USCEE REPORT #403

Digital Color Image Coding and Transmission

by

William K. Pratt

June 1971

Signal and Image Processing Institute
UNIVERSITY OF SOUTHERN CALIFORNIA
Department of Electrical Engineering-Systems
3740 McClintock Avenue, Room 404
Los Angeles, CA 90089-2564 U.S.A.

Preface

This report comprises the final report for a study entitled: "Satellite Data Communication Study", performed by the Image Processing Laboratory of the University of Southern California. The study was sponsored by the National Aeronautics and Space Administration, originally by the NASA Electronics Research Center under the direction of Robert Hinckley, and later transferred to the NASA Goddard Space Flight Center under the direction of Richard Kutz.

DIGITAL COLOR IMAGE CODING AND TRANSMISSION

1. INTRODUCTION AND SUMMARY

1.1 Introduction

Interest in color imagery for scientific, industrial, and military applications is rapidly increasing. Color images are not only more subjectively pleasing than monochrome images, but they also offer true improvements in the detection and discrimination of objects within the image field. Practically all commercial television transmission in this nation is in color, and many of the overseas programs received by satellite have been converted to color. Indeed, it is clear that there will be a growing need for the transmission of color imagery. Presently, a major hinderance to this growth potential is the inherently large bandwidth required for color television signals, especially for digital transmission.

This report is concerned with three basic points of digital color image transmission: techniques of electronically and digitally representing color images; source coding methods for minimizing the number of image code bits; and the effect of channel noise or errors on the digitally coded images. Emphasis has been placed on color images represented in the three primary color system.

Figure 1-1 contains a block diagram of a general color image transmission system. In operation, a color image sensor, such as a color television camera or color flying spot scanner, produces three electronic color signals, R_N, G_N, B_N, that, in a simplified sense, specify the red, green, and blue content of an image point. An electronic, or digital conversion of the R_N, G_N, B_N color signals is then performed to produce three color signals that are more amenable to efficient transmission than the original color signals. After quantization and coding, the composite digital color signal is transmitted through the channel. The receiver then reconstructs three electronic color signals R'_N, G'_N, B'_N for display of the color image. Plate 1a is a reproduction of an Ektacrome photograph that has been digitally coded by a computer controlled color image system simulating the transmission system of Figure 1-1. This photograph, which serves as the original test image in this report, contains 256 points in each coordinate direction. Each R_N, G_N, B_N color component has been linearly quantized to 64 levels. Characteristics of the original are described in Appendix B.

1.2 Summary

The major results of this study are summarized below:

1. A relatively simple model for color vision based upon the trichromatic theory of color vision has been presented. The model is consistent with the known laws and axioms of colorimetry. It provides a convenient bridge between the perception of color and its quantitative specification.

2. A matrix analysis method for analyzing color matching according to the tricolor theory has been developed. Using the matrix approach, the established formulations for computing a color match can be developed in a direct manner from the color model. This systematic technique of quantatively deriving the equations of color matching should lead to simpler methods of analyzing color transmission systems.

3. Physical limitations on the colors reproducible by real primaries in actual sensors have been established. Additionally, the restrictions on the color gamut resulting from reflected illumination of natural non-luminous objects has been incorporated in the specification of color solids for representing true colors. Knowledge of these solids enables the efficient design of electronic quantization and coding systems.

4. Transform image coding techniques have been successfully

A

B

C

D

E

F

G

H

I

2. COLOR IMAGE REPRESENTATION

It has long been known that a light of almost any arbitrary color can be matched by superimposing red, green, and blue lights of variable intensity. This color matching experiment has led to the tricolor theory of color vision, first stated by Thomas Young in 1807 [9]. Young postulated that there are three types of color sensors in the eye which have spectral responses that peak in the red, green and blue spectral regions, respectively. Recently, there has been experimentation [8, 10] that indicates that the human retina does indeed absorb energy in three distinct spectral regions. Figure 2-1 shows spectral sensitivity curves obtained by MacNichol [8].

There is much yet to be understood in the development of a complete theory of color vision. The tricolor theory does not adequately explain some anomalies of the defective color vision [11], the concept of subjective color [12], nor some of the experimental results of Land on two primary systems of color vision [13]. Nevertheless, the tricolor theory has proven adequate for color photography and color television. Hence, for this report, the tricolor theory has been adopted as the basis of a simplified model of color vision.

2.1 Trichromatic Color Matching Model

Consider the block diagram of Figure 2-2 as a perceptual model of color vision. An arbitrary light*, $\{C\}$, of spectral energy distribution, $C(\lambda)$, in the visible region defined by the wavelength limits λ_L and λ_U, is incident on the eye. The light $\{C\}$ striking the eye arises from an object point that is either self-luminous, transmissive, or reflective. In the latter two cases, the spectral energy distribution $C(\lambda)$ can be factored into the product of the energy distribution of an illuminating

*The brackets $\{\cdot\}$ indicate a colored light.

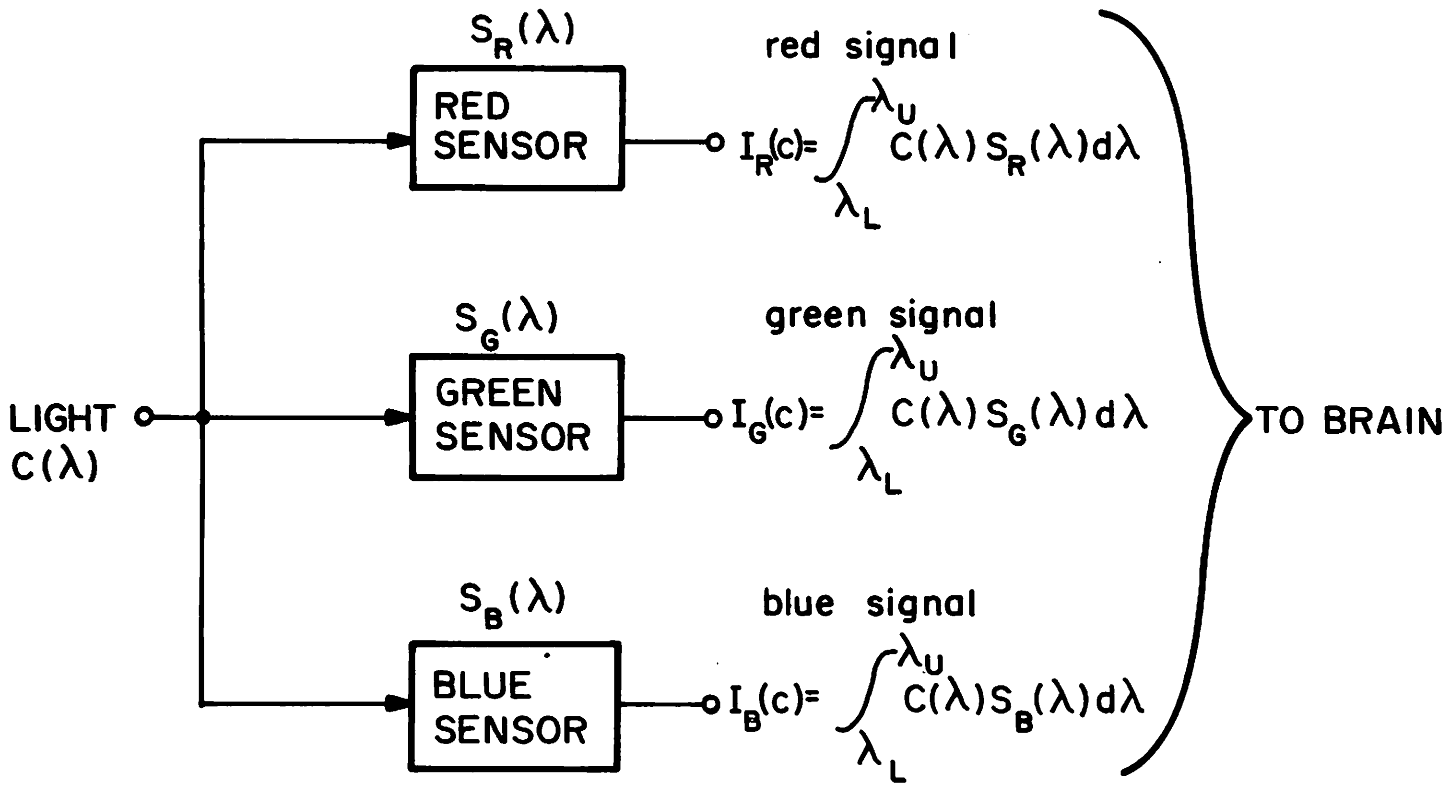


Figure 2-2. Perceptual Model of Color Vision

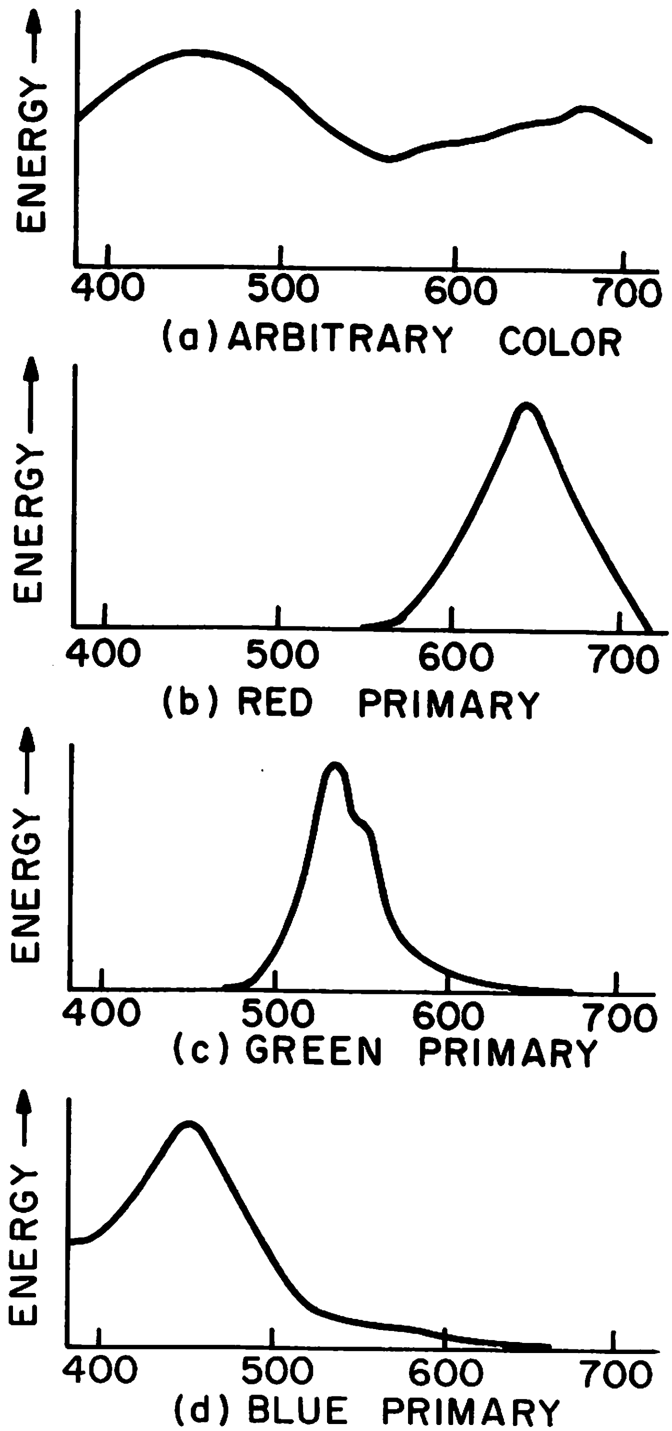


Figure 2-3. Spectral Energy Distributions

$$T_2(C) = \frac{A_2(C)}{A_2(W)} \tag{2-5b}$$

$$T_3(C) = \frac{A_2(C)}{A_3(W)} \tag{2-5c}$$

are called the <u>tristimulus values</u> of the color. Note that for the reference white, the tristimulus values are all unity. Combining eqs. (2-4) and (2-5) results in a set of three equations relating the color signals $I_R(C)$, $I_G(C)$, $I_B(C)$ and the measureable tristimulus values $T_1(C)$, $T_2(C)$, $T_3(C)$ of a color.

