solutions (Summaries)]
Q1: I still need help with counterfactual analysis. I didn't find what I was looking for in the last reference that you suggested. So, let me be very specific.

I am going to use Table 5.7 from your 2009 book. If I use a consistency level of >0.8, then only two causal combinations survive, namely [Note: *D* is developed country, *U* is urban country, and *L* is literate country] *dul* and *duL*. Their union is *du*. Four causal combinations are listed as remainders, namely: *dUl*, *dUL*, *Dul* and *DUl*. I assume that these 4 remainders are used in the counterfactual analysis. I realize that this is a very simple example and that *du* may already be the parsimonious solution, but this should also fall out from the counterfactual analysis. Can you please show me the steps (and the thought process) of counterfactual analysis for *dUl, dUL, Dul* and *DUl*?

[At a later date (September, 2010), I understood that counterfactual analysis is not done using the remainders, so my question made no sense to Ragin. He never came out and said so, but instead gave another example.]

A1: This example is not very useful for a demonstration because the parsimonious and intermediate solutions are the same, so there is no counterfactual analysis possible. They are the same because the expectation is that it is the absence of the three conditions that should be linked to the outcome.

I've attached a document that I sent to my programmer describing how to program the *intermediate solution*. Take a look at it and let me know if you have questions [it is at the end of this report].

At a very basic level, the intermediate solution is the complex solution stripped of causal conditions that “don't make sense.” The parsimonious solution exists simply to bar the stripping of conditions that must be retained. For example, if the complex solution had been *duL* and the parsimonious solution had been *d*, we could strip the nonsensical *L*, but not the “sensical” *u*. [Mendel: I think what this means is that if a country is not developed (*d*) it cannot be literate (*L*), which is why Ragin refers to *L* in this situation as “nonsensical.” Ragin: Actually, what I mean by nonsensical is that the causal condition is inconsistent with theoretical and/or substantive knowledge, in the causal combination in which it appears. The parsimonious solution indicates whether or not the nonsensical element can be dropped.] If the complex solution had been *duL* and the parsimonious solution had been *dL*, we could strip nothing because we are barred from stripping *L*, but *u* is still “sensical.”

It is more difficult to explain when working at the remainder level [Mendel: This seems to be an understatement, and reinforces the problem that we are having in mathematically understanding counterfactual analysis using remainders. My “guess’ is that this is done automatically by a computer program, and how it is done is not really understood. Additionally, it seems that intermediate solutions are more important than the parsimonious solution, and they are obtained by starting with the most complex solution and not with the parsimonious solution; however, the parsimonious solution contains what cannot be eliminated from the most complex solutions (it's the censor).]

Again, referring back to Espresso, choosing between tied prime implicants has an impact on the derivation of intermediate solutions. file://localhost/message/%253C93EACE5D5C8B9D4BB2DDE8F90595081A4D8B14E4A3@MAVMAIL2.uta.edu%253E.

A further note: The way I approach the intermediate solution, the selected remainders are, in effect, named after the fact and not examined one at a time. It can still be done one at a time, but it is more difficult because it is a three-way relation that is the focus: the two solutions and the researcher's inputted knowledge regarding causal conditions.

Q2: What do you mean by "sensical" and "nonsensical?"

A2: The intermediate solution is based on substantive or theoretical knowledge. An element in a combination in the complex solution is "sensical" if it is consistent with this knowledge. If it is sensical then it cannot/should not be removed. If an element in a combination in the complex solution is nonsensical (i.e., inconsistent w/ substantive or theoretical knowledge), then it can be removed ONLY IF it does not appear in the parsimonious solution.

Q3: We are now applying counterfactual analysis to our data and this has led to yet another question. After the intermediate solutions have been obtained, their consistency is also obtained. We have a situation where the consistency of an intermediate solution is less than our consistency threshold of 0.8. The consistencies of the other intermediate solutions are greater than 0.8. My inclination is to discard the intermediate solution whose consistency is less than 0.8, so that all solutions are "technically consistent." Is this correct?

A3: Your impulse is correct. Generally I tell people to pay less attention to the solutions with lower consistency scores—below whatever threshold has been set. "Discard" is OK, but perhaps a bit strong. I wouldn't be too rigid about the 0.80, especially if a solution with (say) 0.78 consistency looks really interesting.

Q4: Is it possible for an intermediate solution to not have at least one best instance? If so, what does this mean, and what do you do with such an intermediate solution?

A4: This can happen, though it is rare if the frequency threshold used to code the truth table spreadsheet is greater than 1. Generally, if there are no strong instances of a recipe, I would probably ignore it.

Q5: This is about intermediate solutions versus complex solutions and parsimonious solutions. From an information viewpoint it seems to me that a complex solution contains more information than either an intermediate or parsimonious solution. If so, then why do you favor intermediate solutions?

A5: In my experience complex solutions often include causal conditions that don't make sense. The reason is that there were no paired cases with the condition negated. For example you might expect *ABC* to be a solution. However, the complex solution is *ABc*. The problem is that there are no cases of *ABC* and the complex solution defines all remainders (including *ABC*) as false. The intermediate solution, however, defines *ABC* as an acceptable counterfactual and yields the (more reasonable) solution *AB*.

The only conditions that can be removed from the complex solution to create the intermediate solution are those that are permitted by the researcher via the intermediate solution dialogue box.

- **Intermediate Solutions With Low Consistencies**

Q: After QM, it can happen that the consistency of the parsimonious solution is below the acceptable consistency threshold, but the consistency of the complex solution is at or above the threshold. It can also happen that after CA the consistency of the intermediate solution (s) is (are) below the acceptable consistency threshold. In this case, does one revert to the complex solution?

A: Consistency scores are only a general guide. If the solution consistency is still OK, I tend not to worry too much about the consistency scores for the different recipes. But your general impulse is correct—if consistency scores drop too much, then too much simplification has occurred.

- **Parsimonious Solutions With Low Consistencies**

Q: If a parsimonious solution has a consistency of less than "around 0.8" should it be discarded prior to performing Counterfactual Analysis? My gut feeling is the answer should be "YES."

A: The automated fsQCA procedure does not make any such distinction, using the consistency scores of the parsimonious solutions.

The basic idea I've been working with is that parsimonious solutions, in general, cannot be trusted because they incorporate difficult counterfactuals and must therefore be "corrected" via the intermediate solution routine. From this viewpoint, a low consistency score for a parsimonious solution could simply reflect its over-inclusiveness, which is then corrected. And this is in fact what often happens. [For example,] *AB* in the parsimonious solution becomes *ABCD* in the intermediate. Its consistency rises and its coverage drops.

Your suggestion still makes sense, however, to the degree that the parsimonious solutions are trusted as meaningfully valid solutions.

- **Substantive Knowledge About Causal Conditions That are in the Parsimonious Solution**

Q1: In describing CA as a collection of thought experiments, I like to state this in an algorithmic way (engineers like algorithms), as follows ("*A*" is a causal condition):

> *In the thought experiments one asks: Based on my expert knowledge, (1) Do I believe that "A" strongly influences the desired output? If the answer is YES, then stop, and "A" is put on the list of substantive knowledge. On the other hand, if the answer is NO or DON'T KNOW, then one asks: (2) Is it, instead, "a" that strongly influences the desired output? If*

the answer is YES, then "a" is put on the list of substantive knowledge. If the answer is NO or DON'T KNOW, then neither "A" or "a" are put on the list of substantive knowledge, i.e. the substantive knowledge is silent about the causal condition or its complement. This process is repeated for each of the causal conditions.

I don't know if I should do this for a causal condition (or its complement) that is in the parsimonious solution. My gut feeling is that I should not [I was wrong about this], because doing so could eliminate the causal condition (or its complement) from the intermediate solution, which would violate the requirement that a parsimonious solution must be included in the intermediate solution.

But there still is a problem. Suppose, e.g., the causal conditions are *A*, *B*, *C*, *D*, *E* and *F*, the parsimonious solution is *B*+*E*, and that one of the complex solutions is *AbdEf*. I understand how to apply my 3 questions to *A*, *C*, *D* and *F*, but I am not sure if they should even be applied to *B* and *E*.

If, for example, I am "silent" about *B* and *E*, then they are not used during CA, and [the assumptions are *A*, *C*, *D*, *F*] the intermediate solution for *AbdEf* is *AbE*, which I think is the correct solution.

On the other hand, if I interpret the presence of *B* and *E* in the parsimonious solution as evidence that they must strongly influence the desired output (which could be an error in my thinking, because it does not represent expert knowledge [it was]), then I could interpret that to mean that *B* and *E* are part of the expert knowledge, and then an intermediate solution for *AbdEf* is *AE*.

So, if a causal condition (or its complement) is in the parsimonious solution, does one remain silent about it during CA? If not, then how is it handled?

A1: The basic principle to keep in mind is that the parsimonious is (must be) a superset of the intermediate and the intermediate is (must be) a superset of the complex. Thus, there may be several intermediate solutions, all valid. The issue is which intermediate solution makes the most sense, given whatever expert knowledge we bring to the analysis.

In the example you gave, the parsimonious is *B* + *E*; one of the combinations in the complex is *AbdEf*. The assumptions are *A*, *C*, *D*, *F* (the expectation is that it is their presence that should be connected to the outcome).

Whatever is in the parsimonious trumps everything else, so you can't eliminate anything that is in the parsimonious; it should be seen as the limit on what can be dropped from the complex.

In your example *AbdEf* is a subset of *E* (in the parsimonious) but not of *B* (also in the parsimonious). So the key connection is *AbdEf* <= *E*.

To maintain the subset relation, element *E* must be kept. Knowledge tells us that "*A*" makes sense, but "*d*" and "*f*" do not. So *d* and *f* can be dropped. This leaves us with *AbE*. If, as you say, we are silent about *B*/*b*'s connection to the outcome, then yes it must be kept.

Let's pretend that you had also assumed that *B* is connected to the outcome. Based on this expert knowledge, we could further reduce *AbE* to *AE*, but this would be based on expert knowledge, not on the parsimonious solution. The parsimonious solution sets a limit to the simplification of the complex to the intermediate. That is its fundamental (only) role in CA.

Q2: But, if you assumed *B* is connected to the outcome, then *AE* would no longer contain *B*, and doesn't this violate the parsimonious solution?

A2: As long as the intersection of the intermediate solution and the parsimonious solution returns the intermediate solution, it is valid: $AE(B + E) = ABE + AE = AE$

(In the example, I am assuming that there are other recipes in the intermediate solution besides AE, not shown in the example.)

An aside: If the solution is $AE + B$, then it can be re-written $AbE + B$, without any loss of coverage. Y happens if B happens; if B doesn't happen (b), then AE is required for Y.

Q3: So, if a causal condition is in the parsimonious solution, then must one remain silent about it during CA? I would like to believe/hope that the answer to this is "Yes," or else I need further understanding of the role of a parsimonious solution and CA [I did].

A3: CA must be driven by expert knowledge, not the content of the parsimonious solution. However, I do not consider it a sin to update expert knowledge based on the parsimonious solution. The real purpose of CA is to remove nuisance terms from the complex solution, creating a superset of the complex solution that is a subset of the parsimonious.

In a typical analysis, I am vocal (i.e., not silent) regarding my expectations for every single causal condition. I click the last column (present or absent) only for those conditions that I truly think could go either way. Usually, this strategy yields the simplest (and most interpretable) intermediate solution.

Q4: Suppose B is the parsimonious solution (or a parsimonious term). There are 3 possibilities for the substantive knowledge about B: (a) Present, (b) Absent (meaning b is present), and (c) Don't Know. Is it true that only (a) and (c) are valid choices for the substantive knowledge (because to assume b could remove B from an intermediate solution)?

A4: All are valid choices. Even if you assume "b," "B" cannot be removed from the complex solution to generate the intermediate. Example:

Complex: ABc + ...
Parsimonious: B + ...
Assumptions [Substantive Knowledge]: a, b, c
Intermediate: Bc + ...

Q5: Suppose b is the parsimonious solution (or a parsimonious term). There are 3 possibilities for the substantive knowledge about b: (a) Present, (b) Absent (meaning B is present), and (c) Don't Know. Is it again true that only (a) and (c) are valid choices for the substantive knowledge (because to assume B could remove b from an intermediate solution)?

A5: Answer parallels the first. The intermediate solution MUST be a subset of the parsimonious, which sets a limit on what can be removed from the complex.

Q6: Does your software protect against this, i.e. for Q1 if a user chooses "Absent" as substantive knowledge, does the software compare this against the parsimonious solution, B and reject "Absent" as substantive knowledge? Or, does it automatically block the choice of "Absent," so that a user can then only choose "Present" or "Don't Know?" If the former, does it then assume a "Don't Know" for b?

A6: It doesn't really reject the substantive knowledge per se, it simply prevents removals from the complex solution that would produce an intermediate solution violating the subset relation with the parsimonious. In effect, it lets "*B*" stand.

No blocking, per se. Each parsimonious solution is compared with each complex solution to determine which complex terms are subsets of which parsimonious terms. Then the candidates for removal are established—single conditions that appear in the complex but not in the corresponding parsimonious. Then the user's inputted knowledge is brought to bear on those candidates. In my example, the two solutions in question have a subset relation; "*B*" appears in both; it is not a candidate for removal, regardless of what substantive knowledge is brought to bear.

• **Extracting Substantive Knowledge From Data**

Q1: I now have a question about obtaining substantive knowledge from data. Many times we do not have access to an "expert," but we do have access to data. We are developing a way to extract "substantive knowledge" from the data so that this knowledge can then be used during counterfactual analysis. Have you extracted such knowledge from data? If so, how do you do it?

A1: I need to more a little bit more about your extraction method. The short answer is that I draw information from the data all the time (in interaction with knowledge of specific cases). For example, I will often examine the distribution of an interval-scale variable and the location of specific cases before calibrating it as a fuzzy set.

Q2: We are trying to extract substantive knowledge from data that can then be used as part of counterfactual analysis. Otherwise we cannot perform CA.

Here are two approaches that we are taking. Both are based on consistency.

1. Using all of the cases that we then use in our fsQCA, we compute the consistency of each causal condition and its complement separately. If such a single causal-condition consistency is greater than 0.80, we conclude that the causal condition (or its complement) influences the desired outcome during CA. On the other hand, if neither the causal-condition consistency or it's complements consistency are greater than 0.8, we assume that substantive knowledge about them is UNKNOWN during CA.

We tried this very recently on the UCI Repository MPG data set for a desired outcome of LOW MPG. We only obtained one piece of useful substantive knowledge, which has caused us to rethink this approach.

2. It seemed to us that substantive knowledge should be extracted only for the subset of cases that are closer to full membership in the desired outcome than for all of the cases that are used in our fsQCA. So in our second approach we will only use the subset of cases whose membership function values in the desired outcome is >0.50.

What do you think about these approaches? Do you have a better way of doing this?

A2: I have tried the first approach but not the second. As I imagine the plot using the second approach, you have essentially eliminated the possibility that cases might plot in the forbidden quadrant, the lower right. But this is where the action is from a sufficiency viewpoint. Cases in the bottom right corner offer the best refutations of sufficiency (shared outcomes).

The first approach could be recast as a regression through the origin problem (instead of using consistency scores). The issue would be which conditions (considering their negations as well) offer the steepest positive slopes?

This is a random, off-the-cuff suggestion!

Here's another one: Compute average membership in the (1,0) corner: sum(min($x,\sim y$))/N. You want this number to be as low as possible. Usually, I just study the scatterplots when I'm trying to use data to aid CA.

<u>fsQCA</u>

- **Best Approach**

Q: Because we have a limited amount of time to complete our projects, we would like to use what you believe to be the most powerful version of fsQCA. There seem to be three ways to analyze data in the two books, although in the 2008 [2] book you dismiss the way described in the 2000 [1] book. The case study at the end of the 2008 book provides a way to approach a problem. Is this the way you are still using or do you have an even better way?

A: The 2008 book describes my current approach. Two things: (1) the description is partial. For example, I usually test for necessary conditions before using the fuzzy truth table procedure. I also usually check for what I call "substitutable" necessary conditions. (2) There is a new exploratory procedure in fsQCA called "subset/superset" analysis that analyzes all possible subsets (of conditions) of a user's (arbitrary) causal "recipe." Still, the 2008 book is by far the best guide.

- **Best Instances (see, also Counterfactual Analysis: Intermediate Solution and No Best Instances, and Necessary Conditions)**

Q1: Once the intermediate solutions are found, we would like to associate best instances with them. I don't think the software does this, does it? How would one do this?

A1: The most recent version includes this option [as of August 24, 2010], when specifying the causal conditions and the outcome. The program searches for string variables (must have letters in data cells) and assumes that one of them is a case ID variable; then lets you specify it as such. Cases are then listed with the each solution, if they have greater than 0.5 membership in the causal recipe. This is a relatively new feature and may be buggy, so double check some cases against the data spreadsheet.

Q2: After CA, it is possible that a case may be a "Best Instance" for more than one intermediate solution. What does one make (or do) of (about) this?

A2: Yes, this happens more often than I would like. Sometimes the two solutions are very close, and you just merge them into one. For example $ABc + ABD + Cd$ becomes $AB(c{+}D) + Cd$ (i.e., two solutions instead of three). Sometimes, I factor solutions in order to separate them, and this may solve the problem. For example, $AB + CD$ might become $AB + CD(a + b)$ or $AB(c + d) + CD$ or $AB(c + d) + CD(a + b) + ABCD$. There's nothing published on this, however.

Q3: I am able to find best instances at different stages of fsQCA, as follows:

1. After the "winning" causal combination (i.e., the one causal combination whose membership function values is > 0.50) has been computed for each case, there is a direct connection of a case to such a causal combination (as in Table 5.7 of your 2010 book).
2. After the subset of causal conditions is found whose consistencies are greater than, e.g. 0.8, there is still a direct connection of a case to such a causal combination (again as in Table 5.7 of your 2010 book).
3. Now we come to the part that I am murky about: after QM for both the complex and parsimonious solutions, I have been starting with the previous set of best instances (Item

2) and checking each one to see if it contains (as a subset) a complex or parsimonious solution, and, if it does, calling its associated best instances a best instance for that complex or parsimonious solution. While that seems to be correct for the complex solution, since it can be obtained by performing set-theoretic operations on the union of the primitive Boolean expressions from Item 2, I am not sure it is correct for the parsimonious solution, because the parsimonious solution has invoked causal combinations not in, e.g., Table 5.7. How do you compute the best instances for the two ends of QM?

4. Finally, there is counterfactual analysis. I am doing the same thing to find the best instances for an intermediate solution as I have done for the complex and parsimonious solutions, as just explained in Item 3. How do you compute the best instances for the intermediate solutions?

I would like to understand "best instances" before resolving my understanding (or misunderstanding) of coverage.

A3: When I use the term best instances, I am usually referring to a solution, for example, $Y >= AB + CD$. It doesn't matter which solution is the focus (complex, intermediate, parsimonious), but it's usually the case that one of the three is the preferred solution. (For me, the intermediate is most often the preferred solution.) You can compute the degree of membership of every case in the two combinations, *AB* and *CD*, and then determine which of these two scores is greater (again for each case) (some may be tied) and then also see if the greater score (or greatest score, if there are many combinations in the solution) is greater than 0.5. Among those with scores greater than 0.5, the next question is the outcome membership score. (This is why the program prints the combination membership score and the outcome membership score for the cases in each combination in each solution.) If the combination score (which is > 0.5) is less than the outcome score (or at least not too much greater than the outcome score, e.g., $X = 0.8$ and $Y = 0.65$), then I consider the case to be a good instance of the combination. If, however, it contributes a lot to inconsistency (e.g., $X = 0.8$ and $Y = 0.3$), then it is not a "best instance" even though Y (according to the calculation of coverage) is fully covered.

Q4: In your method for determining a Best Instance (BI), if a case looks to be a BI for more than one intermediate solution term, then you focus only on the one intermediate term that has the largest membership function. When more than one intermediate term has a MF value substantially larger than 0.5, couldn't this be ignoring important information (i.e., for such a case, knowing that two different intermediate solutions are possible to explain a desired outcome could suggest that more work needs to be done for that case to understand what's really going on)? What are your thoughts about this?

A4: My usual thinking is that if a case has strong membership in more than one intermediate term, then its strong membership in the outcome is "over-determined." Let's say you can construct several different accounts of why Peru had protest against the International Monetary Fund, some focused on economic conditions and some on social movement actors and labor unions. My conclusion is not that one account is correct and the others incorrect (although this is certainly possible). Instead I prefer to think that it was more or less inevitable for Peru to have IMF protest, with many reinforcing mechanisms.

Another thing worth mentioning is that QCA will produce solutions like $AB + CD$, even when there are many-many cases of $ABCD$ and only a few of $AB(c + d)$ and $CD(a + b)$. So the real action may be in the intersection of recipes, as in $ABCD$.

Q5: Do you sometimes think that the concept of a "best instance" is too severe? A case may be "close" but does not become a BI. Of course, you could twiddle delta to make it a BI, but how do you justify doing this?

A5: It's usually a good idea, especially in social science applications, to know which cases have high scores in both a recipe and the outcome. The usual idea is that a recipe has implied mechanisms and that these implied mechanisms should be manifested (best) in the above cases. However, I could see expanding the definition of relevant "good" cases to virtually the entire quadrant of the scatterplot ($X > .5$ and $Y > .5$).

• **Binary Classifier**

Q: In (binary) pattern recognition (classification), it is common to begin with a set of training data (e.g., cases) and to use them to design a classifier that maps features into a class. Either a datum is in the class or is not in the class. Can csQCA/fsQCA be thought of as a binary classifier?

A: I don't see why not. My only concerns would be the diversity of the training data—does it reflect the diversity of possible empirical situations?

• **Boilerplate for**

Q: Do you have a "boilerplate" description of fsQCA that one can use in a new article? This would be very helpful to me.

A: I've never tried to condense it to a few paragraphs. Take a look at the description in the fuzzy chapter of Configurational Comparative Methods [3]. If there is any subset of that description that would be useful for you, I would be happy to send it in WORD

• **Consistency and Frequency Cut-offs**

Q1: We did a study for **low production rate** (lots of the wells are low producers) in which we used all six causal conditions and wanted to see how many (and which) causal combinations would survive as function of two parameters: *consistency level* and *cut-off frequency* (f). Here is a table with some results:

TABLE
NUMBER OF SURVIVING CAUSAL COMBINATIONS FROM FSQCA

	Cut-off frequency	
Consistency level	$f > 9$	$f > 12$
0.70	14	7
0.75	12	5
0.80	5	2
0.85	4	1
0.90	4	1

We thought that it was interesting that the number of surviving causal combinations was the same for the last two situations, and that maybe this suggests that their results (which where exactly the same) should be used. Any advice about this?

A1: My general recommendation is to use two different frequency cut-off and two consistency cut-offs (four analyses), to see the impact of varying these criteria. Note that there is a subset relation between the solutions using the different consistency cut-offs within each of the frequency cut-offs. This subset relation is often interesting to exploit (conceptually).

The next two entries were between J. Mendel and Peer Fiss (email, Oct. 28-30, 2010)

Q2: Ragin and you (Chapter 11 of [2]) state (for large N): "The fuzzy-set analysis that follows uses a frequency threshold of at least ten strong instances. <u>This value was selected because it captures more than 80 percent of the cases assigned to [causal] combinations [in their works]</u>." Can you please explain what the underlined items mean?

A2: Setting the threshold at 10 and then deleting all the rows in the truth table where there are fewer than 10 cases left us with 80% of the cases. In other words, there is a tradeoff between requiring more cases per causal condition and covering the whole population. Does that make sense?

Q3: You must have had a lot of cases (as I recall, you had 700) to have set the threshold at 10. So far, in our work we only have 60 cases.

A3: Yes, that kind of approach would probably not be feasible for you. However, with perhaps only one case in some of the rows it becomes even more important to go back to the understanding of the causal condition and knowing what each means and how they interact (i.e. probably go back to the sponsor/engineers).

- **Consistency (see, Counterfactual Analysis: Intermediate Solutions; fsQCA: Software—Coverage and Consistency Computations)**

- **Coverage[6] (see, also fsQCA: Software—Coverage and Consistency Computations)**

Q1: In your 2008 book [2] you have a lot of discussions about "coverage," but in Chapter 5 of your 2009 textbook [3] there is no discussion about it. By focusing first on "necessity" (as you have advised in an earlier e-mail) is it true that "coverage" is handled?