$$I_R(C) = \int_{\lambda_L}^{\lambda_U} C(\lambda) S_R(\lambda)\, d\lambda$$

$$= \int_{\lambda_L}^{\lambda_U} \Big[\underbrace{A_1(W)T_1(C)}_{A_1(C)}P_1(\lambda) + \underbrace{A_2(W)T_2(C)}_{A_2(C)}P_2(\lambda) + \underbrace{A_3(W)T_3(C)}_{A_3(C)}P_3(\lambda)\Big] S_R(\lambda)\, d\lambda \tag{2-6a}$$

$$I_G(\lambda) = \int_{\lambda_L}^{\lambda_U} C(\lambda) S_G(\lambda)\, d\lambda$$

$$= \int_{\lambda_L}^{\lambda_U} \Big[A_1(W)T_1(C)P_1(\lambda) + A_2(W)T_2(C)P_2(\lambda) + A_3(W)T_3(C)P_3(\lambda)\Big] S_G(\lambda)\, d\lambda \tag{2-6b}$$

$$I_B(\lambda) = \int_{\lambda_L}^{\lambda_U} C(\lambda) S_B(\lambda)\, d\lambda$$

$$= \int_{\lambda_L}^{\lambda_U} \Big[A_1(W)T_1(C)P_1(\lambda) + A_2(W)T_2(C)P_2(\lambda) + A_3(W)T_3(C)P_3(\lambda)\Big] S_B(\lambda)\, d\lambda \tag{2-6c}$$

Equation (2-6) can be written more compactly in matrix form by defining

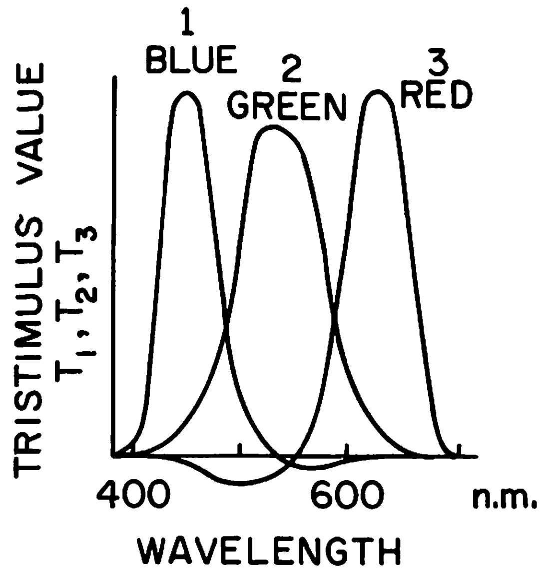


Figure 2-4. Tristimulus Values of Typical Red, Green, and Blue Primaries Required to Match Unit Energy Throughout the Spectrum

$$T_1(C) = \int_{\lambda_L}^{\lambda_U} C(\lambda_k)T_1(\lambda_k)\,d\lambda_k \qquad (2\text{-}13a)$$

$$T_2(C) = \int_{\lambda_L}^{\lambda_U} C(\lambda_k)T_2(\lambda_k)\,d\lambda_k \qquad (2\text{-}13b)$$

$$T_3(C) = \int_{\lambda_L}^{\lambda_U} C(\lambda_k)T_3(\lambda_k)\,d\lambda_k \qquad (2\text{-}13c)$$

From Figure 2-4, it is seen that at some wavelengths the tristimulus values obtained from solution of eq. (2-6) may be negative. Since the tristimulus values represent units of energy, the physical interpretation to this mathematical result is that a color match can be obtained by adding the primary with the negative tristimulus value to the original color and then matching this resultant color with the remaining primary. In this sense, any color can be matched by any set of primaries. However, from a practical viewpoint, negative tristimulus values are not physically realizable, and hence, there are certain colors that cannot be matched in a practical color display (e.g., a color television receiver) with fixed primaries. Fortunately, it is possible to choose primaries so that only a few rarely occuring natural colors cannot be matched.

For those natural colors that can be matched by positive tristimulus values, the match may be indicated by the colorimetric equation*

$$K\{C\} \equiv T_1\{P_1\} + T_2\{P_2\} + T_3\{P_3\} \qquad (2\text{-}14)$$

where the equivalence sign (≡) represents a color match to the human viewer.

---

*It is understood that $T_i = T_i(C)$.

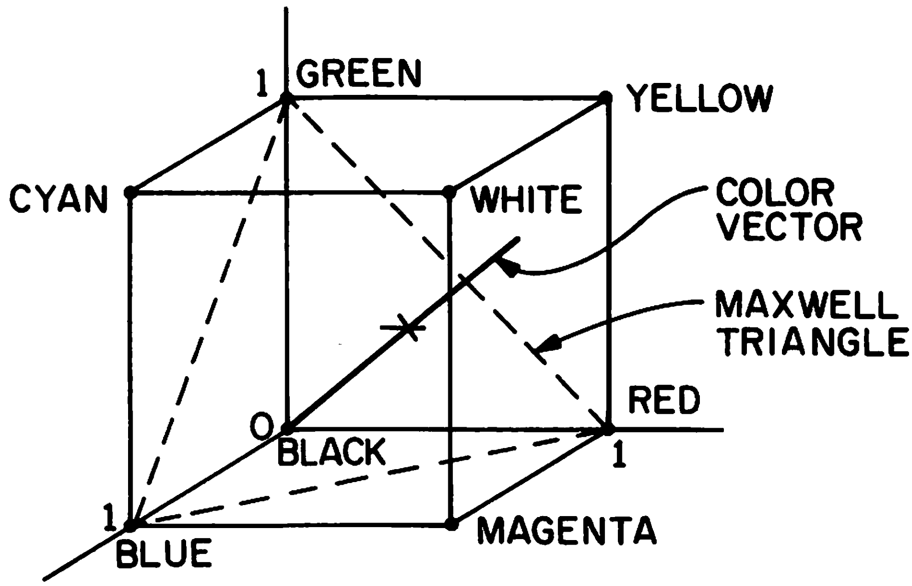


Figure 2-5. Color Space for Typical
Red, Green, and Blue Primaries

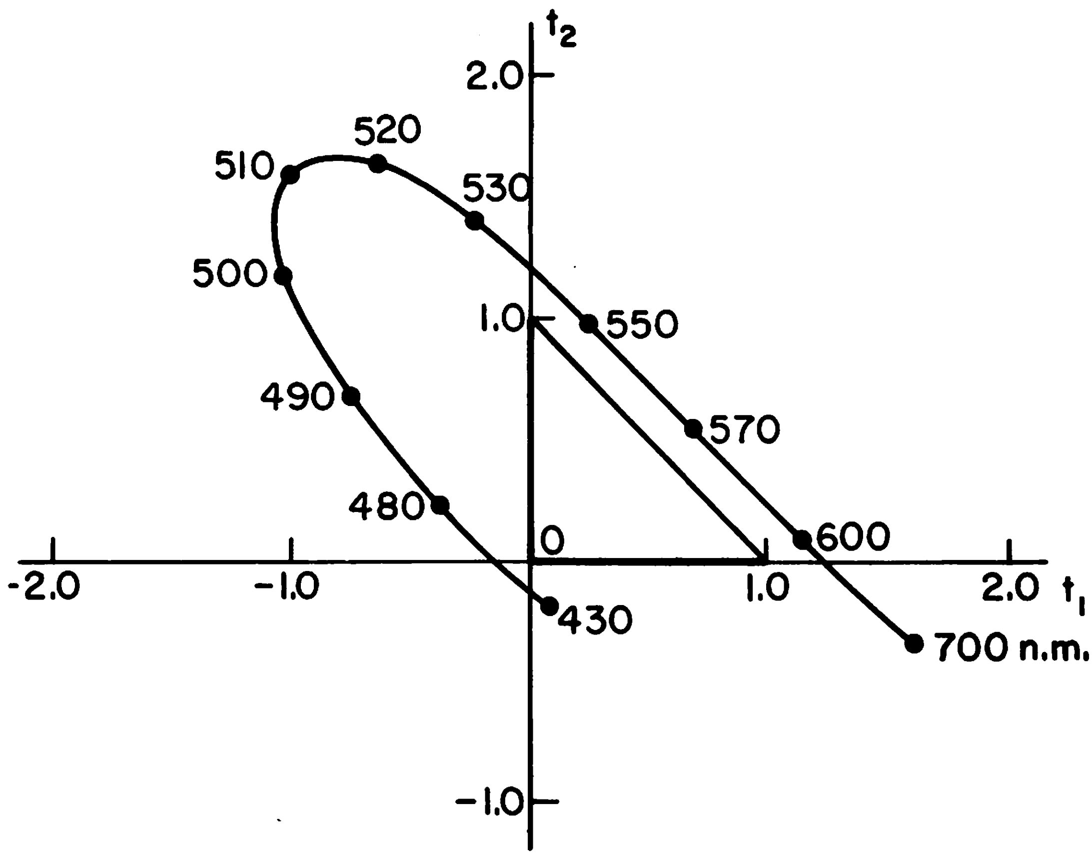


Figure 2-6. Chromaticity Diagram for Typical Red, Green, and Blue Primaries

obtain a relationship between the tristimulus values of the original and new primary system giving

$$\begin{bmatrix} T_1'(C) \\ T_2'(C) \\ T_3'(C) \end{bmatrix} = \begin{bmatrix} \begin{vmatrix} T_1(C) & T_1(P_2') & T_1(P_3') \\ T_2(C) & T_2(P_2') & T_2(P_3') \\ T_3(C) & T_3(P_2') & T_3(P_3') \end{vmatrix} \Bigg/ \begin{vmatrix} T_1(W') & T_1(P_2') & T_1(P_3') \\ T_2(W') & T_2(P_2') & T_2(P_3') \\ T_3(W') & T_3(P_2') & T_3(P_3') \end{vmatrix} \\ \\ \begin{vmatrix} T_1(P_1') & T_1(C) & T_1(P_3') \\ T_2(P_1') & T_2(C) & T_2(P_3') \\ T_3(P_1') & T_3(C) & T_3(P_3') \end{vmatrix} \Bigg/ \begin{vmatrix} T_1(P_1') & T_1(W') & T_1(P_3') \\ T_2(P_1') & T_2(W') & T_2(P_3') \\ T_3(P_1') & T_3(W') & T_3(P_3') \end{vmatrix} \\ \\ \begin{vmatrix} T_1(P_1') & T_1(P_2') & T_1(C) \\ T_2(P_1') & T_2(P_2') & T_2(C) \\ T_3(P_1') & T_3(P_2') & T_3(C) \end{vmatrix} \Bigg/ \begin{vmatrix} T_1(P_1') & T_1(P_2') & T_1(W') \\ T_2(P_1') & T_2(P_2') & T_2(W') \\ T_3(P_1') & T_3(P_2') & T_3(W') \end{vmatrix} \end{bmatrix} \tag{2-21}$$

Equation (2-22) then may be written in terms of the chromaticity coordinates $t_i(P_1')$, $t_i(P_2')$, $t_i(P_3')$ of the new set of primaries referenced to the original primary coordinate system. With this revision

$$\begin{bmatrix} T_1'(C) \\ T_2'(C) \\ T_3'(C) \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} T_1(C) \\ T_2(C) \\ T_3(C) \end{bmatrix} \tag{2-22}$$

where

relative to a match of a reference white. Often, it is necessary to determine the absolute, rather than the relative, amount of light from each primary needed to produce a color match. This information is found from luminance measurements of a color. The luminance of a colored light is a measure of the brightness of the light as perceived by a human viewer. Let $L(P_1)$, $L(P_2)$, $L(P_3)$ which are denoted as luminosity coefficients of the primaries, be the luminances of unit amounts of the three primaries $\{P_1\}$, $\{P_2\}$, $\{P_3\}$. Since $T_1(C)$, $T_2(C)$, $T_3(C)$ units of each primary are involved in the color match, the luminance of a colored light is given by

$$L(C) = T_1(C)L(P_1) + T_2(C)L(P_2) + T_3(C)L(P_3) \qquad (2\text{-}23)$$

If the color to be matched is one watt of a spectral color at a wavelength λ_k, then the luminance of the spectral color is defined as [1, p. 371]

$$680V(\lambda_k) = T_1(\lambda_k)L(P_1) + T_2(\lambda_k)L(P_2) + T_3(\lambda_k)L(P_3) \qquad (2\text{-}24)$$

where $V(\lambda_k)$ is called the luminosity function $V(\lambda)$ evaluated at $\lambda = \lambda_k$. Following eq. (2-23), the luminance of a reference white is

$$L(W) = T_1(W)L(P_1) + T_2(W)L(P_2) + T_3(W)L(P_3) \qquad (2\text{-}25)$$

Multiplying both sides of eq. (2-25) by $T_1(W)$ and rearranging gives

$$T_1(W) = \frac{L(W)T_1(W)}{T_1(W)L(P_1) + T_2(W)L(P_2) + T_3(W)L(P_3)} \qquad (2\text{-}26)$$

Or, in terms of the chromaticity coordinates, $t_1(W)$, $t_2(W)$, $t_3(W)$ of the white

$$T_1(C) = \frac{t_1(C)L(C)}{t_1(C)L(P_1) + t_2(C)L(P_2) + t_3(C)L(P_3)} \tag{2-29a}$$

$$T_2(C) = \frac{t_2(C)L(C)}{t_1(C)L(P_2) + t_2(C)L(P_2) + t_3(C)L(P_3)} \tag{2-29b}$$

$$T_3(C) = \frac{t_3(C)L(C)}{t_1(C)L(P_1) + t_2(C)L(P_2) + t_3(C)L(P_3)} \tag{2-29c}$$

A third approach in specifying a color is to represent the color by some linear or nonlinear invertible function of its tristimulus or chromaticity values. From equation (2-22) it is evident that a linear function of the tristimulus values is simply a conversion to a new set of primaries. Appendix A describes methods of linear color coordinate conversion.

In practice many different coordinate systems are employed for the specification and analysis of color. Table 2-1 summarizes the notation for the color coordinate systems to be considered in this report. Appendix B contains examples of an image displayed in these color coordinates. These coordinate systems are discussed in greater detail below.

C.I.E. Spectral Primary Color Coordinate System [1,3]

In 1931 the Commission Internationale de l'Eclairage, an international body established to provide common standards for colorimetry, developed a standard primary reference system with three monochromatic primaries at wavelengths: red = 700nm.; green = 546.1nm.; and blue = 435.8nm. The units of the tristimulus values are such that the tristimulus values R_c, G_c, and B_c are equal when matching an equal-energy white throughout the visible spectrum. The primary system is defined by tristimulus curves of the spectral colors, as shown in Figure 2-7, similar to the curves of Figure 2-4. These curves have been obtained indirectly by experimental color matching experiments performed by a number of

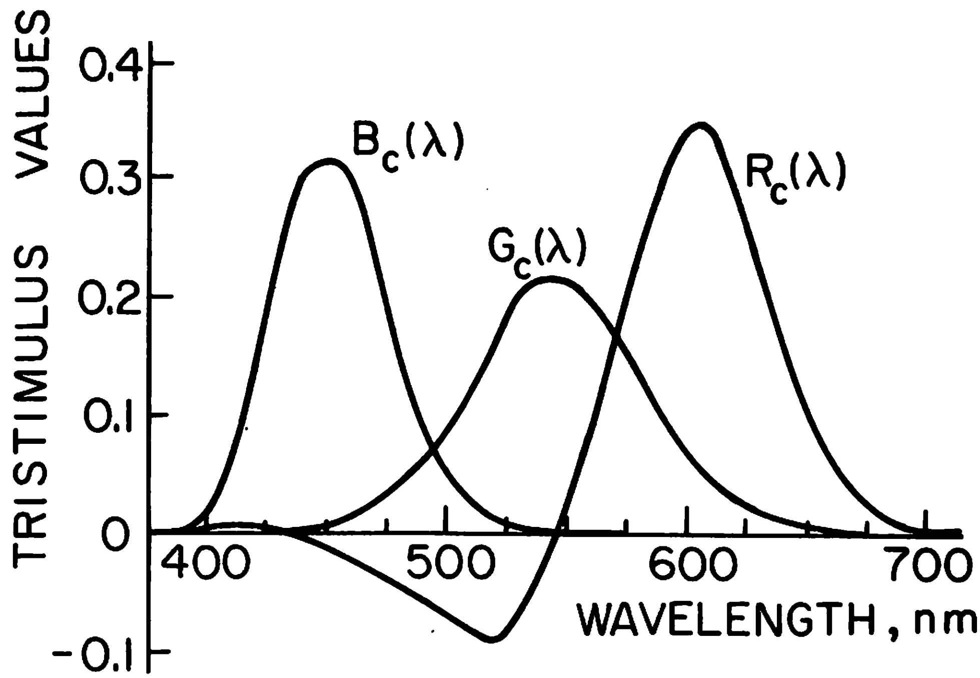


Figure 2-7. Tristimulus Values of C.I.E. Spectral Primaries Required to Match Unit Energy Throughout the Spectrum Red=700nm., Green=546.1nm., and Blue=435.8nm.

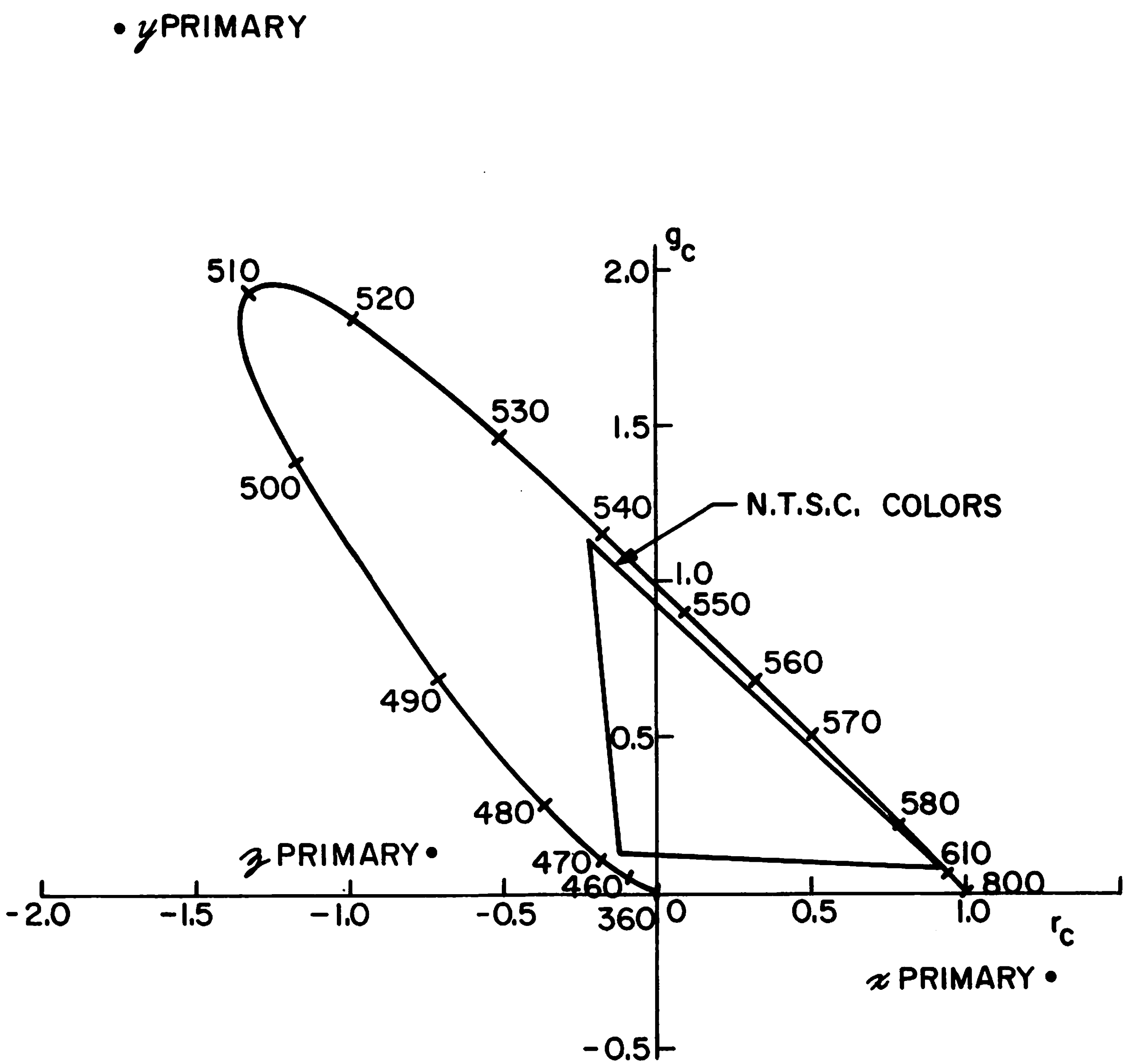


Figure 2-8. Chromaticity Diagram for C.I.E. Spectral Primary System

artificial primary coordinate system in which all tristimulus values required to match the spectral colors are positive. These artificial primaries are shown in the C.I.E. spectral primary chromaticity diagram of Figure 2-8. The XYZ system primaries have been chosen so that the Y tristimulus value is equivalent to the luminance, L(C), of the color. Figure 2-10 contains the chromaticity diagram for the C.I.E. XYZ primary system. The XYZ coordinate system primaries are referenced to the equal-energy white.

N.T.S.C. Transmission Color Coordinate System [2]

In the development of the United States color television system, the N.T.S.C. formulated a color coordinate system composed of three tristimulus values Y,I,Q. The Y tristimulus value is the same as the Y of the X,Y,Z coordinate system; it is a measure of the luminance of the color. The remaining two tristimulus values I and Q jointly describe the hue and saturation of the image. The reasons for transmitting the Y,I,Q tristimulus values rather than the R_N, G_N, B_N tristimulus values directly from the color camera are twofold; the Y signal alone can be used with existing receivers without modification to display monochrome images; and, it is possible to limit the bandwidth of the I and Q signals, without noticeable image degradation. As a result of this latter property, by a clever modulation scheme, the bandwidth of the analog color television carrier can be limited to the same bandwidth as a monochrome carrier.