A1: The 2009 textbook has a limited presentation of fsQCA. Coverage always refers to the degree to which a subset covers (fills up) the superset. When looking at sufficiency, this is straightforward, and allows the identification of the most empirically important causal "recipes." When looking at necessity, the outcome is a subset of the causal conditions, so the coverage calculation is really about the degree of constraint on the outcome that is imposed by the necessary condition. Thus, in this context is it more reasonable to think of the coverage

[6] Coverage [2, p. 45] indicates the empirical relevance or importance of a set-theoretic connection.

calculation as "relevance."

In general, the inclusion of a necessary condition in the calculation of a sufficient combination's coverage should have little impact (unless consistency is relatively low) because:

membership in sufficient combination <= outcome <= necessary condition

so the necessary condition will rarely provide the minimum membership score, if included in the recipe. [I think this is because a necessary condition, e.g., *a*, will always appear combined with a sufficient condition, e.g. *b*, so that $MF(ab) \leq MF(a)$.]

Q2: In your 2008 book [2], "Coverage" is computed by using a set-theoretic formula, but in Chapter 5 of the 2009 textbook [3] there is not the same formulaic level of emphasis on it. Is that because coverage is now related to (i.e., is the same as) the number of cases that survive (pass) the frequency threshold test? If not, then where and how is "coverage" used?

A2: I didn't write about coverage in CCM [3] because it was not addressed in the other chapters (on csQCA and mvQCA). (Consistency was addressed in the other chapters via the concept of "contradictions.") So this was an editorial decision. Already, the fuzzy chapter in CCM [3] was long and well advanced conceptually, compared to the rest of the book. You are correct that for the other chapters, coverage is based simply on the count of cases. I tried to be consistent with the other chapters for the sake of continuity. In practical terms, you could think of there being two possible measures of coverage in fuzzy set analyses, one based on membership scores and the other based on counts of cases. ***I would still prefer the first*** [emphasis is by Mendel].

Q3: In fsQCA I see that authors report "coverage," with values ranging from small to large; but, I can find no guidelines on exactly what to do with these numbers in relation to keeping or discarding an intermediate solution. In addition, you have explained that there are three kinds of coverage, but I don't find authors using the three. Can you shed some light on which coverage to compute and why?

A3: In general, I use coverage only descriptively, although sometimes I use it to exclude an intermediate solution, as you suggest.

Unique coverage is almost always very low in solutions with several recipes, so I generally don't pay much attention to it. Raw coverage is a better guide in most contexts.

What would be really useful (and sometimes I compute these by hand) is a table showing pair-wise overlapping coverage, using all the different recipes in a given intermediate solution. When two recipes overlap a lot, it is often possible (and reasonable) to combine them into a single recipe, as in $ABC + ABD = AB(C + D)$. There are no guidelines regarding what is "good coverage" because it depends on the nature of the evidence. This parallels the use of measures of "explained variation" in statistical analyses. Sometimes 10% is considered adequate.

Q4: Why use a fuzzy measure of coverage rather than a crisp measure of it, since a case is a crisp concept?

A4: I developed a fuzzy calculation for coverage because it is possible to have some "best instances" (in terms of scores on causal conditions) that have low scores on the outcome. Their contribution to coverage, fuzzy version, is trivial. It's not clear how you plan to handle these in a crisp calculation. My guess is that you would ignore them, but there are other possibilities.

I think you should use whatever measure you find most useful. The crisp version can also be partitioned (raw, unique, overlapping). But what would you do with a case that has 0.55 membership in recipe #1 and 0.95 membership in recipe #2?

• **Describing the Results Linguistically**

Q. In a paper or conversation about QCA, how do you linguistically refer to the final results from fsQCA, e.g., "summarization of …," or what?

A. The term "summarization" is not used by social scientists. Usually, the results are described simply as "the (main) combinations of causal conditions linked to the outcome in question, based on a logical minimization (or simplification) of the case configurations."

[This seems too long. Maybe, one could refer to the results as "A logical summarization."]

• **Displaying the Results**

Q: While it is easy to display the results from one step of fsQCA to another by using tables, when the number of causal conditions is small (as in your books), using tables seems impossible when this is not true, e.g. in our case of 6 causal conditions we have $2^6 = 64$ causal combinations. On the one hand, we can just present the final results. On the other hand, engineers like data and want to see calculation results ("seeing is believing"). Have you developed a way to show calculations for such a complex situation? Any suggestions?

A: Sorry, no. The general practice in social science (unfortunately) is to cherry pick the most interesting results and present only those results. One option is to do a crisp truth table analysis (in effect a meta-analysis) of the results of the different analyses—e.g., under what conditions do we find this or that connection . . . .

• **"Explanation" versus "Describe"**

Q: On page 155, paragraph 2 of the 2009 textbook [3], there is a short discussion about "explanation" versus "describe." To me, these words seem similar. Can you please elaborate or clarify the distinctions?

A: In social science most causal mechanisms are unobserved, due to the abstractness of the concepts (e.g., "stable" [country]). An explanation invokes the researcher's theory about the causal mechanisms; a description does not invoke mechanisms but sticks to statements about which conditions are linked (i.e., less speculation about why and how).

• **fsQCA versus csQCA versus mvQCA**

Q: I am curious to learn how your colleagues feel about dismissing cs and mv analyses in QCA (I agree with you about this). I ask because we in the engineering field have some (many) people who absolutely hate fuzzy sets, whereas others really like them.

A: Regarding the book *Configurational Comparative Methods* [3]: In general, my position is that if you have a solid interval-level indicator of set membership, it is a mistake to dichotomize, or even to convert it to a multichotomy. So it is natural for me to dismiss the crisp-set and multi-value analyses in CCM.

• **Intermediate Solutions (see Counterfactual Analysis: Intermediate Solutions)**

• **Misspecification**
Q: How does this show up in the truth table?

A: Misspecification (in social research) usually shows up in the truth table as weak consistency scores. Another possible signal is low coverage of solutions. There is no hard and fast rule or procedure. A completely misspecified set of causal conditions could still yield good consistency scores. However, more than likely, the recipes would not make good sense. These techniques were developed to help researchers interact with their empirical evidence, using their best ideas about how things happen.

• **Necessary Conditions (see also fsQCA: Necessary and Sufficient Conditions)**
Note: There are many questions and answers under this topic and over the course of time Prof. Ragin's viewpoints about Necessary Conditions has changed. So beware that the early questions and answers may have been replaced by later questions and answers.

Q1: Can fsQCA automatically establish the necessary conditions?

A1: Not automatically. There is a procedure for evaluating them, one at a time (or joined via logical or), but they are not automatically filtered out of the truth table analysis. If you find one that you think is consistent enough to be considered a superset of the outcome, then you may simply set it aside as something to be reported in the results, but excluded from the analysis of sufficient combinations of conditions (via the truth table analysis).

Q2: I have been going through the numerics of testing for *necessity*, using the data in Table 5.2 of your 2009 book [3]. In my example (as in your example in Table 5.7) I am only working with 3 causal conditions, D, U and L. I created a table for the 18 countries (cases) that lists the membership function (MF) values for D, U and L and then for d, u and l. I then counted the number of cases for which each of these MFs is > 0.5, and I got: 10, 5 and 13 for D, U and L, and 8, 13, 5 for d, u and l. In your book you have stated that you cannot have both a causal condition and its complement be a necessary condition, so I didn't know how to choose a frequency threshold for these counts. How do you do this?

A2: I don't use frequency thresholds for necessary conditions. Generally, there are enough cases to go ahead because social scientists generally conceptualize their causal conditions so that there are good instances (>.5) of each one. Frequency becomes more of an issue when you are taking the min of a number of conditions. [I don't see the connection of the last two sentences to the answer to my question.]

Q2 (Continued): Instead, I compared the number of cases for D versus d, (10 versus 8), U

versus *u* (5 versus 13), and *L* versus *l* (13 versus 5), and chose the "winner" as the causal condition or its complement that had the largest count. The "winners" were *D*, *u* and *L*. Is this what you would have done? If not, what would you have done?

A2 (Continued): What you have done is totally reasonable. You can accomplish much the same simply by calculating the average membership score for *X*; the average membership score for ~*X* will be 1-*X*'s average. In general, for a condition to be necessary, it needs to have a decent average membership score. In practical terms, however, I simply compute all the necessary condition scores for all my causal conditions, both in their original form and negated. [The latter is what I have also done.]

Q2 (Continued): Next, I computed the subsethood of *D*, *L* and *u* in *Breakdown*, and obtained:

Subsethood of *D* in *Breakdown* = 0.322
Subsethood of *L* in *Breakdown* = 0.573
Subsethood of *u* in *Breakdown* = 0.857

What threshold would you use in order to decide which of these are necessary conditions? It looks to me like *u* is a necessary condition for *Breakdown*, and *D* and *L* are not. For sufficiency, I used a threshold of 0.8. Is this also okay for necessity? If not why?

A2 (Continued): My general argument is that once you have identified a condition as necessary (or really "necessary enough"), you remove it from the sufficiency analysis. So my usual practice is to set a high consistency threshold for necessity (e.g., .9). The evidence should be strong and it should "make sense" as a necessary condition.

Q2 (Continued): Finally, let's assume that *u* is a necessary condition. My prior fsQCA for sufficiency led to only one term, namely *du*. So now it seems that *u* is both necessary and sufficient for *Breakdown*. [Mendel: The rest of this is based on my threshold of 0.8; if I used Prof. Ragin's threshold of 0.9, then *u* would not be a necessary condition, and the rest of this question and answer would be irrelevant. It is relevant for the threshold of 0.8.] How do you linguistically describe this? I ask because in fuzzy logic we would express a sufficient condition as an IF-THEN rule, e.g., IF *d* and *u*, THEN *Breakdown*. In the present case, where *u* is both necessary and sufficient, I would have a rule expressed as: IFF *u* and IF *d* THEN *Breakdown*. Make sense?

A2 (Continued): Following the rule I just mentioned [for the threshold of 0.8], "*u*" would not be included in the sufficiency analysis. The result would be:

$u >= Y >=$ results of the sufficiency analysis (without "*u*")

That is, *u* is necessary for *Y* (a superset of *Y*) and the combinations from the truth table analysis are sufficient for *Y* (i.e., subsets of *Y*).

[This is very important because it explains how to linguistically summarize the results.]

Comment by Prof. Ragin: I've gone back on forth on these questions for several years, and as far as I can tell there is no straightforward resolution. One big issue centers on the fact that we have allowed inconsistency, so a given causal condition can be consistent with necessity say at the .95 level, but still provide a useful min ($z <= y$) (for a few rows) in a truth table (TT) analysis, because of the 0.05 inconsistency.

It is also the case, as you point out, that the negation of necessary condition (~*z*) could be included in a causal recipe, resulting from the inclusion of *z* in the TT analysis. But is this "error" or is it simply a recipe for the outcome when a quasi-necessary condition has not been met (again, given that consistency with necessity is less than 1.0). One reason that I started excluding necessary conditions from the TT analysis is this very problem. On the one hand I was arguing that *z* was a (virtual) necessary condition, but on the other hand, I was getting recipes from the TT analysis that included ~*z* as a condition. If you think about it, ~*z* is a very good candidate for providing the min when you consider that for most cases $z >= y$.

Q3: In the use of rules in fuzzy systems, we always formulate them as "IF $x_1 = F_1$ and ... and $x_p = F_p$, THEN $y = G$." Such rules focus only on sufficiency. They can be "activated" by measured values of x_1, ..., and x_p using Zadeh's sup-min composition (which can be thought of as producing the outcome of a rule when it is excited by the measured input values). These rules are in the spirit of causality as used by engineers, meaning that a rule provides no output until it is activated by measured values of its antecedents.

We don't focus on necessity, which would be associated with a rule stated as, e.g., ONLY IF $x_1 = F_1$, THEN $y = G$. Such a rule requires knowledge of the consequent prior to knowledge about the antecedent, which in engineering terms is associated with an "inverse problem." Some people may be interested in inverse problems, but I can't recall seeing papers about this, which raises a red flag to me.

So here is a radical thought, motivated by my experiences with things that either don't make sense or don't seem to have a resolution: Ignore necessity!