C.I.E. Uniform Chromaticity Scale Color Coordinate System

A desirable property for any color coordinate system is that any unit change in the chromaticity diagram of that system should be perceived as an equivalently noticeable color shift to an observer. Figure 2-11 illustrates the contours of twenty just noticeable color differences about a point in the xy chromaticity diagram. This figure and other experimental results [14, 15] indicate that the human viewer is most sensitive to color

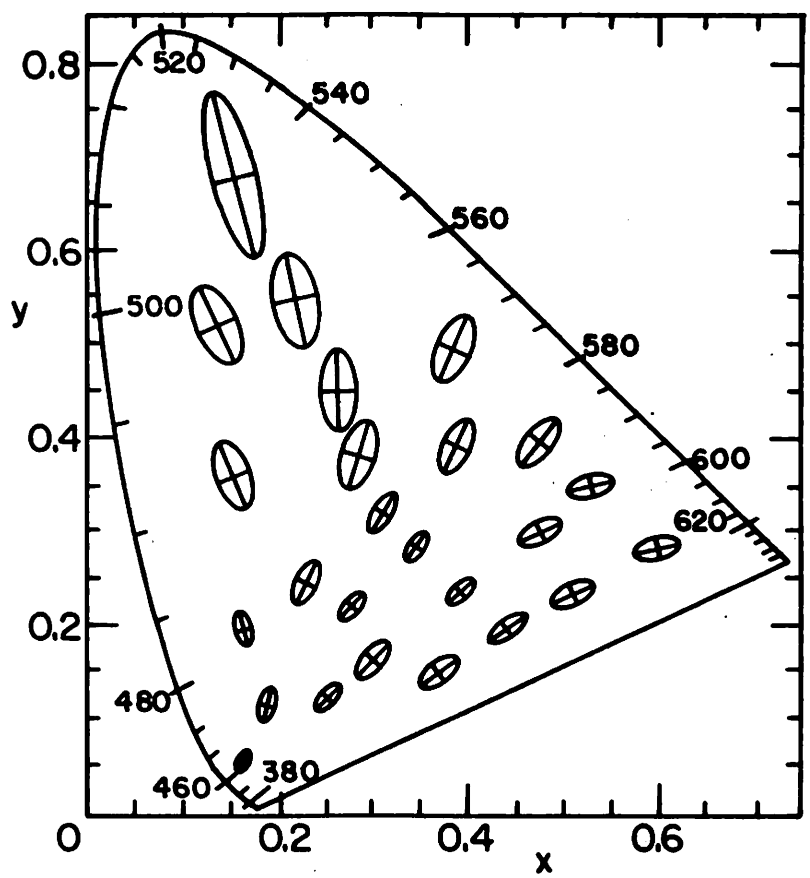


Figure 2-11. Contours of Just Noticeable Color Differences in the xy Chromaticity Diagram (after MacAdam, Ref. [14])

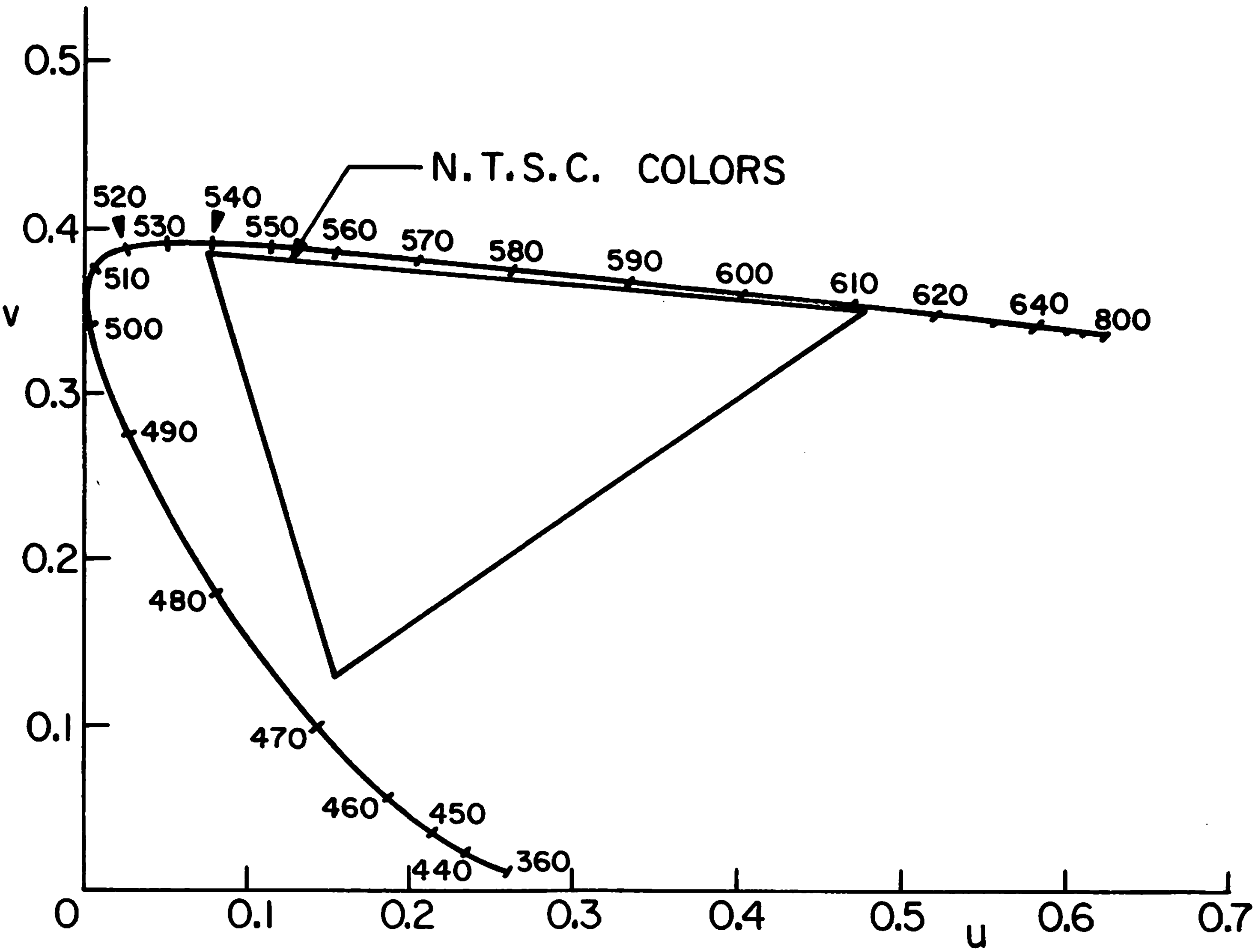


Figure 2-12. Chromaticity Diagram for C.I.E. Uniform Chromaticity Scale Primary System

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{12} & u_{22} & u_{23} \\ u_{13} & u_{23} & u_{33} \end{bmatrix} \begin{bmatrix} m_{11} & m_{21} & m_{31} \\ m_{12} & m_{22} & m_{32} \\ m_{13} & m_{23} & m_{33} \end{bmatrix} = \begin{bmatrix} \Lambda_1 & 0 & 0 \\ 0 & \Lambda_2 & 0 \\ 0 & 0 & \Lambda_3 \end{bmatrix} \quad (2\text{-}32)$$

where $\Lambda_1, \Lambda_2, \Lambda_3$ are the eigenvalues of the covariance matrix and

$$u_{11} \equiv E\{(R_N - \bar{R}_N)^2\}$$
$$u_{22} \equiv E\{(G_N - \bar{G}_N)^2\}$$
$$u_{33} \equiv E\{(B_N - \bar{B}_N)^2\}$$
$$u_{12} \equiv E\{(R_N - \bar{R}_N)(G_N - \bar{G}_N)\}$$
$$u_{13} \equiv E\{(R_N - \bar{R}_N)(G_N - \bar{G}_N)\}$$
$$u_{23} \equiv E\{(G_N - \bar{G}_N)(B_N - \bar{B}_N)\}$$

2.3 Color Coordinate Selection

It is evident from the previous section that there are a number of color coordinate systems that could be employed for the representation of a color. Unfortunately, there appears to be no technique for determining an "optimum" coordinate system for most applications. Rather, one must state the characteristics of the desired coordinate system, and then

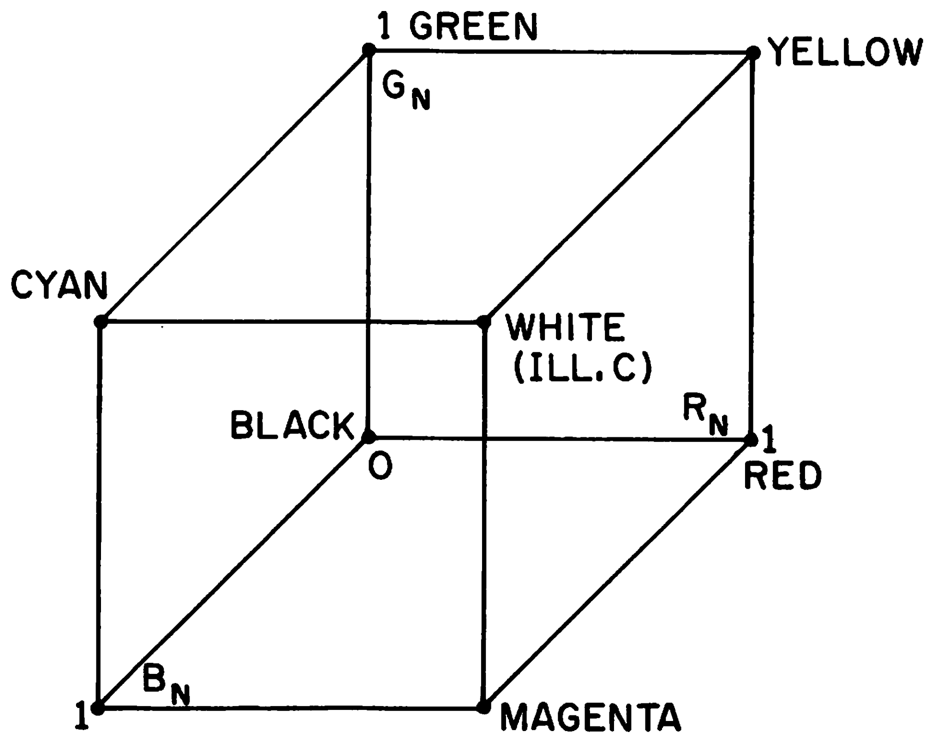


Figure 2-13. Tristimulus Color Solid
for N.T.S.C. Receiver Primary System

Table 2-2 Continued

U_R	0.000	0.054	0.919	0.974	0.919	0.054	0.974	0.000
V_R	0.513	3.469	1.796	5.264	2.309	3.982	5.777	0.000
W_R	2.474	1.800	0.236	2.036	2.710	4.273	4.510	0.000
u_R	0.000	0.010	0.312	0.118	0.155	0.007	0.086	-
v_R	0.172	0.652	0.608	0.636	0.389	0.479	0.513	-
w_R	0.828	0.338	0.080	0.246	0.456	0.514	0.401	-
Y	0.300	0.591	0.120	0.711	0.421	0.891	1.000	0.000
I	0.602	-0.279	-0.317	-0.596	0.285	0.323	0.000	0.000
Q	0.210	-0.520	0.312	-0.208	0.522	-0.310	0.000	0.000
U*	217.358	-130.319	-24.558	-108.566	100.385	22.144	0.278	-
V*	35.461	79.703	-90.508	-17.913	-61.336	80.125	-0.121	-
W*	60.594	80.160	39.265	86.081	69.415	94.451	99.040	-
S	220.232	152.760	93.781	110.034	117.640	83.128	0.303	-
Θ(rad)	0.161	2.609	4.445	3.304	5.733	1.280	5.869	-
W*	60.594	80.160	39.265	86.081	69.415	94.451	99.040	-
K_1	0.575	0.615	0.540	1.155	1.114	1.190	1.730	0.000
K_2	0.608	0.120	-0.785	-0.665	-0.177	0.728	-0.057	0.000
K_3	0.548	-0.779	0.305	-0.474	0.853	-0.232	0.073	0.000

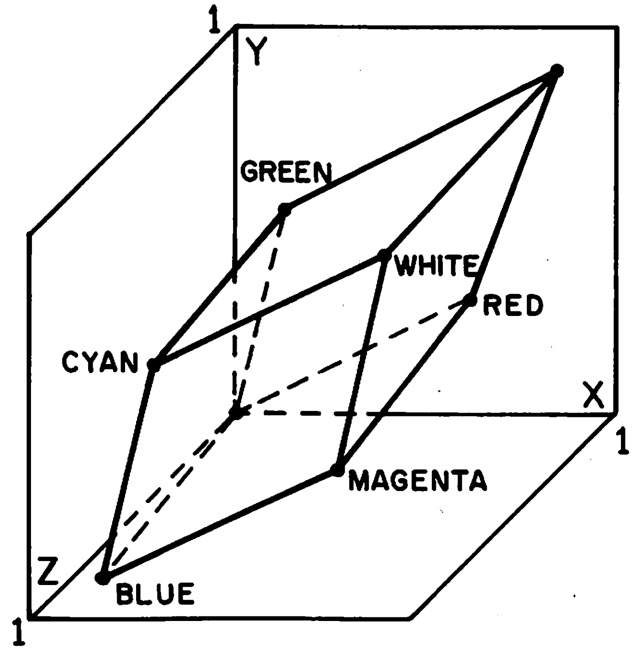


Figure 2-14. Tristimulus Color Solid for C.I.E. X,Y,Z Primary System

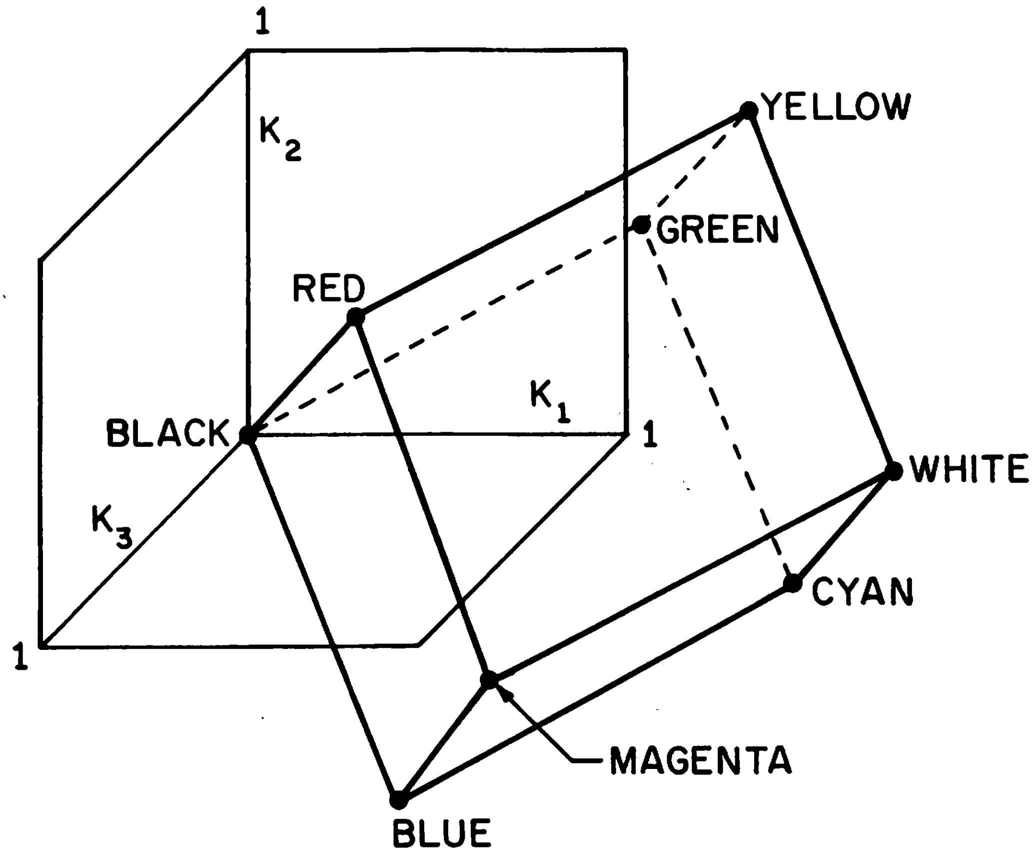


Figure 2-16. Tristimulus Color Solid for Karhunen-Loeve Primary System

MacAdam [5, 6] has proven that the maximum color gamut of an object occurs when the object possesses a reflection, or transmission, characteristic of the type shown in Figure 2-18. Such objects do not physically exist, hence the gamut of physical object colors will be even less than that indicated by objects with ideal reflection or transmission characteristics. For simplicity, consideration will only be given to reflective objects. The same results also apply to transmissive objects.

Consider a color $\{C\}$ to be specified by its X, Y, Z tristimulus values. Following eqs. (2-1) and (2-13), the tristimulus values are given by

$$X = \int_{\lambda_L}^{\lambda_U} r_o(\lambda_k) E(\lambda_k) X(\lambda_k)\, d\lambda_k$$

$$Y = \int_{\lambda_L}^{\lambda_U} r_o(\lambda_k) E(\lambda_k) Y(\lambda_k)\, d\lambda_k$$

$$Z = \int_{\lambda_L}^{\lambda_U} r_o(\lambda_k) E(\lambda_k) Z(\lambda_k)\, d\lambda_k$$

where $r_o(\lambda_k)$ is the object reflectance as a function of wavelength, $E(\lambda_k)$ is the spectral energy density of the object illumination, and $X(\lambda_k)$, $Y(\lambda_k)$, $Z(\lambda_k)$ are the tristimulus values of one unit of monochromatic light as a wavelength λ_k. The visual efficiency, β, is defined as the ratio of the luminance of an object with a reflectance, $r_o(\lambda_k)$, to the luminance of an object with unity reflectance over all wavelengths. Hence

$$\beta \equiv \frac{\int_{\lambda_L}^{\lambda_U} r_o(\lambda_k) E(\lambda_k) Y(\lambda_k)\, d\lambda_k}{\int_{\lambda_L}^{\lambda_U} E(\lambda_k)\, Y(\lambda_k)\, d\lambda_k}$$

If the object has a reflectance characteristics of the type shown in Figure 2-18a, then the luminance of the object is

$$Y = \int_{\lambda_1}^{\lambda_2} E(\lambda_k) Y(\lambda_k) d\lambda_k = \beta \int_{\lambda_L}^{\lambda_U} E(\lambda_k) Y(\lambda_k) d\lambda_k$$

Suppose that the spectral energy distribution, $E(\lambda)$, of the object illuminant is known. Then, for a given value of the visual efficiency, β, the object luminance may be computed. For an arbitrary lower limit on the wavelength of the object's reflectance, λ_1, the upper limit may be found by numerical techniques to satisfy

$$\int_{\lambda_L}^{\lambda_2} E(\lambda_k) Y(\lambda_k) d\lambda_k = Y + \int_{\lambda_L}^{\lambda_1} E(\lambda_k) Y(\lambda_k) d\lambda_k$$

With this value for λ_2, the tristimulus values X and Z may be found from

$$X = \int_{\lambda_1}^{\lambda_2} E(\lambda_k) X(\lambda_k) d\lambda_k$$

$$Z = \int_{\lambda_1}^{\lambda_2} E(\lambda_k) Z(\lambda_k) d\lambda_k$$

The chromaticity values x and y may then be determined from the tristimulus values. If this procedure is followed for all λ_1 in the visible region ($\lambda_L \leq \lambda_1 \leq \lambda_U$) for given values of the visual efficiency, β, it is possible to obtain the locus of object colors in the tristimulus or luminance/chrominance color solids. Figure 2-19 shows a perspective view of the object colors in the luminance/chrominance color solid for illumination by Illuminant C. It should be noted that the maximum luminance only occurs for the reference white illuminant.

2.3.2 Color Difference

A recurring problem in colorimetry is to quantitatively determine the difference between colors based upon their tristimulus or luminance/ chrominance values. There are two possible types of color difference measure: deterministic and statistical. Furthermore, the color difference measure may be global, i.e., valid for all colors; or may be local, i.e., restricted to small color differences.

A deterministic color difference measure may be obtained by considering the tristimulus color solid to be an Euclidean space. Then, a color $\{C_1\}$ specified by tristimulus values T_1, T_2, T_3 is separated by an incremental distance $\Delta\{C_1, C_2\}$ from a color $\{C_2\}$ with tristimulus values $T_1 + \Delta T_1, T_2 + \Delta T_2, T_3 + \Delta T_3$ as given by

$$\Delta^2\{C_1, C_2\} = (\Delta T_1)^2 + (\Delta T_2)^2 + (\Delta T_3)^2 \qquad (2\text{-}35)$$

If this color difference measure is to be subjectively valid, the incremental color difference $\Delta\{C_1, C_2\}$ should correspond to equal "just noticeable color differences", (jncd), for equal shifts $\Delta T_1, \Delta T_2, \Delta T_3$ in the tristimulus values for all tristimulus values. In other words, the equation, $\Delta^2\{C_1, C_2\} \equiv 1$, should represent a sphere of unit radius of one jncd about T_1, T_2, T_3. This is not the case when the tristimulus values correspond to red, green, and blue primaries. Subjective testing indicates that the human viewer is most sensitive to unit changes in the blue tristimulus value and least sensitive to unit changes in the green tristimulus value.