This may not be as radical as it seems. fsQCA for sufficiency treats all of the *k* causal conditions on a level playing field. If a causal condition is indeed necessary then wouldn't it appear in all of the surviving causal combinations? If so, then wouldn't this be a tip- off that the causal condition is also necessary?

What do you think?

A3: What you have described is in fact what I routinely do. Ignore necessity. If an element appears in all the recipes for an outcome, however, that is no guarantee that it is a necessary condition (unless coverage of the outcome approaches 1.0). Its necessity still has to be established separately.

That being said (ignore necessity), there is still a lot of interest in necessary conditions in the social sciences. Is "prior state breakdown" (e.g., fiscal crises) a necessary condition for "social revolution"? If it is, then the outcome will be a subset of the cause when we look across historical cases. It takes the form: can you identify a case of social revolution not preceded by state breakdown?

Also, sometimes a student will be baffled because the TT [truth table] analysis results are weak. Often, if we go back and look at the data in terms of necessary conditions, we find a lot to talk about. Necessary conditions, however, are almost always one at time. If you start compounding them via the min, you almost always degrade your consistency score.

One other thing: there is a link between product of sums expressions and the notion of substitutable necessary conditions $(z_1 + z_2) * (x_1 + x_2)$, but I haven't finished thinking it through. In any event, the product of sums way of expressing results never seems tidy enough to permit the substitutability idea.

Q4: For one of our studies we observed that 4 out of 6 causal conditions (or their complements) were chosen by fsQCA as necessary conditions, which seems high. We used a consistency threshold of 0.90 and two of the four causal conditions (or their complements) had consistencies above 0.98, whereas a third was around 0.94 and the fourth was around 0.91. Would you accept all four or would you raise the threshold higher?

A4: For me, the real question is: Do they make sense as necessary conditions? The 0.90 threshold simply serves to generate a list of potential necessary conditions. Once some kind of filter has been applied, the "does it make sense" criterion applies.

Of course, a filter closer to 1.0 is always better from a set theoretic point of view. The 0.90 is arbitrary.

Sometimes you see a lot of "potentially necessary" conditions because the mean membership in the outcome is relatively low, which is to say that the finding of multiple necessary conditions could be partially an artifact of coding/calibration. So you may want to review your calibration of the outcome and scoot the membership scores higher—if it makes good substantive sense to do so.

Q5: We performed fsQCA for an outcome (**high production rate**) and its antonym (**low production rate**) and exactly the same causal condition passed the 0.90 consistency threshold as a necessary condition. For **low production rate** it had a consistency level of 0.914, and for **high production rate** it had a consistency level of 0.924. Our substantive knowledge about the relationship between this causal variable and production rate made us suspicious about its validity as a necessary condition for **high production rate**. It does not seem "fair" though to throw out one of the results and not the other when their consistency levels are so close. Have you seen such a situation in your works? Based on you experiences, what do you suggest?

A5: This appears to be an issue in the calibration of the causal condition [the membership function for the causal condition]. Specifically, most if not all cases probably have very high membership scores in the causal condition. In general, a causal condition that has uniformly high membership scores can be viewed as trivial (e.g., was air present?). Usually, this trivialness will be reflected in the coverage scores (high consistency with necessity, but low coverage). My strategy would be either to classify it as a trivially necessary causal condition or recalibrate it. I would not arbitrarily throw out one of the two findings [underlining by Mendel].

Q6: Because our causal conditions **each** have two linguistic terms associated with them ("high this" and "low this," e.g., **high sand volume** and **low sand volume**), if one of the terms passes the necessity test and we choose to remove that causal condition from the sufficiency test, does this mean we should remove it completely (i.e., remove both terms, e.g., **high sand volume** and **low sand volume**)? This may not be as trivial as it sounds, because we have to consider **low this**, **not low this**, **high this** and **not high this** as candidate necessary conditions. So if **low this** is a necessary condition, clearly **not low this** cannot be a necessary condition, but it is possible that **high this** could be a necessary condition because **high this** contains **not low this**. Have you experienced this situation before? What is your take on this?

A6: In general, I treat causal conditions that have been calibrated in different ways using the same source variable as different conditions, though still recognizing that there is a subset relation among them. It is quite reasonable for example, that "not-low" income might be a necessary but not sufficient condition for getting a loan, while "high income" might be sufficient, by itself. In other words, I wouldn't throw out the alternate calibrations because one way of calibrating is necessary but not sufficient. Still, it might not be all that interesting to know that not-low income is a necessary condition for getting a loan. It all depends on the knowledge and interests you bring to the analysis.

Q7: Earlier you had informed us that you remove necessary conditions from sufficiency-fsQCA, thereby reducing the number of causal conditions that have to be considered during the latter. What do you then do with necessary conditions at the end of all of this, e.g. do you summarize your results in a statement such as "For this desired outcome, it is necessary that ... and it is also sufficient that ..."?

A7: I no longer make this recommendation. It is valid to remove a necessary condition from a truth table analysis only if the other causal conditions are also subsets of the necessary condition. If this condition is met, then the outcome is seen as a subset of the necessary condition (which can be considered a shared antecedent condition) and the sufficient combinations are subsets of the outcome (they are cases that share the outcome).

Q8: Do you find best instances for a necessary condition, and if so how?

A8: I've never applied the concept of best instances to a necessary conditions argument. In a way, cases that have been constrained from expressing the outcome by virtue of low membership in the necessary conditions are the "best" cases in support of the argument of necessity.

Q8 (Continued): Can you please clarify the second statement [in A8]?

A8 (Continued): Necessity means no outcome without the condition. In short, the condition enables but does not determine the outcome, i.e., creating the context for its possible occurrence. The cases most supportive of necessity are the ones constrained to not exhibit the outcome by virtue of their low membership in the necessary condition. I know this sounds a bit odd, but it's what popped into my head when you asked about best cases for necessity.

Q9: Suppose that I did not remove a necessary causal condition from sufficiency-fsQCA. If such a causal condition appears in every one of the intermediate solutions for fsQCA, can it be concluded that it is a necessary condition? Or, if such a causal condition appears in every one of the complex, parsimonious and intermediate solutions, can it be concluded that it is a necessary condition? Put another way, is it possible to deduce whether a causal condition is a necessary condition from sufficiency-fsQCA?

A9: No. This type of inference can be made ONLY IF coverage is very-very high, e.g., > .95. My usual argument is that any necessary condition argument requires an explicit test (e.g., using the necessary conditions procedure or the *XY*-plot).

Q9 (Continued): What do you mean by an "*XY* plot?" Is this explained in one of your books?

A9 (Continued): *XY* plot is simply the scatterplot of two fuzzy sets in fsQCA. The bottom right number shows consistency with necessity (the degree to which *X* as a superset of *Y*).

Q10: Suppose that we examine the necessity of *high MPG* for each of *low acceleration*, *moderate acceleration* and *high acceleration*, and find that *low acceleration* is indeed a necessary condition. Would you remove only *low acceleration* from sufficiency fsQCA or would you remove all of the acceleration causal conditions, e.g., it may be possible for *moderate acceleration* or *not high acceleration* to combine with other causal conditions to lead to *high MPG*? [Note: Based on Q7/A7, this question is superfluous, because Ragin no longer recommends removing necessary conditions from fsQCA.]

A10: This is an interesting question. There may be some conditions that are sufficient for *high MPG* only when combined with *low acceleration*. If you dropped *low acceleration* from the analysis, you would miss these (sufficient) recipes for *high MPG*. And yes, there may be causal recipes involving *moderate* and *not high acceleration* even when *low acceleration* is a "necessary condition." It is important not to get tripped up by the language of necessity. Just because instances of *high MPG* form a subset of *low acceleration* does not imply that there cannot be subsets of *high MPG* involving *not high acceleration*. (In any event, *not high acceleration* is a superset of *low acceleration*.)

• **Necessary and Sufficient Conditions**

Q1: In an earlier e-mail, you stated: "My general argument is that once you have identified a condition as necessary (or really 'necessary enough'), you remove it from the sufficiency analysis." If you do this, then it is not possible to discover if a condition is both necessary and sufficient, so why would you remove it from the sufficiency analysis?

A1: *I have changed my position on this issue*[italics added by Mendel.] The reason I was removing them was because (1) I wanted to reduce the dimensionality of the truth table and (2) most of the supersets I had identified tended to dwarf the outcome and therefore were unlikely to also be sufficient. However, I have started putting them back into the truth table analysis for the reason you mentioned, but also for the reason that some of the other conditions may be sufficient only when intersected with the necessary condition. This seems obvious to me now, so I regret having given the advice to drop the necessary conditions from the truth table analysis.

Q2: I am having second thoughts about my question. Once a condition has been found to be "necessary" I think it can only also be found to be sufficient if it is not combined with other causal conditions, i.e. it has to be tested by itself for sufficiency. Is this correct?

A2: I don't think so. Of course, it can be tested by itself, but if a condition is both necessary and sufficient, that implies that the plot of the membership scores shows them all near the main diagonal. In my experience this is very rare.

In any event, it seems to me that one argument is that a sufficiency analysis that excludes a necessary condition may not give the right recipes. Imagine outcome *y* is a subset of necessary condition *z*; represent this intersection as *y* filling half of circle (vertical line); set *x* intersects the half of *z* that is shaded *y*, but there are also cases of *~zx*. *zx* is sufficient for *y*, but you wouldn't have been able to see the subset relation without including *z* in the analysis.

Q3: Also, if a subset of conditions is then found to be both necessary and sufficient, it is no longer necessary to perform sufficiency tests on causal combinations. Is this correct?

A3: Again, to find a single condition that is both necessary and sufficient in my experience is quite rare. Still, I think your impulse is correct. The one thing I worry about is that fsQCA allows the TT [Truth Table] reduction (e.g.) to "*a* <= *y*" using conditions *a*, *b*, and *c* only if all the included combinations have been satisfied either empirically or via counterfactual analysis:

ABC	*Y*
100	1
101	1
110	1
111	1

Q4: My understanding of your reply is that once you have established that a condition (e.g., *C* or *c*) is necessary, its complement (*c* or *C*, respectively) is never used in the tests for sufficiency. So this is a way to cut down on the total number of causal combinations that have to be tested during sufficiency. Is this correct?

A4: Yes, this was my original position, augmented by the observation that fsQCA (when a necessary condition is included in the TT analysis) will often provide recipes with its negation as one of the ingredients. Although, I should mention that this problem is less common in intermediate solutions.

In fsQCA, one option is simply to delete the truth table rows (in the truth table spreadsheet) with the necessary condition negated and the outcome coded 1. There are other tricks. For example, the user-inputted outcome of don't care ("-") is also a valid entry in the truth table spreadsheet. This feature is not documented.

Q5. Is it true that if a causal condition appears in all surviving causal combinations from fsQCA it can be considered as a necessary condition? An example in your 1987 book indicates that the answer is "Yes."

A5. The correct answer is "sometimes." Coverage would have to approach 1.0 for the condition to be necessary. As long as there are unknown paths, the claim that a condition is necessary can be established only be showing that it is a superset of the outcome. It still has to pass the "makes sense" test, however. If a condition appears in all the known recipes for an outcome and coverage is substantially less than 1.0, then it is still an "important" condition, because of its commonality.

• Necessary but No Sufficient Conditions
Q: Is it possible to find some necessary conditions but no sufficient conditions? What does this mean?

A: This happens. At a practical level, it suggests that most cases have low membership in the outcome, suggesting a recalibration/re-labeling of the outcome may be in order. The issue with multiple necessary conditions is that they are "quasi necessary" (not 1.0 consistency), which means that when you make them joint (taking the min of several), the consistency scores generally drop (for the combined quasi-necessary conditions). So the individual conditions are "usually necessary" but their combination may not be.

• No Best Instances for a Believable Simplified Intermediate Solution
Q1: During our processing of the mpg data, fsQCA obtained an intermediate solution of high consistency but it had no best instances. Is this possible? If it is, what can you say about such a rule?

A1: Generally, it is important to set a frequency threshold when coding the truth table. This becomes more important when the total *N* is greater. Still, it is possible that the cases with high membership in a row have low membership in the outcome and you can still get a just-acceptable consistency. When this happens I usually raise the consistency threshold.

Q2: Do you think that this could be a result of the way in which you choose a best instance (BI)? Your "delta" factor seems quite important.