As a next step to correct eq. (2-35) to better correspond to the results of subjective testing, one might consider the tristimulus color solid to be a Riemannian space. In a Riemannian color space, the incremental color difference is defined as and the loci of jncd are spheres centered about $\tilde{T}_1, \tilde{T}_2, \tilde{T}_3$. It has been shown [7, p. 513] that, in general, no coordinate conversion for the desired mapping exists. However, several linear coordinate conversions with approximately the desired properties have been postulated. The C.I.E. has adopted a tristimulus coordinate system, called the U, V, W system, in which the incremental distance between the chromaticities of a color is uniform. The incremental chromaticity difference between colors with chromaticities u_1, v_1, and u_2, v_2 is defined as

$$\Delta^2\{u_1, v_i, u_2, v_2\} = k_1(\Delta u)^2 + k_2(\Delta v)^2 \qquad (2\text{-}39)$$

where $\Delta u = u_1 - u_2$, $\Delta v = v_1 - v_2$, and where k_1 and k_2 are scaling constants. In this coordinate system the incremental color distance

$$\Delta^2\{C_1, C_2\} = k_1(\Delta U)^2 + k_2(\Delta V)^2 + k_3(\Delta W)^2 \qquad (2\text{-}40)$$

is not uniform; the human viewer is more sensitive to color changes of dim colors than to bright colors. In an effort to accommodate this characteristic, the C.I.E. has provisionally adopted the U*, V*, W* coordinate system as a standard in which chromaticity and brightness changes are expected to be uniformly noticeable. The color difference in the U*, V*, W* coordinate system is defined as

$$\Delta^2\{C_1, C_2\} = k_1(\Delta U^*)^2 + k_2(\Delta V^*)^2 + k_3(\Delta W^*)^2 \qquad (2\text{-}41)$$

where

$$\Delta U^* \equiv U_1^* - U_2^*, \quad \Delta V^* \equiv V_1^* - V_2^*,$$

and

$$\Delta W^* \equiv W_1^* - W_2^*$$

One difficulty with the U*, V*, W* color difference measure is that the measure becomes

$$E\{f(d\{C_1, C_2\})\}$$

The usual statistical measure for the color difference is the mean square color difference. The reason for the choice is twofold: the mean square color difference expression is much more analytically tractable than other functions that might be chosen; and also there is a great deal of statistical information available on mean square difference measures.

In the next section, use will be made of the mean square incremental luminance/chrominance difference and color difference expressions in the U, V, W and U*, V*, W* coordinate systems, respectively. These expressions are defined below. In the U, V, W coordinate system the mean square incremental chromaticity difference is given by

$$E\{\Delta^2\{u_1, v_1; u_2, v_2\}\} \equiv k_1 \overline{(\Delta u)^2} + k_2 \overline{(\Delta v)^2} \tag{2-44}$$

where the overbar indicates the expectation operator. The corresponding mean square incremental luminance difference is

$$E\{\Delta^2\{Y_1, Y_2\}\} \equiv k_3 \overline{(\Delta Y)^2} \tag{2-45}$$

The mean square incremental color difference in the U*, V*, W* coordinate system is defined as

$$E\{\Delta^2\{C_1, C_2\}\} = k_1 \overline{(\Delta U^*)^2} + k_2 \overline{(\Delta V^*)^2} + k_3 \overline{(\Delta W^*)^2} \tag{2-46}$$

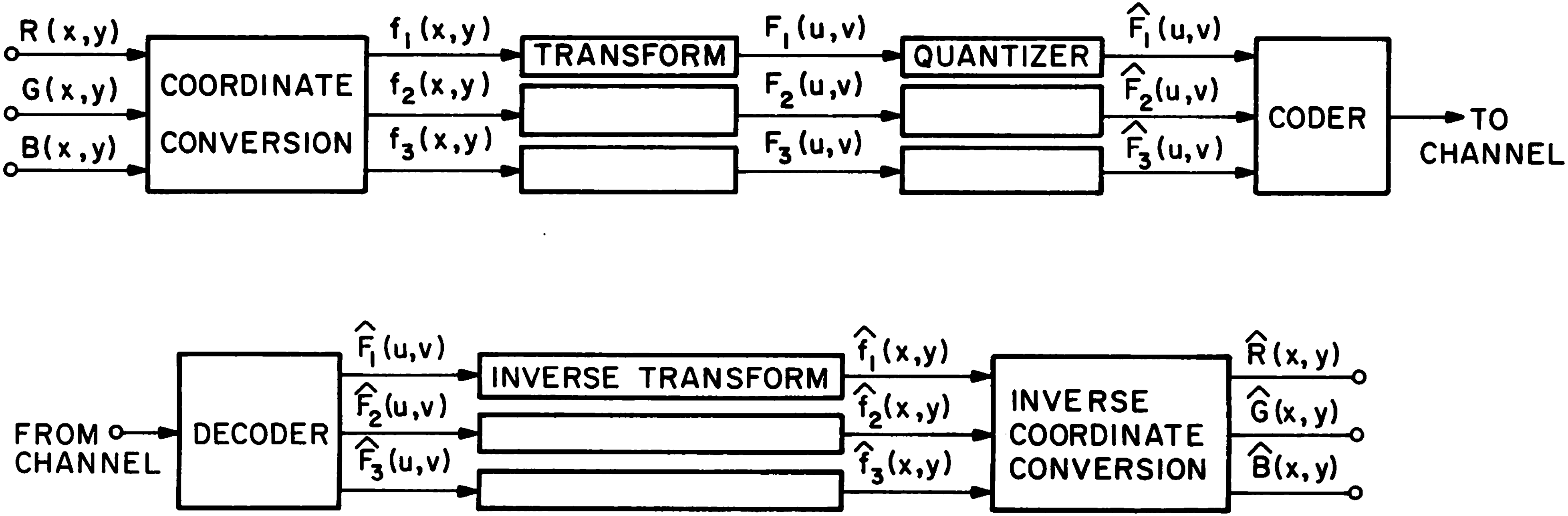


Figure 3-1. Color Image Transform Coding System

the data array, respectively. If $\underline{A}_x$ or $\underline{A}_y$ is an identity matrix, the transformation becomes one dimensional. Table 3-1 defines several unitary transforms that have been used for various data processing and coding applications.

In the transform coding system, it is the quantization process that enables a bandwidth reduction to be achieved. There are two basic strategies for efficiently quantizing and coding transform domain samples: zonal sampling and threshold sampling [26]. With zonal sampling, various zones are established in the transform domain, and each sample within the zone is coded with the same number of bits. The number of bits assigned to each zone is proportional to the assumed variance of samples within the zone. A simpler, but suboptimal procedure, is to employ only two zones, e.g., zonal low pass filtering in which the low frequency samples are all coded with the same number of bits, and the high frequency samples are discarded. Threshold sampling is an adaptive technique in which a threshold is established, and only those transform domain samples whose amplitudes are greater than the threshold are coded. With either technique, zonal or threshold sampling, it is generally possible to digitally code the transform plane with a fewer number of bits than the data array, f(x,y), requires, while limiting the quantization error to acceptable limits.

In the performance evaluation of transform coding techniques, it is useful to consider the data matrix $\underline{f}$ as a discrete two dimensional random process with known second order statistics. For separable transforms, it is only necessary to consider the statistical properties of $\underline{f}$ along its rows and columns. Let $\underline{C}_{f_x}$ and $\underline{C}_{f_y}$ denote the covariance matrices of rows and columns of $\underline{f}$, respectively. Then, the covariance matrices of the rows and columns of the transformed data array, $\underline{F}$, are given by

$$\underline{C}_{F_x} = \underline{A}_x \underline{C}_{f_x} \underline{A}_x^{-1} \qquad (3\text{-}2)$$

and

$$\underline{C}_{F_y} = \underline{A}_y \underline{C}_{f_y} \underline{A}_y^{-1} \tag{3-3}$$

A matrix of the variances of the elements of $\underline{F}$ may then be defined as

$$\underline{V}_F = \underline{V}_{F_x} \underline{V}_{F_y}^T \tag{3-4}$$

where

$$\underline{V}_{F_x}^T = \left[C_{F_x}(1,1),\ C_{F_x}(2,2), \ldots, C_{F_x}(N,N) \right]$$

and

$$\underline{V}_{F_y}^T = \left[C_{F_y}(1,1),\ C_{F_y}(2,2), \ldots, C_{F_y}(N,N) \right]$$

It is common to evaluate the performance of transform coding systems by considering the mean square error between the original data array $f(x,y)$ and the reconstructed data array $\hat{f}(x,y)$ as defined by

$$\mathcal{E}^2 = \frac{1}{N^2} E \left\{ \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} [f(x,y) - \hat{f}(x,y)]^2 \right\} \tag{3-5}$$

For a given sampling and quantization scheme, the mean square error can be expressed in terms of the variances $V_F(u,v)$, of the transform domain samples as

$$\mathcal{E}^2 = \frac{1}{N^2} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} V_F(u,v) \; [Q(u,v)] \tag{3-6}$$

where $Q(u,v)$ is a factor proportional to the quantization error of each transform domain sample. As a result of the summing operations of the unitary spatial transforms, the transform domain samples are approximately Gaussianly distributed. Under this condition, the quantization error measure, for uniformly spaced quantization levels, is given by [17,27]

$$
\begin{aligned}
T_1(x,y) &= m_{11}R(x,y) + m_{12}G(x,y) + m_{13}B(x,y) \\
T_2(x,y) &= m_{21}R(x,y) + m_{22}G(x,y) + m_{23}B(x,y) \\
T_3(x,y) &= m_{31}R(x,y) + m_{32}G(x,y) + m_{33}B(x,y)
\end{aligned} \tag{3-9}
$$

where the m_{ij} are constants, and T_1, T_2, T_3 are considered tristimulus values for a new set of primaries. For an electrical representation, the new set of primaries need not be physically realizeable since an inverse coordinate conversion will be performed before display. Another common type of color representation is in terms of the luminance of an image point given by

$$Y(x,y) = \alpha_1 R(x,y) + \alpha_2 G(x,y) + \alpha_3 B(x,y) \tag{3-10}$$

where the α_k are constants; and by the chromaticity values

$$t_1(x,y) = \frac{T_1(x,y)}{T_1(x,y) + T_2(x,y) + T_3(x,y)} \tag{3-11}$$

$$t_2(x,y) = \frac{T_2(x,y)}{T_1(x,y) + T_2(x,y) + T_3(x,y)} \tag{3.12}$$

A final type of color image representation to be considered is some set of nonlinear, but invertible, function of the image sensor tristimulus values.

The color image coordinate system employed for transform coding should ideally:

a) provide color signal planes that can be spatially filtered and coded with a minimal loss of apparent spatial resolution;

b) provide color signals that are uncorrelated so that they can be efficiently processed separately;

Table 3-2
Color Image Representations

I. Source Tristimulus Values

N.T.S.C. receiver phosphor primaries:

R, G, B $\qquad 0 \le R \le 1$, $0 \le G \le 1$, $0 \le B \le 1$

II. Tristimulus Conversion

C.I.E. uniform chromaticity scale:

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} .405 & .116 & .133 \\ .299 & .587 & .114 \\ .145 & .827 & .627 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

note: V = Y = luminance signal

N.T.S.C. transmission primaries:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .299 & .587 & .114 \\ .596 & -.274 & -.322 \\ .211 & -.523 & .312 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

Karhunen-Loeve coordinate system: (experimental example)

The rows of m_{ij} are the eigenvectors of the covariance matrix of the R,G,B tristimulus values.

$$\begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} .575 & .615 & .540 \\ .608 & .120 & -.785 \\ .548 & -.779 & .305 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

III. Luminance/chrominance conversion

C.I.E. uniform chromaticity scale:

$Y = .299R + .587G + .114B \qquad 0 \le Y \le 1$

$$u = \frac{U}{U + V + W}$$

$$v = \frac{V}{U + V + W}$$

IV. Nonlinear conversion

U*, V*, W* coordinate system:

$U^* = 13W^*(u - 0.201)$

$V^* = 13W^*(v - 0.307)$

$W^* = 25(100Y)^{1/3} - 17 \qquad 0 \le Y \le 1$

difference, between images. Many different color difference measures have been postulated, but no single measure has been adopted as standard. Perhaps, the most universally accepted color difference measure, and the one adopted here, is the measure based upon the U*, V*, W* color coordinate system [1]. By this measure, the color difference between two colors is defined as

$$D^2 \equiv (U_1^* - U_2^*)^2 + (V_1^* - V_2^*)^2 + (W_1^* - W_2^*)^2 \tag{3-13}$$

A cumulative average color difference over the entire color image may be defined as

$$D_C^2 \equiv \frac{1}{3N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left[(U_1^* - U_2^*)^2 + (V_1^* - V_2^*)^2 + (W_1^* - W_2^*)^2\right] \tag{3-14}$$

If the color signal planes are considered as stochastic planes of data, then a mean square color difference

$$\overline{D_C^2} \equiv E\{D_C^2\} \tag{3-15}$$

becomes meaningful.