A2: Yes it is. Sometimes good BIs just don't exist. Sometimes this is signaled by the coverage calculations. Some users set raw coverage thresholds, e.g., 20 or 25 percent. Solutions that don't meet the threshold are ignored.

• No Sufficient Conditions
Q1: It is possible, isn't it that no causal combinations survive fsQCA? This would mean that none of the causal combinations are sufficient for an outcome, and we need to look at other causal indicators.

A1: Absolutely.

Q2: When we ignored necessary conditions, and focused only on sufficiency, we had a situation where fsQCA led to no sufficient conditions. None of the consistency scores exceeded 0.6 (we are using consistency thresholds of 0.75 or 0.80). We did this for six causal conditions each described by the two terms **low this** and **high this**. Our thinking about this is:

a. Maybe we do not have enough causal conditions. The problem is that these are the only ones for which we have data and we have no idea what other causal conditions might be. So, increasing the number of causal conditions is not a viable option for us.

A-a: Sometimes it is surprising what can be added as causal conditions. For example, it is possible to mix crisp and fuzzy causal conditions. There may be some crisp differences between your cases (or their contexts) that may be relevant.

b. Maybe we are using too many causal conditions and should remove some and repeat fsQCA. Is this a viable strategy? Is it mathematically possible that, e.g., one "bad/incorrect" causal condition can mess up fsQCA?

A-b: Yes, this is possible. There is also a subtle interplay with limited diversity and what you do with the "remainder" combinations. There is a new procedure in fsQCA, called subset/superset analysis, which allows users to specify an arbitrary recipe and then the procedure assesses not only the consistency and coverage of this recipe (with respect to the outcome), but also assesses all possible subsets of conditions. If you have "too many" causal conditions, then this should reveal the problem. IN GENERAL, however, more conditions produces higher consistency scores (and lower coverage).

c. Maybe the data really does not reveal anything. Obtaining no sufficient conditions from fsQCA happened for the outcome of **high production rate** and we know that our data comes from wells that are in a very difficult oil field, one that has been active for more than 100 years, has more than 2 billion barrels of reserve, but only 5-7% of the petroleum has been able to be removed from this field during the 100 + years. Going to our sponsor and telling them that we have learned nothing from the data is not exactly what they want to hear. Have you ever had a problem where you could not learn anything from the data using fsQCA?

A-c: Usually, my strategy is to look at the cases with high membership in the outcome and try to figure out what I've missed. The other strategy is to admit that that the high production wells are few and far between and focus instead on "at least moderate" productivity (i.e., recalibrate and re-label the outcome).

d. We are thinking about backing down from **high production rate** to something more inclusive, e.g., **moderate production rate**, or **moderate through high production rate**, and then redoing fsQCA for it. Have you ever had such a situation and had to do something similar? Any advice?

A-d: OK. That's what I just recommended: recalibration/re-labeling.

• QCA or fsQCA: Engineering Applications of

Q: To your knowledge, has QCA or fsQCA been applied to engineering problems?

A: Not to my knowledge. There may be a few applications in industrial engineering, but none comes to mind at the moment.

• Strategy

Q: We assign terms to our conditions (which can then be treated as new conditions), e.g., *low pressure*, *moderate pressure*, *high pressure*. Each of these terms then does have a natural "yes" or "no" connotation (see *Adjective Terms*). My thoughts are to focus attention on one possible outcome (e.g., *high production rate*) and then on one term at a time for each condition (e.g., *high pressure*, *low volume*), and perform fsQCA. (To include all of them at one time would cause huge limited diversity problems.) This would lead to a "partial summarization." I could then repeat this for another set of conditions (e.g., *high pressure*, *moderate volume*), obtaining another partial summarization. I could repeat this until I ran out of conditions that I wanted to explore. As a result of doing this, I would have a collection of partial summaries that could then be combined using the union. Does this make sense?

A: It makes perfect sense. Sometimes I like to put the different calibrations in the same analysis, for example "high" pressure and "low" pressure. One of the four corners of the space formed by these two is not logically possible (both high and low). So the increment is really from 2 to 3, not 2 to 4. And then one of the corners (not-high, not-low) is the same as "moderate." At least, that is how I've been doing the calibrations.

[*Comment:* At a later date we switched over to treating all terms as independent causal conditions.]

• Subsethood: Probabilistic Criteria

Q: This is about footnote 7 in Chapter 5 of your 2009 textbook [3]: Do you advocate the incorporation of probabilistic criteria into the assessment of the consistency of subset relations; or, do you advocate choosing a threshold ≥ 0.75?

A: Again, I almost always try to stay close to the evidence and rely less and less on probabilistic criteria. Generally, people who use these probabilistic techniques are trying to identify a formal rule (e.g., benchmark = .65, alpha = .10) that certifies what they have already decided, based on looking at the spreadsheet.

• Subsethood: Threshold

Q: On page 107 of the 2009 textbook [3], you state: "For example, a researcher's rule might be that there must be at least 5 or at least 10 cases with greater than 0.5 membership in a causal combination in order to proceed with the assessment of the fuzzy subset relation." Now that more people are gaining experience using fsQCA are there newer and more definitive guidelines for the number of cases? I ask because in our work, we are in the large *N* situation.

A: I have worked on this, but don't have a definitive rule. My general practice is to assign cases to configurations, sort the configurations according to frequency (as in the truth table spreadsheet) and then use two cutoffs, one at about 75-80% of the cases and another at about 85-90%. I then do two analyses, recognizing that there will be a subset/superset relationship between the two solutions because the higher frequency cut-off value dumps more configurations into the counterfactual/ remainder pool (i.e., is somewhat less constrained by the evidence).

I have also developed statistical rules and set-theoretic rules, but generally find that sticking close to the data works best (and also using two cut-offs, as a check on robustness). In general, I don't find striking differences between the two solutions.

• Subset/Superset Analysis (procedure)

Q: Is subset/superset analysis in the software version that is presently on-line? If not, when do you think it will be available on-line?

A: The subset/superset procedure is included in the current software download. It is accessed from the "Analyze" menu.

• Subset/Superset Analysis Publications

Q: I am very interested to learn more about "subset/superset" analysis that analyzes all possible subsets (of conditions) of a user's (arbitrary) causal "recipe." Do you have a publication that describes this? Is it already in the fsQCA code?

A: It is already a procedure in fsQCA. I just added it. There is nothing published on it. I hint at it on pp. 114-121 of my book Redesigning Social Inquiry [2].

• Substitutable Necessary Conditions

Q: What exactly are "substitutable" necessary conditions?

A: If two (or more) conditions make sense as substitutable ways to satisfy a more general requirement (a necessary condition), then they may be joined via logical or and used as a single (encompassing) condition. Example: either possession of assets or a steady income might be considered (by a loan officer) as substitutable conditions for getting a loan. The necessary conditions routine in fsQCA allows the exploration of substitutable necessary conditions.

I am currently working on some tricks to speed the identification of substitutable necessary conditions. Still, the key is that it all makes conceptual sense, as in my credit-worthiness example.

• Validation of Results from fsQCA

Q1: Our sponsor keeps asking us "how can we 'validate' the fsQCA results?" How do you validate your results?

A1: In macro-sociology, you take the results back to the cases (e.g., "France") and see if they help illuminate the cases. If the cases are anonymous, of course, this is impossible.

Another tactic is to compute membership in the causal recipes and use these as measures in a multiple regression, predicting membership in the outcome. In general, I don't recommend this. There are some nuances in how to do this properly (e.g., overlapping recipes eating the same variation in the outcome).

If you have a lot of cases, you can randomly assign cases to two groups, use one for the QCA and then test the results on the other half (i.e., computing consistency and coverage scores for the derived recipes).

Q2: What does "computing consistency and coverage scores for the derived recipes" mean, i.e. how do these computations "validate" fsQCA?

A2: Your impulse is correct. What I described does not validate the QCA. It simply provides evidence of its robustness. In social science applications validation comes as described in the email I just sent—verifying that the mechanisms implied in a causal recipe are evident in the relevant cases ($X > .5$ and $Y > .5$). The closer min (X,Y) is to 1.0, the better the case and the more apparent the implied mechanisms should be. Case study and analysis is central to QCA in social science applications.

<u>Membership functions</u>

- **Logarithmic Transformations**

Q: I am not familiar with the logarithmic transformations that are mentioned in Footnote 5 in your 2008 book [2]. Can you give me examples of such transformations?

A: It happens in regression analysis. An example using country level data-dependent variable: proportion literate; independent variable: GNP/capita. Much of the variation in literacy is among the poor countries, yet they vary relatively little in GNP/capita (e.g., from $200 to $1000). The rich countries vary little in literacy (e.g., 95-99), but they vary tremendously (by comparison) in GNP/capita (e.g., $20,000 versus $35,000). To achieve a better fit, researchers will log GNP/capita to expand the variation among the poor countries and contract the variation among the rich countries.

- **Log-Odds Transformation Method for Constructing a Membership Function**

Q: In your 2009 book [3] you explain a method for constructing a membership function using a log-odds transformation. I am curious to know why you use this method. If one specifies the three anchor points, it is easy to connect them by using piece-wise linear functions. Have you compared the two approaches to conclude that the log-odds method gives superior results (I am not sure how one can quantify "superior results"), and, if so, why is that so?

A: The log odds function is useful because it stays within the 0-1 interval and also conveniently compacts variation at the two ends of the distribution that the user defines as (near) irrelevant, giving these cases scores from 0 to .05 on one end and .95 to 1.0 on the other.

- **Membership Functions: Calibration of**

Q1: In our problems we usually have terms between **low** and **high**, such as **moderate** income. Can fsQCA be used for such a term, and if so how is the membership function obtained (calibrated) for it?

A1: Example: When I want to calibrate degree of membership in "middle" income (with high income and low income both low membership), I first calibrate degree of membership in high income (focusing on the right tail), then calibrate degree of membership in low income (focusing on the left tail), and then compute membership in middle = min [(1-membership in high), (1-membership in low)] that is, middle is the intersection of not-high with not-low. Of course, if high income and/or low income are calibrated too inclusively, then the max membership in middle income is truncated accordingly, so I pay close attention to the calibration anchors for low and high. Hope this is clear. In effect, I calibrate all three at once.

[(1) I think this will only work if the MFs for low and high income are widely separated, so that the resulting MF for middle income will be a normal fuzzy set, i.e. have a maximum value of one. A non-normal fuzzy set would not be used by us.
(2) One does not have to use Prof. Ragin's procedure for establishing the MF of a fuzzy set. He/she can choose the MF to be anything he/she wants, and fsQCA can still be used.]

Q2: Maybe you have run into the following situation. Surprisingly (to us) there are not many practitioners in our sponsor's company who are very knowledgeable about our data set and who can provide us with reliable MF data. So, our sponsor came up with an interesting suggestion, namely: "guide" the practitioner by providing the key MF "breakpoints" as obtained first from the data approach. The practitioners can then move the breakpoints to the left or right of these "starting" values. What do you think of this?

A2: I think this kind of back and forth is very useful. The iterative process also helps to clarify the vocabulary, so that researchers and practitioners are on the same page.

• **Membership Functions: Dependence on Size of Vocabulary**

Q: In the works of yours [Prof. Ragin] that I have read, you always have a membership function for a causal condition or output that is S-shaped. In a prior e-mail, I mentioned to you that we frequently choose some terms with adjectives for a causal condition (e.g., low pressure, moderate pressure, high pressure—a "vocabulary" for pressure) and will then do an fsQCA for each of these. The following is a question about membership function (MF) data collection for causal conditions or outcomes that can be described by a vocabulary of terms.

We have found that the MF data depends on how many terms are used for a causal condition or outcome, i.e. are in the vocabulary. This is something one would not see if there was only one term for the causal condition or outcome.

Let me be specific. If a person knows that pressure will be described by just two terms (low pressure and high pressure), then the data the person provides (about the membership function) for low pressure and high pressure are different from the data they provide if they know that pressure is described by three terms. Have you experienced this before?

It has consequences for us. If, for example, we do an fsQCA for three-term vocabularies (e.g., low pressure, moderate pressure, high pressure) for each causal condition and outcome, and then decide to focus on using only the "low" and the "high" adjectives (from the three terms), those results may (will) be different from the ones we would get if from the very beginning we only used a two-term vocabulary, because the MFs for "low" and "high" will be different for the two different size vocabularies. Hope this is clear.

This is very interesting to us, because we think that this represents a "new" kind of uncertainty that could be modeled by using type-2 fuzzy sets, and is now on our "research agenda."

A: This is very interesting and promising. In general, I am in agreement that the vocabulary that is used must be viewed holistically—"low" means something different depending on the size and nature of the vocabulary it is embedded in and thus motivates different calibrations.

> *Question* [*Ragin*]: Who is doing the calibration, the researcher or the practitioner? In survey research, we see "wording effects" all the time—the wording of the question has a huge impact on the response. So if it is someone in the field who is rating pressure on a fuzzy scale [*Mendel*: it is not], then the vocabulary will have an even bigger impact. Of course, it also has an impact on the researcher then calibrating an objective measure (e.g., of pressure) using different vocabularies. (It was unclear from your example who the "person" was and whether the issue centered on people in the field or on researcher practices.)
>
> *Answer* [*Mendel*]: The answer to the question: "Who is doing the calibration, the researcher or the practitioner?" is "Both." I don't mean to be cavalier about this, but we (the researchers) have developed what we believe to be a very novel way of synthesizing

MFs from data. It uses one of the most important and widely used clustering methods in fuzzy logic—fuzzy c-means. We have been driven to creating MFs directly from data by our sponsor, because they are “passionate” about their data. In addition, we are developing a way to extract MF data from the “practitioner.”

Reply [*Ragin*]: I have some (very minor) experience with fuzzy c-means clustering. In general, I have tried to stay away from clustering (in general) because the solutions can differ substantially from one method (and one option in a single method) to the next. But these techniques can address a key problem, which is data reduction. Truth tables get so unwieldy with many inputs.

I'd be interested to learn more about how you use clustering. Do you use it to reduce the number of inputs?

In terms of fsQCA users in the social sciences, very few have followed my lead and attempted multiple calibrations of the same source variable. So I have not dealt with this issue before, except in my own experience. My general argument is that calibrations must be grounded as much as possible in substantive knowledge, but this rule does not circumvent the impact of vocabularies.

You stated: “This is very interesting to us, because we think that this represents a ‘new’ kind of uncertainty that could be modeled by using type-2 fuzzy sets, and is now on our ‘research agenda.’ I would love to learn more about your thinking along these lines. As I mentioned before, I have not done much work with type 2 fuzzy sets.

Miscellaneous

- **Causality**

Q1: Zadeh has gone on record (in talks, at least) that causality is not black and white but instead is a matter of degree. I can't recall if you have stated that fsQCA is about causality, but it sure seems like it is to me, especially when one has to choose "causal conditions." Causality though is a big can of worms (snakes) with many detractors. What are your counter-arguments to the detractors of fsQCA as a method for establishing causality?

A1: My usual argument is that causality can be studied only at the case level. QCA results provide leads for this case-level investigation, based on the candidate causal conditions that go into QCA.

I also argue that most "causal recipes" have direct implications regarding what kinds of causal mechanism are at work, which in turn provides additional guidance at the case level.

So QCA is relevant to the investigation of causation, but not a direct examination of causation, per se.

Q2: Seems to me there is a very fine line (or perhaps no line at all) between a sufficient "condition" for a desired outcome and a "cause" for that outcome. Can you explain the difference to me?

A2: You are correct. There's no difference. The term sufficiency is really too strong, as is the term necessity. In truth, it makes more sense to substitute cases with "widely shared outcomes" for "sufficiency" and cases with "widely shared antecedent conditions" for necessity. It's pedagogically useful to make the sufficiency/necessity distinction, but in practical terms, it's a real stretch to make any kind of true causal inference.

- **csQCA: Comparison With Other AI Rule-Generation Methods**

Q: Has anyone compared csQCA with other AI rule-generation methods (e.g., trees, chains, etc.)?

A: Not to my knowledge; sounds like something I should look into. Suggestions welcome.

- **Forecasting**

Q1: In some of [our] works we use cases to create a "forecaster" (not by QCA). In the 2009 book, Box 1.4, you give 5 types of uses for QCA techniques. Can it also be used for forecasting, e.g., using a final set of QCA rules/truth table, obtained from what is called a "training sample" of cases, if a new set of cases become available, can the rules be used to provide a consequent?

A1: Regarding forecasting: I think this is an interesting application. The main issue I see is that the initial set of cases might be more limited in its diversity (in relation to the number of truth table rows it is able to populate with empirical cases). In short, if the two sets of cases populate the same or at least very similar rows, then forecasting seems quite feasible.

Q2: How can the results from fsQCA be used for forecasting? I don't know the answer yet, but I am inclined to believe that best forecasting results will be obtained by using the most complex results from fsQCA because they will provide the largest number of rules. The mathematics of fuzzy logic will then let us provide a "forecast." What do you think?

A2: In general, I agree. It is no more hazardous to use fsQCA results, based on existing data, than it is to use regression results. I might ignore causal recipes with low raw coverage, however. Still, keep in mind that the complex solutions sometimes include what I consider nuisance terms, especially when the diversity of cases is low and the number of causal conditions is great. For example, suppose the outcome is staying out of poverty and one of the conditions included in a successful combination is having "low income parents." If the reason that that this condition is included is simply because there was no matched row with "not-low-income parents" (i.e., there were no cases with this specific combination), then having "low income parents" as part of a combination linked to staying out of poverty is truly a nuisance. This is why I almost universally prefer intermediate solutions.

• fsQCA Results Compared With Those Obtained by Another Method

Q: Our sponsor then asks "How do the results from fsQCA compare with those obtained from another method?" How do you address such a question?

A: In social science, the usual comparison is multiple-regression. The usual finding is that QCA offers much more nuanced results. Still, the conditions that seem most important are usually the same in both analyses. For example, a multiple regression might show that $x1$ has a very strong net effect, while the QCA results would show several recipes with $x1$ joined with other conditions.

[JMM: After obtaining the three fsQCAs for each of the 10 folds, we could then compare the fsQCA-classification results with those obtained by other classifiers; and, we can compare the fsQCA prediction results with those obtained by other predictions (such as regression).

CR: In principle, this sounds good. The only problem I typically encounter is the fact that regression is symmetric and focused on Y versus $\sim Y$ at every step of the analysis, while set theoretic analysis separates the analysis of Y from the analysis of $\sim Y$. So the results of both set theoretic analyses should be entered into the regression analysis. This is probably true for the classification analyses as well.]

• Scores: Negative

Q: In Table 5.2 of the 2009 textbook [3], "Survival" has positive and negative numbers. Why is that? How are they determined?

A: These are index scores published by a political scientist more than a few years back. It's a combined measure calculated as the difference between two separate measures, one of democracy (1-10) the other of autocracy (1-10). The measure is simply democracy score minus autocracy score.

• Textbook [3]: Resource Site

Q: I tried to access the textbook resource site for your 2009 textbook at http://www.compasss.org/Textbook.htm; but, I got an error message: The requested URL /Textbook.htp was not found on this server. Is there a new URL for this site?

A: Benoit Rihoux is putting this together. It's still in progress: http://www.compasss.org/pages/links/ccmtextbook.html just repeats stuff from the edited book.

• Using fsQCA Results

Q1: Our sponsor also wants to know "What can you do with the results from fsQCA?" How do you answer such a question?

A1: When conditions are combined in a recipe, there are often implications about (unobservable) causal mechanisms. If there are competing arguments about mechanisms, fsQCA results often speak to these arguments.

fsQCA results also can be predictive in the usual sense, providing a guide for securing good outcomes (and separately, avoiding bad outcomes). The asymmetry of set theoretical analysis with respect to the outcome provides greater nuance.

Q2: In particular our sponsor keeps thinking about the mpg [mile per gallon] problem [data obtained form the UCI Data Repsotitory] from a classification point of view, i.e. once the rules are extracted from fsQCA how successful will they be in correctly classifying a new vehicle (one that was not included in the fsQCAs) as low mpg, moderate mpg or high mpg? In order to answer this question we would have to perform a statistical study using, e.g. 10-fold cross-validation. Have you encountered such responses to your works? If so, what do you normally do?

A2: I have not encountered this. Most social science applications do not involve prediction. QCA was developed in part because of the difficulty of a statistical analysis applied to configurational/combinatorial evidence. Not quite sure what you mean by **10-fold cross-validation**. The purpose of QCA is to allow complexity, despite data that are limited in their diversity.

[JMM: In **10-fold cross validation** (which is widely used in pattern recognition) the cases would be put into 10 "folds" by a random assignment method. Nine folds (90% of the cases) would be used for each fsQCA and the remaining fold (10% of the cases) would be used for validation. This is done 10 times, each time using a different set of 9 folds. Then validation statistics are computed across the 10 experiments to provide average "success" and a standard deviation for it. One hopes that the average will be high and the standard deviation will be low.

One idea we have is to obtain three fsQCAs for each of the 10 folds, one each for low mpg, moderate mpg and high mpg. Each of these fsQCAs leads to one complicated rule with different intermediate solutions connected by "OR." We would then have three big rules. The mathematics of fuzzy logic, applied to these big rules, can then be used to obtain a highly nonlinear mapping of the causal conditions into mpg (either a numerical value or a linguistic value). We could then apply the 10-fold cross-validation approach to classify the cars in the 10% fold into one of the three categories. Or, we could also use the 10-fold cross-validation approach to predict the numerical mpg of the cars in the 10% fold. These are the ways that we are thinking about "validation" for this application.

CR: This sounds very interesting and promising. I especially like the fact that the procedure allows production of a standard deviation.]

QM Algorithm

- **Appearance of Both a Causal Condition and its Complement in Different Parsimonious Terms (see Causal Conditions/Combinations: Appearance of Both a Causal Condition and its Complement in Different Parsimonious Terms)**

- **Fuzzy Truth Table: Information Loss**

Q: Someone asked me the following question and I am curious to know how you would answer it: What information is lost when going from the fuzzy truth table to the Boolean truth table?

A: There are several answers to this question, depending on what's behind the question. First of all, fuzzy logic minimization allows for expressions like *AaB*, which are rarely useful in social science research. (Maybe they are in your applications.) So I developed the fuzzy truth table approach, as a way to bypass this possibility. The inclusion algorithm (in *Fuzzy Set Social Science* [1]) does not rely so heavily on Boolean logic minimization, but it also is not nearly as robust and often gives results like $ABC + ABc \rightarrow Y$ which is troubling to most social scientists.

Second, the crisp truth table I use is based on the statements that follow from multiple fuzzy set analyses. Some readers have the impression that the fuzzy sets are dichotomized in the process of constructing the truth table spread sheet. They are not. The crisp truth table spreadsheet is simply a listing of the different logically possible combinations of fuzzy set causal conditions and the fuzzy-set based consistency scores that go with each combination of conditions. I argue that the truth table spreadsheet summarizes *statements* about the property space defined by the causal conditions (and it is permissible to use a conventional truth table to array statements).

Third, there *is* loss of information when a frequency cut-off is imposed. In social science research, data are often very noisy, and statistical theory argues against placing too much faith in very small *N*s, so I consider this loss trivial (and beneficial). As always, multiple cut-offs can be used to generate fine-grained versus coarse grain analyses of the same table.

Fourth, there is also loss of information when (in effect) the fuzzy consistency scores are dichotomized. My approach to this problem is to use multiple cut-offs (like level sets) and nest the results. For example, the results for consistency $\geq .90$ will be a subset of the results for consistency $\geq .80$.

- **Logical Minimization: Loss of Relevant Information**

Q: When logical minimization is used, sometimes relevant-information may also be lost. This can happen sometimes in your least parsimonious analysis and almost always in your most parsimonious (counterfactual) analysis. For example, using the simplified statement, given on page 136 of your 2000 book [1]—"IF (large and growing) or (wealthy) THEN ethnic political mobilization"—the information that "linguistic ability" was (also) used in the QCA has been lost. In order to restore that information, one could restate the rule as: "Regardless of linguistic ability, IF (large and growing) or (wealthy) THEN ethnic political mobilization." In our potential engineering applications for fsQCA, I think it would be important to restore such lost information so that petroleum engineers know that it was considered during the analysis. Do social scientists have similar concerns?

A: Generally, social scientists are accustomed to being mindful of what has been eliminated, without making it explicit in summary statements. In any event, I usually encourage publication of the truth table spreadsheet for each result, making it clear what the starting point was. But still your point is a very good one, especially in engineering and other practical applications.