3.4 Color Image Transform Coding

Consider the color image transform coding system of Figure 3-1 in which the color coordinate conversion, as given by eq. (3-7), is linear. The three color signals planes can be expressed as

$$\begin{aligned} \underline{f1} &= m_{11}\underline{R} + m_{12}\underline{G} + m_{13}\underline{B} \\ \underline{f2} &= m_{21}\underline{R} + m_{22}\underline{G} + m_{23}\underline{B} \\ \underline{f3} &= m_{31}\underline{R} + m_{32}\underline{G} + m_{33}\underline{B} \end{aligned} \tag{3-16}$$

Again, the order of the inverse color coordinate conversion and the inverse spatial transformation is unimportant.

The design procedure for the color image transform coding system of Figure 3-1 consists of: (a) the selection of the color coordinate conversion matrix; (b) the choice of the unitary transform for each color signal plane; and (c) the specification of the quantization law for transform domain samples. Consideration will first be given to analytical design techniques. This will be followed, in the next section, by a presentation of experimental results and a subjective assessment of performance.

Assuming the mean square color difference measure of eq. (3-12) to be a valid measure of color image quality, then the system design should be performed to minimize the error

$$\mathcal{E}_c^2 = \frac{1}{3N^2} E\left\{ \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left[(U^*(x,y) - \hat{U}^*(x,y))^2 + (V^*(x,y) - \hat{V}^*(x,y))^2 + (W^*(x,y) - \hat{W}^*(x,y))^2 \right] \right\} \tag{3-20}$$

where U^*, V^*, W^* and $\hat{U}^*$, $\hat{V}^*$, $\hat{W}^*$ are the color coordinates of the original and reconstructed image, respectively, in the U^*, V^*, W^* coordinate system. This minimization does not appear to be analytically feasible because of the nonlinear relationship between the U^*, V^*, W^* and the R, G, B coordinates. As an alternative approach the analytic design procedure adopted has been to select the combination of the color coordinate conversion matrix, and the spatial, unitary transform matrices to provide the greatest compaction of image energy into the fewest transform domain samples. Then, the quantization law has been selected to minimize the total mean square error

$$\mathcal{E}_f^2 = \frac{1}{3N^2} \sum_{k=1}^{3} E\{[f_k(x,y) - \hat{f}_k(x,y)]^2\} \tag{3-21}$$

of the color signal planes.

$$K_{22} \equiv m_{21}^2\sigma_R^2 + m_{22}^2\sigma_G^2 + m_{23}^2\sigma_B^2 + 2m_{21}m_{22}C_{RG} + 2m_{21}m_{23}C_{BR} + 2m_{22}m_{23}C_{GB}$$

$$K_{33} \equiv m_{31}^2\sigma_R^2 + m_{32}^2\sigma_G^2 + m_{33}^2\sigma_B^2 + 2m_{31}m_{32}C_{RG} + 2m_{31}m_{33}C_{BR} + 2m_{32}m_{33}C_{GB}$$

$$K_{12} \equiv m_{11}m_{21}\sigma_R^2 + m_{12}m_{22}\sigma_G^2 + m_{13}m_{23}\sigma_B^2 + (m_{11}m_{12} + m_{12}m_{21})C_{RG} + (m_{11}m_{23} + m_{13}m_{21})C_{RB} + (m_{12}m_{23} + m_{13}m_{22})C_{GB}$$

$$K_{13} \equiv m_{11}m_{31}\sigma_R^2 + m_{12}m_{32}\sigma_G^2 + m_{13}m_{33}\sigma_B^2 + (m_{11}m_{32} + m_{12}m_{31})C_{RG} + (m_{11}m_{33} + m_{13}m_{31})C_{RB} + (m_{12}m_{33} + m_{13}m_{32})C_{GB}$$

$$K_{23} \equiv m_{31}m_{21}\sigma_R^2 + m_{32}m_{22}\sigma_G^2 + m_{33}m_{23}\sigma_B^2 + (m_{31}m_{22} + m_{32}m_{21})C_{RG} + (m_{31}m_{23} + m_{33}m_{21})C_{RB} + (m_{32}m_{23} + m_{33}m_{22})C_{GB}$$

From eqs. (3-2) and (3-3) the covariance matrices of the rows and columns of the three transform domain planes are

$$\underline{C}_{P_x} = \underline{Ak}_x \underline{C}_{p_x} \underline{Ak}_x^{-1} \qquad (3\text{-}24)$$

and

$$\underline{C}_{P_y} = \underline{Ak}_y \underline{C}_{p_y} \underline{Ak}_y^{-1} \qquad (3\text{-}25)$$

Then, the variances of the transform planes are given by

$$\begin{aligned} \underline{V}_{F1} &= K_{11}\underline{V}_{P1_x} \underline{V}_{P1_y}^T \\ \underline{V}_{F2} &= K_{22}\underline{V}_{P2_x} \underline{V}_{P3_y}^T \\ \underline{V}_{F3} &= K_{33}\underline{V}_{P3_x} \underline{V}_{P3_y}^T \end{aligned} \qquad (3\text{-}26)$$

energy compaction, and greatly simplifies the system implementation. Figure 3-2 shows a comparison of the energy compaction properties of the Fourier, Hadamard, and Karhunen-Loeve transforms as a function of block size for data modeled as a Gauss-Markov process. A block size of about 16 by 16 elements appears adequate for most natural images.

The next problem of interest is to analyze various quantization and coding schemes that might be employed to achieve the bandwidth reduction. With the error measure of eq. (3-21) and the result of eq. (3-6), the total mean square error in the color signal planes as a result of quantization is seen to be

$$\mathcal{E}_f^2 = \frac{1}{3N^2} \sum_{k=1}^{3} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} V_{Fk}(u,v) \, [\, Q_k(u,v)] \tag{3-27}$$

where from eq. (3-7), for uniformly spaced quantization levels,

$$Q_k(u,v) = \exp \{ -\tfrac{1}{2} n_k(u,v) \ln 10 \} \tag{3-28}$$

The total number of bits allotted to the three color signal planes, N_T, is equal to

$$N_T = \sum_{k=1}^{3} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} n_k(u,v) \tag{3-29}$$

The bit assignment algorithms developed for monochrome images [28] can easily be modified for color images. Further study should be undertaken to determine bit assignment strategies that minimize the mean square error and provide subjectively good quality reconstructions.

Thus far, consideration has only been given to color image transform coding systems employing a linear color coordinate conversion. With the nonlinear systems listed in Table 3-2, the color coordinate conversion and the unitary, spatial transformation operations are not commutable. Furthermore, the nonlinear systems are not amenable to statistical analaysis. Hence, their performance evaluation can only be made on the basis of subjective tests.

Experimental Results

A series of experiments has been performed to subjectively determine the performance of the color image transform coding system using: a) various linear and nonlinear color coordinate systems; b) the Fourier, Hadamard, and Karhunen-Loeve spatial transforms; c) limitations in data block size; and, d) different quantization and coding techniques.

The first set of experiments has attempted to subjectively evaluate the performance of the transform coding system for the (R, G, B), (Y, I, Q) and (Y, u, v) color coordinate systems. In these experiments, each color signal plane was Fourier transformed over its full size, and zonal low pass filtering was then performed on the transform planes. For the (R, G, B) case the lowest spatial frequencies were deleted to obtain a sample reduction factor of 4.8:1. In the (Y, I, Q) and (Y, u, v) experiments, the Y sample reduction was 2:1 and the u, v sample reductions were 16:1 each. Thus, all three color images have been coded with an average of 3.75 bits per element, compared to the 18 bits/element of the original. Subjectively, the (Y, I, Q) and Y, u, v) processed images appear to be equivalent and of higher quality than the (R, G, B) processed image. The R, G, B, Y, I, Q, and Y, u, v results are shown in Plates 1b to 1d.

The next set of experiments has sought to evaluate the performance of the Hadamard transform for color coding. Plate 1e shows a color image reconstruction of the Hadamard coded (Y, I, Q) planes with zonal low pass filtering on each plane. The Y plane underwent a sample reduction of 2:1 and the I, Q planes, a reduction of 16:1 each. The resultant image is coded with an average of 3.75 bits/element. The experiment was repeated with a 16 x 16 element block size. The result shown in Plates 1f and 1g indicates no noticeable difference between the Fourier and Hadamard transforms for the full size image, nor any appreciable difference when the block size is

4. EFFECTS OF CHANNEL ERRORS FOR PCM TRANSMISSION

In the transmission of color images over digital communication links, consideration must be given to the effect of errors in the reconstructed images as a result of receiver noise. The general solution to this problem is many faceted: it depends on the type of images to be transmitted and their use; the error criterion; the methods of source and channel coding; and the statistical model for the communication channel. Consideration will be given to the practical case of color image transmission over a binary symmetric channel for color images represented by the standard three color primary system. Constant word length pulse code modulation (PCM) coding is assumed for each primary signal. No source coding for redundancy removal nor channel coding for error correction is considered.

Figure 4-1 contains the model of the color image transmission system under investigation. The camera produces three source tristimulus signals R_N, G_N, B_N. A coordinate conversion is performed to obtain three color signals, E_1, E_2, E_3, that are independently coded and transmitted over binary symmetric channels. The error corrupted color signals, E'_1, E'_2, E'_3, then undergo an inverse coordinate conversion to produce the reconstructed tristimulus signals R'_N, G'_N, B'_N.

4.1 Binary Symmetric Channel Errors

A binary symmetric channel may be characterized as reversing the state of transmitted bits with probability p, and correspondingly providing correct transmission with probability 1-p. Bit errors are independent of one another.

Consider an N element binary vector, K, that represents a single, linearly quantized color component, and is an input to the binary symmetric channel.

$$K = \sum_{i=0}^{N-1} k_i 2^i \qquad k_i \in \{0, 1\} \tag{4-1}$$

The channel output is given by

$$H = \sum_{i=0}^{N-1} h_i 2^i \qquad h_i \in \{0, 1\} \tag{4-2}$$

and the channel error is defined as

$$\mathcal{E} = H - K = \sum_{i=0}^{N-1} \mathcal{E}_i 2^i \qquad \mathcal{E}_i \in \{0, 1\} \tag{4-3}$$

In analyzing the effects of channel errors on color transmission, the prime interest is the expected color shift and the variance of the color shift rather than the average color of an image point. Therefore, the statistics of interest are the first and second conditional moments of the output binary vector.