• **Minimal Prime Implicants (see Quine-McCluskey Algorithm)**

• **Prime Implicants (see Quine-McCluskey Algorithm)**

• **Quine-McCluskey Algorithm**
Q1: I bought your 1987 book. It has a very nice chapter on the basic concepts of Boolean algebra. Does the Quine algorithm provide the prime implicants or the minimal set of prime implicants or both?

A1: The minimal set [Actually, it can be used in two different ways to provide both; see A2 and A3.].

Q2: Are the prime implicants what you now refer to as "parsimonious solutions" and the minimum set of prime implicants what you now refer to as the "complex solutions"?

A2: The parsimonious solution is the minimal set, treating all combinations of conditions (truth table rows) without cases as "don't cares." The complex solution is the minimal set, treating all combinations of conditions without cases as "false."

Q3: [For counterfactual analysis] it seems that it is absolutely essential to determine the "parsimonious solution," since it censors the intermediate solutions. How do you get that solution (if it is not the one found from the Quine algorithm)?

A3: The parsimonious solution in fsQCA is the Quine minimal set, using the maximum possible number of don't care combinations (as just described).

Q4: Some Boolean functions may have more than one minimum prime implicant. Have you encountered this? If so, how do you or would you handle more than one minimal prime implicant?

A4: I have a little dialogue box that pops up whenever two (or more) prime implicants are "tied." The user gets to choose (and can choose more than one from among those that are tied). I avoid any automated procedure because the data are not switching circuits; that is, the choice of prime implicants is not without substantive impact. Another solution is to simply print all the different solutions and choose the one(s) that make(s) the most sense.

This is my gripe with the program Espresso (developed at Berkeley, which—by the way—can be utilized with a home-grown, customized user interface). When faced with choices, it makes its own, without user input. I haven't figured out how to get it to show all the equivalent solutions or to allow choice.

[Note: Mohammad Korjani (Ph. D. student of Dr. Mendel) uses Espresso.

JMM (email, Dec. 5, 2010): Do you get the same results from QM when using Prof.

Ragin's fsQCA software [and Espresso]?

MK (email, Dec. 6, 2010): I got the same results using both software. The difference between Espresso and Prof. Ragin's software is that if there are two or more ways to minimize a Boolean expression then Espresso randomly selects one of the ways while Prof. Ragin's software asks the user to select the way.]

Q5: Now I have a question about what a **prime implicant** is. Although I am an EE, I am not a logic designer (computer engineers do this), and when I went on the Internet searching for explanations of prime implicants, nothing "easy" to understand came up. Many discussions couch prime implicants in the framework of Karnough maps, and life is too short for me to learn (again) about them. Your 1987 book has an excellent discussion about prime implicants. Do you know of other easy-to-read references for them?

A5: I taught myself much of this stuff using this book:
http://www.amazon.com/Schaums-Outline-Boolean-Switching-Circuits/dp/0070414602/ref=sr_1_1?ie=UTF8&s=books&qid=1285093739&sr=8-1-spell
which is really straightforward.

Q6: Your 1987 book (p. 96) obtains the prime implicants (PI) for the example $S = AbC + aBc + ABc + ABC$, as $S = AC + AB + Bc$, by reducing the 3-term combinations to 2-term combinations. Suppose, instead, I start with, e.g., 5-term combinations and I first reduce them to 4-term combinations, and then continue to reduce the 4-term combinations to a mixture of 3- and 4-term combinations, etc. Which of these reductions do you consider to be the "prime implicants"?

A6: What makes a PI a PI is that it can't be further reduced, given the outcome coding in a given truth table. So it's not a PI until all the relevant pairings and reductions have been implemented. So you can start with 5 elements and reduce all the way down to very simple PIs.

Q7: Your 2009 book [3, pp. 112 and 115] leads me to believe that, since the prime implicants are associated with the "complex solution," they are obtained by performing as many set theory reductions as are possible on the primitive Boolean expressions that survive both the frequency and consistency cut-off tests. Am I correct about this?

A7: Yes. The parsimonious solution is derived the same way, except that it also takes advantage of the remainder rows, to generate simple PIs.

Software

- **Availability of Source Code**

Q1: Is a source code version of fsQCA available?

A1: I've never made it available, but we might be able to work something out, as long as it's a two-way street and my programmer approves. The code has a long history—mixture of tcl/tk and modula 2, originally written in turbo pascal. The modula 2 code has all been translated (by a translator, not a human) into C++. I haven't programmed much since BASIC days.

Q2: Regarding the source code for the fsQCA software: (1) I don't program either; (2) I will check with my student to see if he can work in C++. If he can, then we would provide you with any modifications that we make to the software. Is that what you had in mind?

A2: Yes. We would precede the sharing of code with a discussion of proposed changes, etc., and the possibility of multiple versions.

Q3: Would it be possible for you to provide us with the source code just for the QM portion of your software? Since there can be different solutions for QM we would like to be on the same "wave length" with the solutions obtained with your software (from our software).

A3: I'll do this when I return to Tucson. Actually, I would be happy to have the code checked. It is classic Quine, but I altered it in a modest way, which had the effect of increasing the occurrence of "tied" prime implicants, which are then turned over to the user, to choose between the tied PIs. [We never followed through with this.]

- **Coverage and Consistency Computations**

Q1: Your software computes "raw consistency," "PRI consistency" and "product." What are these?

A1: *Raw consistency* is what I describe in my book [2], namely sum(min(x,y))/sum(x). It is the degree to which x is a consistent subset of y. Also called "inclusion."

PRI consistency is a more refined and conservative measure of consistency and subtracts sum(min(x,y,~y) from the numerator and the denominator. The subtracted quantity is where x is a subset of both y and ~y and is therefore "confounded." PRI stands for *proportional reduction in inconsistency*, moving from not knowing if x is a subset of y or ~y, to asserting that it is a subset of y.

Product is simply the product of these two measures, with the idea that it is good to have both. Could take the square root of the product, to make it look better (closer to 1).

Q2: My student, who is using your fsQCA software, has told me that beside coverage and consistency of each combination of causal conditions for fsQCA solutions, "solution consistency" and "solution coverage" are printed out at the end of the all combinations.

1. How are overall consistency and coverage computed?
2. What do you do with these versus what you do with coverage and consistency of each combination of causal conditions.

A2: 1) You compute degree of membership of each case in all the recipes in the solution and then take the max recipe membership score. This is then entered (as x) into the numerator: sum(min(x,y))

2) It's just a global measure of the value of the results, when the different recipes are combined into a single evaluation. Generally, I don't find solution consistency and coverage as useful as these same calculations for the separate recipes.

- **Intermediate Solutions (Summaries)**

Q: When using counterfactual analysis you are led to the simplest summarization, sometimes so simple that a social scientist might say "Well, that's obvious." Of course, you argue that there is a continuum of summaries between the simplest and least parsimonious. The fsQCA software provides a third intermediate summary. How is it chosen? Why it and not others?

A: The intermediate solution is always based on the researcher's best substantive and theoretical knowledge, implemented through the dialogue box. If it is implausible, e.g., that the negation of a particular condition could be linked to the outcome, then this information is used to identify the "easy" counterfactual cases, among all the combinations of conditions that lack empirical instances (the "remainders"). The procedure basically provides a way to interrogate the "complex" solution. If one of the complex solutions is *ABCd* and the "*d*" seems implausible and *ABCD* is a remainder, then an easy counterfactual is defined (*ABCD*) and it can be used to simplify *ABCd* to *ABC*. *The key is to use the complex solution as the starting point and look for elements that seem anomalous and then see if there is an easy counterfactual handy. The parsimonious solution acts as a censor*.

- **Limits on Number of Causal Conditions**

Q: My student is now using your fsQCA software, and has run into a problem. When we have 11 or fewer causal conditions we obtain the complex, parsimonious and intermediate solutions for the software; however, for 12 or more causal conditions, we only get the complex solutions. Is there a limit on the number of causal conditions one can use?

A: Yes, 11-12 is about the limit. When you get only the complex, it is probably still working on the parsimonious, but seems dead. There is another algorithm, Akers, which is faster and will handle more causal conditions, but not necessarily generate the minimal solution and bypasses the issue of choosing between prime implicants (it makes random choices).

- **Listing of Best Cases**

Q: Does the fsQCA software automatically list the Best Cases for each term in the final result, or does this have to be done by hand?

A: There is an option to have it list the best cases, in the dialogue box where the analysis is specified (outcome and causal conditions). It is a recent addition and may be buggy. There must be a non-numeric case id for this to work.

- **Log-Odds Transformation Method for Constructing a Membership Function**

Q: Does the fsQCA software create a membership function using the log-odds method?

A: Yes, there is a calibrate function in the compute dialogue box, described in my 2008 book [2].

• **URL**
You can find Ragin software at: http://www.u.arizona.edu/~cragin/fsQCA/software.shtml

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Appendix A: New Intermediate Solution Algorithm

Prepared by Charles Ragin
Sent to Jerry Mendel on Dec. 13, 2009

The basic sketch of the new intermediate solution algorithm follows.

The procedure evaluates each term in the parsimonious solution against each term in the complex solution. The number of terms in a solution is the number of combinations of causal conditions joined by "+". For example, if the parsimonious solution is AB + CD, then there are two parsimonious terms. If there are *k* parsimonious terms and *m* complex terms, the procedure cycles *k*m* times, once for each possible pairing.

1. Check to see if the complex term is a subset of the parsimonious term. All the elements of the parsimonious term must appear in the complex term. For example, if the parsimonious term is AB and the complex term is ABcD, then the subset relation is satisfied. However, if the parsimonious term is AB and the complex term is aBcD, then the subset relation is *not* satisfied. If the subset relation is not satisfied, then the procedure stops for this parsimonious/complex pairing and proceeds to the next pairing.

Complex terms:

```
1 GLOBAL*ECOVALPOL*CULTZONE*pwelreg+
2 GLOBAL*ECOVALPOL*CULTZONE*DEMLONG+
3 GLOBAL*ECOVALPOL*CULTZONE*HISTDEV+
4 GLOBAL*ecovalpol*pwelreg*DEMLONG*histdev +
5 ECOVALPOL*CULTZONE*pwelreg*DEMLONG*histdev
```

Parsimonious:

```
6 ECOVALPOL+
7 DEMLONG
```

Are the elements in #6 a subset of the elements in #1? Yes, proceed because #6 is a superset of #1.

. . .

Are the elements in #6 a subset of the elements in #4? No, stop.

. . .

2. Check to see if the user-inputted theoretical term is *consistent* with the parsimonious term. If it is not, then the element(s) in the theoretical term that differ from the parsimonious term are "trumped" by the parsimonious term.

The theoretical term, as inputted by the user, is:

```
CULTZONE (present)
PWELREG (present)
DEMLONG (present)
HISTDEV (present)
```

```
(The other two may be present or absent.)
```

In this example, the parsimonious term (#6) does not contradict with the theoretical term. However, we still need to complexify the theoretical term, yielding, for the #6#1 combination:

```
8 ECOVALPOL*CULTZONE*PWELREG*DEMLONG*HISTDEV
(ECOVALPOL added to theoretical case)
```

3. Compare the complex term (#1) and the theoretical term (using the theoretical term that has been altered in step 2). Use the theoretical term to eliminate any extraneous elements from the complex term (pwelreg/PWELREG):

```
1 GLOBAL*ECOVALPOL*CULTZONE*pwelreg+
8 ECOVALPOL*CULTZONE*PWELREG*DEMLONG*HISTDEV
```

Which yields:

```
9 GLOBAL*ECOVALPOL*CULTZONE
```

4. Save this intermediate term, and then proceed to the next pairing. Save all intermediate terms that are valid.

5. List all valid intermediate terms in the truth table spreadsheet with outcome set to 1. Only these terms should appear in the truth table.

For example, `GLOBAL*ECOVALPOL*CULTZONE` would appear as

GLOBAL	ECOVALPOL	CULTZONE	PWELREG	DEMLONG	HISTDEV
1	1	1	-	-	-

(the dash signals "don't care"/"already eliminated")

6. Simplify this truth table using Quine, setting all remainders to false. The result of this analysis is the full intermediate solution.

Appendix B. Subset/Superset Analysis

Prepared by Charles Ragin
Sent to Jerry Mendel on Dec. 27, 2011

The subset/superset procedure offers a way to both test a given causal recipe and to dissect it. The procedure assesses the consistency and coverage of a user-specified recipe, as well as the consistency and coverage of all subsets of ingredients in the recipe. It is called the subset/superset procedure because each subset of ingredients constitutes a superset of the cases included in the initial recipe. For example, the set of males is a superset of the set of the set of red-headed males. More generally, the fewer the ingredients in a recipe, the larger the set of cases it embraces.

The standard fsQCA procedure is truth table analysis, where the goal is to build a logical statement describing the patterns found in the data. The usual result is a specification of the different combinations of causal conditions linked to an outcome. An important part of truth table analysis is the consideration of "remainders"—the combinations of causally relevant conditions that lack empirical instances. The three different ways of treating these remainders provide the basis for the derivation of the "complex," "parsimonious," and "intermediate" solutions to the truth table. Truth table analysis has an inductive quality, because these statements are constructed from the evidence summarized in the truth table.

Subset/superset analysis, by contrast, has a deductive quality. The causal "recipe" is supplied in advance by the researcher. The goal of subset/superset analysis is to assess not only the recipe as formulated by the researcher, but also to assess all possible subsets of ingredients specified in the recipe.

For example, a researcher might speculate that causal recipe *a~bc~d* is sufficient for outcome *y* (i.e., the speculation is that membership in this recipe is a consistent subset of membership in *y*). Notice that the researcher must specify the **directionality** of each causal condition. That is, the researcher must specify whether it is the causal condition (as calibrated in the data spreadsheet) that is linked to the outcome or its negation.

The number of recipes assessed in a single subset/superset analysis is $2^k - 1$, where k is the number of conditions in the initial recipe. For a four-condition initial recipe, the procedure assesses a total of 15 formulations:

- **the initial recipe (a single combination of four conditions)**

  a~bc~d

- **four three-condition recipes:**

  a~bc
  a~b~d
  ac~d
  ~bc~d

- **six two-condition recipes:**

  a~b
  ac

a~d
~bc
~b~d
c~d

- **four one-condition recipes:**

a
~b
c
~d

(If you are familiar with Pascal's triangle, you already know the pattern.)

For each recipe tested, fsQCA reports the consistency and coverage of the recipe, with respect to its subset relation with the outcome. In other words, the program assesses the causal sufficiency of each recipe—do cases with this recipe constitute a fuzzy subset of cases with the outcome? If consistency is high (e.g., at least 0.80), it is reasonable to evaluate coverage. If consistency is low, then the calculation of coverage is meaningless.

Notice what is **not** assessed by the subset/superset procedure. In the truth table procedure, if the researcher specifies causal conditions *a*, *b*, *c*, and *d* as relevant, then the program assess all 16 presence/absence configurations of these four conditions:

~a~b~c~d	*~a~b~cd*	*~a~bc~d*	*~a~bcd*
~ab~c~d	*~ab~cd*	*~abc~d*	*~abcd*
a~b~c~d	*a~b~cd*	***a~bc~d***	*a~bcd*
ab~c~d	*ab~cd*	*abc~d*	*abcd*

The subset/superset procedure, by contrast, starts with only one of these sixteen (e.g., the one that is in bold italics) and then explores simpler versions of this recipe.

Notice also that when using the truth table procedure the movement to a simpler formulation (e.g., to the three-condition recipe *a~bc*) requires that the two four-condition recipes included in this recipe (i.e., *a~bc~d* and *a~bcd*) both "pass" the test for sufficiency (or its equivalent via counterfactual analysis). The testing of simpler recipes using the subset/superset procedure, by contrast, is mechanical and does not require that all the component configurations pass some sort of sufficiency test.

It is also important to understand that because the subset/superset procedure automatically tests all subsets of causal conditions, it is completely blind to the problem of limited diversity and to the importance of counterfactual analysis.

Recall that the truth table procedure's three solutions result from different treatments of the "remainder" configurations (combinations of causal conditions that lack empirical instances). The complex solution bars the use of remainders. The parsimonious solution uses any remainders that it can, regardless of whether they might be considered "easy" or "difficult" as counterfactual cases (from the perspective of theoretical and substantive knowledge). The intermediate solution uses only those remainders that are considered "easy" counterfactuals.

The subset/superset procedure uses remainders in a knowledge-blind manner. That is, like the derivation of the parsimonious truth table solution, there is no check on the use of remainders that might be considered difficult counterfactuals. Suppose, for example, that a researcher using the subset/superset procedure specifies the recipe *a~bc~d*. As part of the analysis, the subset/superset procedure will also automatically test *a~bc*. But suppose there are no cases of *a~bcd* (i.e., no cases with greater than 0.5 membership in this combination), and our theoretical knowledge tells us that *a~bcd* is a difficult counterfactual. From the perspective of the truth table procedure, the test of the sufficiency of *a~bc* is not warranted.

For this reason, is always best to view the subset/superset procedure as exploratory. Perhaps, based on case knowledge, *a~bc* might be considered a better starting recipe than *a~bc~d*, and the whole issue of the difficult counterfactual *a~bcd*, from this perspective, might be seen as a useless distraction.

As always, it is important to remember that the purpose of both procedures, the truth table procedure and the subset/superset procedure, is to facilitate the interaction between ideas and evidence in the production of social scientific representations of empirical evidence.

An Example of the Subset/Superset Procedure

Cases: European democracies between WWI and WWII
Outcome: democratic breakdown
Causal conditions: low development, low industrialization, low urbanization, political instability

Here's the initial output:

Recipe	Consistency	Coverage
~urbanfz*~develfz*~indusfz*~stablfz	0.914934	0.510548
~urbanfz*~develfz*~indusfz	0.885675	0.678270
~urbanfz*~develfz*~stablfz	0.912249	0.526371
~urbanfz*~indusfz*~stablfz	0.909927	0.522152
~develfz*~indusfz*~stablfz	0.897163	0.533755
~urbanfz*~develfz	0.890306	0.736287
~urbanfz*~indusfz	0.727072	0.694093
~urbanfz*~stablfz	0.913765	0.581224
~develfz*~indusfz	0.865110	0.703586
~develfz*~stablfz	0.894017	0.551688
~indusfz*~stablfz	0.889845	0.545359
~urbanfz	0.673859	0.856540
~develfz	0.837472	0.782700
~indusfz	0.708376	0.722574
~stablfz	0.901592	0.657173

Here it is again, sorted by coverage, deleting recipes with consistency < 0.85.

Recipe	Consistency	Coverage
~urbanfz*~develfz	0.890306	0.736287
~develfz*~indusfz	0.865110	0.703586
~urbanfz*~develfz*~indusfz	0.885675	0.678270
~stablfz	0.901592	0.657173
~urbanfz*~stablfz	0.913765	0.581224
~develfz*~stablfz	0.894017	0.551688
~indusfz*~stablfz	0.889845	0.545359
~develfz*~indusfz*~stablfz	0.897163	0.533755
~urbanfz*~develfz*~stablfz	0.912249	0.526371
~urbanfz*~indusfz*~stablfz	0.909927	0.522152
~urbanfz*~develfz*~indusfz*~stablfz	0.914934	0.510548

A key issue is parsimony: is a more parsimonious solution necessarily better if the less parsimonious solution has about the same coverage?

Sometimes more parsimonious recipes might be preferred; sometimes less parsimonious solutions might be preferred. Not only is it easier to connect *less* parsimonious recipes to empirical cases, it is important to recall that parsimony is often achieved at the expense of necessary conditions. Recall also that consistency (as computed above) is an evaluation of the degree to which a causal combination constitutes a subset of the outcome. If this causal combination includes a necessary condition, then (more than likely) this condition *never* supplies the minimum value. After all, its scores are generally $\geq y$. Thus, removing the necessary condition from a combination is unlikely to contribute to inconsistency.

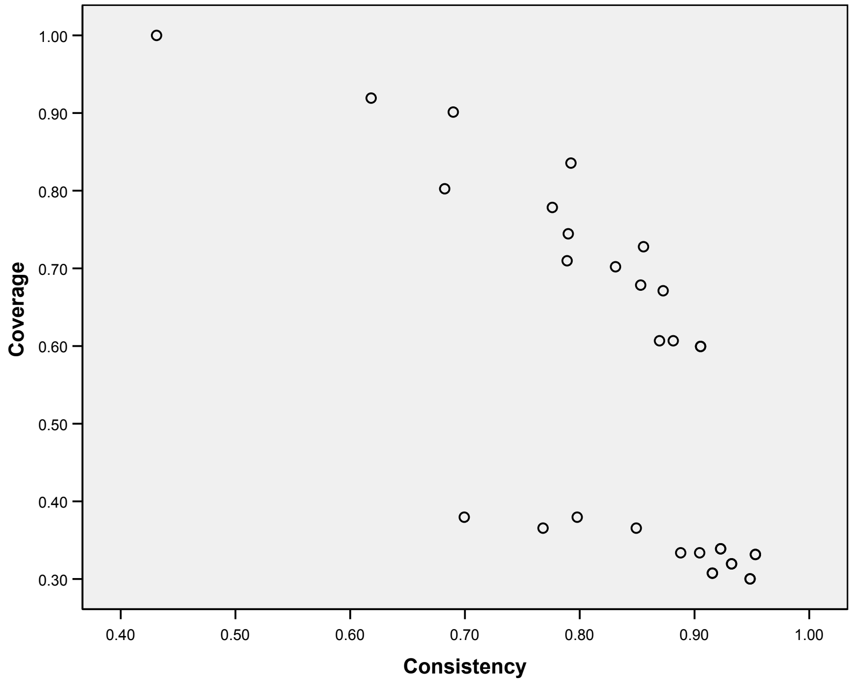
To be more precise, if the consistency of *abc* as a subset of *y* is 0.90, and c is a necessary condition, then the consistency of *ab* will also be about 0.90. Condition *c* never supplies the minimum because *c* is $\geq y$ and abc is $\leq y$. Parsimony would dictate selecting *ab* as the best recipe; common sense dictates the selection of *abc*.

A good example of this issue is shown next. The best solution is, of course, one that maximizes both consistency and coverage. Consistency should be as close to 1.0 as possible; otherwise, the coverage calculation is meaningless. In the spreadsheet that follows, a three-condition recipe and a four conditions recipe have the same coverage and consistency. (The four-condition recipe includes a necessary condition.) Which one is better? From a strict “parsimony” viewpoint, having fewer conditions is better. But from both a logical and an “understand-your-cases” viewpoint, the four-condition recipe is superior. (From a logical viewpoint, it is a bad idea to drop a condition that might be considered necessary.)

In the following spreadsheet, causal combinations with consistency scores greater than 0.85 and coverage scores greater than 0.60 are in bold type.

Combination	Consistency	Coverage
1. press urban hard lib activ	0.95	0.30
2. press urban hard lib	0.95	0.33
3. press hard lib activ	0.95	0.30
4. press urban lib activ	0.93	0.32
5. urban hard lib activ	0.92	0.31
6. press urban hard activ	**0.91**	**0.60**
7. press hard lib	0.95	0.33
8. press lib active	0.93	0.32
9. urban hard lib	0.92	0.34
10. hard lib active	0.92	0.31
11. press hard activ	**0.91**	**0.60**
12. urban lib activ	0.90	0.33
13. urban hard active	**0.88**	**0.61**
14. press urban hard	**0.87**	**0.67**
15. press urban active	**0.86**	**0.73**
16. press urban lib	0.85	0.37
17. hard lib	0.92	0.34
18. lib active	0.89	0.33
19. hard active	**0.87**	**0.61**
20. urban hard	**0.85**	**0.68**
21. press hard	0.83	0.70
22. urban lib	0.80	0.38
23. press urban	0.79	0.84
24. press active	0.79	0.74
25. press lib	0.77	0.37
26. urban active	0.78	0.78
27. hard	0.79	0.71
28. lib	0.70	0.38
29. urban	0.69	0.90
30. active	0.68	0.80
31. press	0.62	0.92

Here's a plot of the consistency and coverage scores from the spreadsheet. [The (0.4, 1) point has been added as a graphic anchor.] It's clear that there are two groups of coverage scores, those that include political liberalization (lib) as a causal condition (the low-coverage group) and those that exclude it (the high-coverage group).


Coverage
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
Consistency