The conditional mean of the output vector can be expressed in terms of the conditional mean of each bit of the output vector as

$$E\{H|K\} = \sum_{i=0}^{N-1} 2^i E\{h_i | k_i\} \tag{4-4}$$

The channel input bit, k_i, is reversed in state to, $(1-k_i)$, with probability (p) and kept in the same state, (k_i), with probability (1-p). Therefore, the conditional mean of the channel output is

$$E\{h_i | k_i\} = p(1-k_i) + (1-p)k_i \tag{4-5}$$

Upon summing the series

$$E\{H|K\} = (1-2p)K + p(2^N - 1) \tag{4-6}$$

The channel error conditional mean is simply

and

$$\sigma^2\{\mathcal{E} \mid K\} = p(1-p)\left(\frac{4^N - 1}{3}\right) \qquad (4\text{-}14)$$

Upon taking the expectation of the mean and mean square error, as given by equations (4-7) and (4-13), with respect to the input vector, K, the variance of the error is found to be

$$\sigma^2\{\mathcal{E}\} = \left(\frac{4^N - 1}{3}\right) p(1-p) + 4p^2\sigma^2\{K\} \qquad (4\text{-}15)$$

The error variance is minimized when the level of each color component is uniformly distributed, i.e., $P\{k_i = 0\} = P\{k_i = 1\} = \frac{1}{2}$. Then,

$$E\{K\} = \frac{2^N - 1}{2} \qquad (4\text{-}16)$$

and

$$\sigma^2\{K\} = \frac{4^N - 1}{3}\, p \qquad (4\text{-}17)$$

Under this condition the expected error is

$$E\{\mathcal{E}\} = 0 \qquad (4\text{-}18)$$

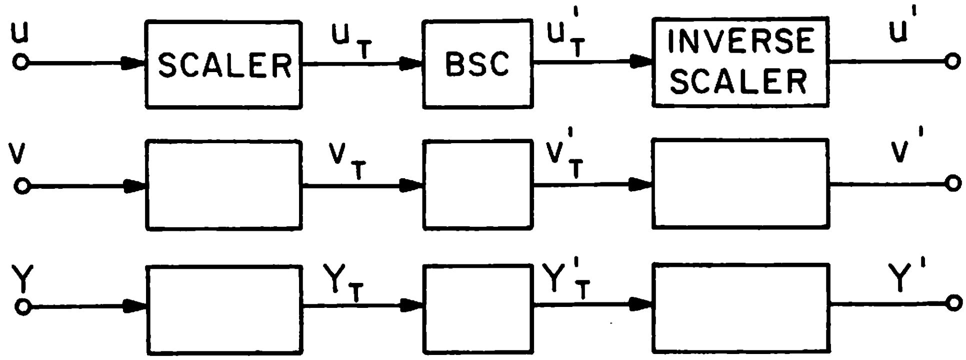
and the variance of the error is [33]

$$\sigma^2\{\mathcal{E}\} = \frac{4^N - 1}{3}\, p \qquad (4\text{-}19)$$

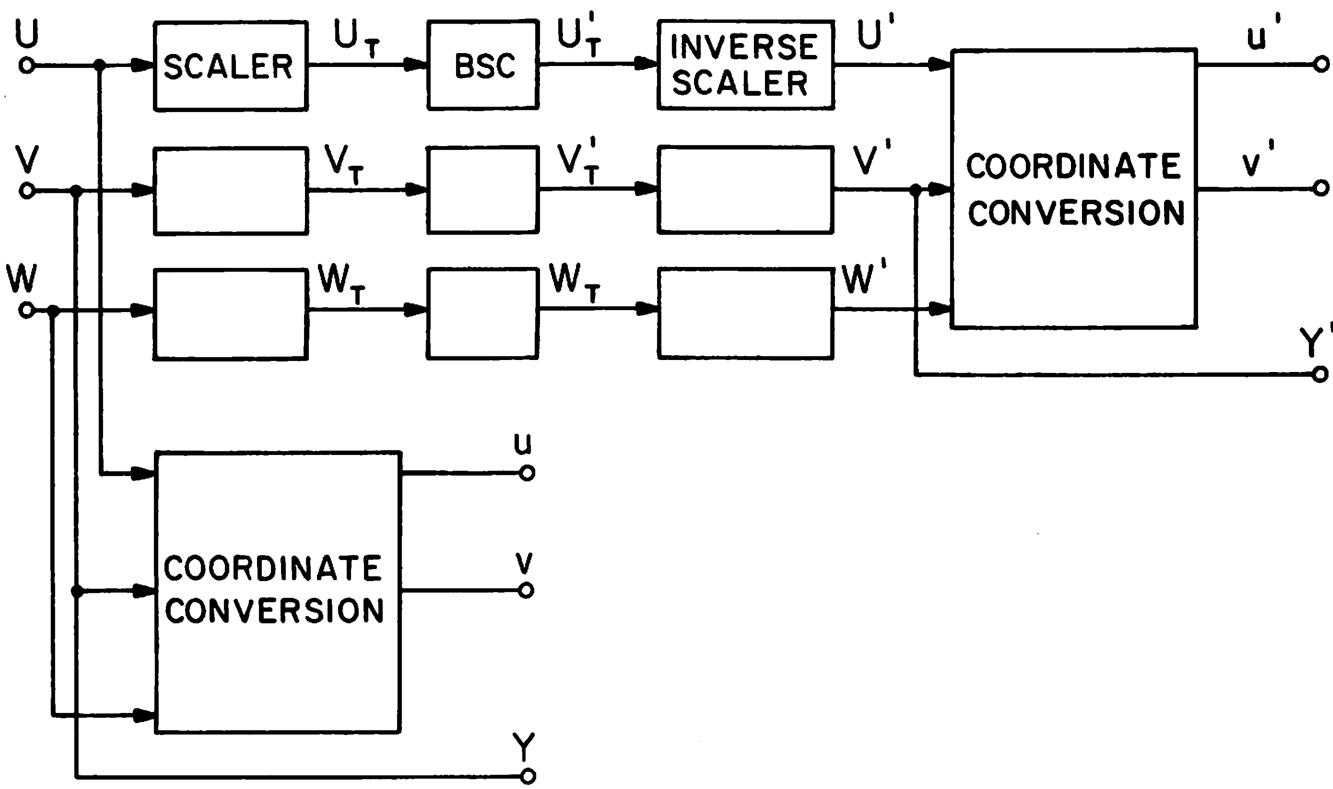
It should be noted, however, that for natural color images, the level of the tristimulus values or luminance/chrominance values is usually not uniform.

4.2 Color Image Transmission Errors

The effect of channel errors on the transmission of color images is dependent upon the color component system chosen for transmission and the analysis reference coordinate system. The color coordinate system



a. Case 1: U,u,v transmission



b. Case 2: U,V,W transmission

Figure 4-2. Models for Analysis of Channel Errors conditional expectation of eq. (4-21) with respect to the luminance signal, Y, one obtains

$$E\{Y'|Y\} = \left(\frac{Y_M - Y_N}{2^N - 1}\right) E\{Y_T'|Y\} + Y_N \tag{4-22}$$

Then, from eqs. (4-6) and (4-20) it is found that the mean luminance shift can be written as

$$E\{Y'|Y\} = (1-2p)Y + p(Y_M + Y_N) \tag{4-23}$$

In a similar manner the variance of the received luminance component conditioned on the transmitted luminance is

$$\sigma^2\{Y'|Y\} = \left(\frac{Y_M - Y_N}{2^N - 1}\right)^2 \sigma^2\{Y_T'|Y\} \tag{4-24}$$

From eq. (4-12) the conditional variance of the luminance shift is given by

$$\sigma^2\{Y'|Y\} = \left(\frac{Y_M - Y_N}{2^N - 1}\right)^2 p(1-p)\left(\frac{4^N - 1}{3}\right) \tag{4-25}$$

Figure 4-3 shows the effect of channel errors on the luminance of an image point. The luminance errors tend to produce a shift in the luminance toward a mid grey level.

The expected shifts in the u and v chromaticity values can be written directly from eq. (4-23) as

$$E\{u'|u\} = (1-2p)u + p(u_M + u_N) \tag{4-26}$$

and

$$E\{v'|v\} = (1-2p)v + p(v_M + v_N) \tag{4-27}$$

Similarly, from eq. (4-25) the variances of the u, v shifts are

$$\sigma^2\{u'|u\} = \left(\frac{u_M - u_N}{2^N-1}\right)^2 p(1-p)\left(\frac{4^N-1}{3}\right) \qquad (4\text{-}28)$$

and

$$\sigma^2\{v'|v\} = \left(\frac{v_M - v_N}{2^N-1}\right)^2 p(1-p)\left(\frac{4^N-1}{3}\right) \qquad (4\text{-}29)$$

Figure 4-4 contains a u, v chromaticity diagram illustrating the mean color shifts for several colors, including the N.T.S.C. major colors (red, green, blue, cyan, magenta, yellow, white). The channel errors are seen to shift the color toward magenta.

Case 2: U, V, W channel input

The transmitted luminance component, (V=Y), for case 2 is the same as for case 1. The measurable chromaticity coordinates are given by

$$u' = \frac{U'}{U' + V' + W'} \qquad (4\text{-}30)$$

$$v' = \frac{V'}{U' + V' + W'} \qquad (4\text{-}31)$$

After inverse scaling the receiver chromaticity coordinates become

$$u' = \frac{a_1 U'_T + a_2}{a_3 U'_T + a_4 V'_T + a_5 W'_T + a_6} \qquad (4\text{-}32)$$

$$v' = \frac{a_7 V'_T + a_8}{a_3 V'_T + a_4 V'_T + a_5 W'_T + a_6} \qquad (4\text{-}33)$$

where

$$a_1 = a_3 = U_M - U_N$$

$$a_2 = (2^N - 1)U_N$$

$$a_4 = a_7 = V_M - V_N$$

$$a_5 = W_M - W_N$$

$$a_6 = (U_N + V_N + W_N)(2^N - 1)$$
$$a_8 = (2^N - 1)V_N$$

Since eq. (4-32) and (4-33) are of the same analytic form, the analysis will only be continued for the u chromaticity component. The conditional mean of u' with respect to the transmitted components $\{M\} = \{U_T, V_T, W_T\}$ can be obtained in approximate form by expanding u' in a Taylor's series, and keeping only terms of second order [34, p. 212]. Following this technique, the mean shift in the u chrominance component is found to be

$$E\{u'|M\} = \frac{a_1 E\{U'_T|M\} + a_2}{a_3 E\{U'_T|M\} + a_4 E\{V'_T|M\} + a_5 E\{W'_T|M\} + a_6} \quad (4\text{-}34)$$
$$+ \frac{1}{2}\sigma^2\{U'_T|M\}\frac{\partial^2 u'}{\partial U'^2_T} + \frac{1}{2}\sigma^2\{V'_T|M\}\frac{\partial^2 u'}{\partial V'^2_T}$$
$$+ \frac{1}{2}\sigma^2\{W'_T|M\}\frac{\partial^2 u'}{\partial W'^2_T}$$

where the conditional means $E\{U'_T|M\}$, etc., and the conditional variances $\sigma^2\{U'_T|M\}$ are found from the scaling equations and the channel first and second order conditional statistics. With a similar approximation to an accuracy of second order moments, the variance of the u chrominance shift is obtained as

$$\sigma^2\{u'|M\} = \sigma^2\{U'_T|M\}\left[\frac{\partial u'}{\partial U'_T}\right]^2 + \sigma^2\{V'_T|M\}\left[\frac{\partial u'}{\partial V'_T}\right]^2$$
$$+ \sigma^2\{W'_T|M\}\left[\frac{\partial u'}{\partial W'_T}\right]^2 \quad (4\text{-}35)$$

For this case, the chrominance shift resulting from channel errors is

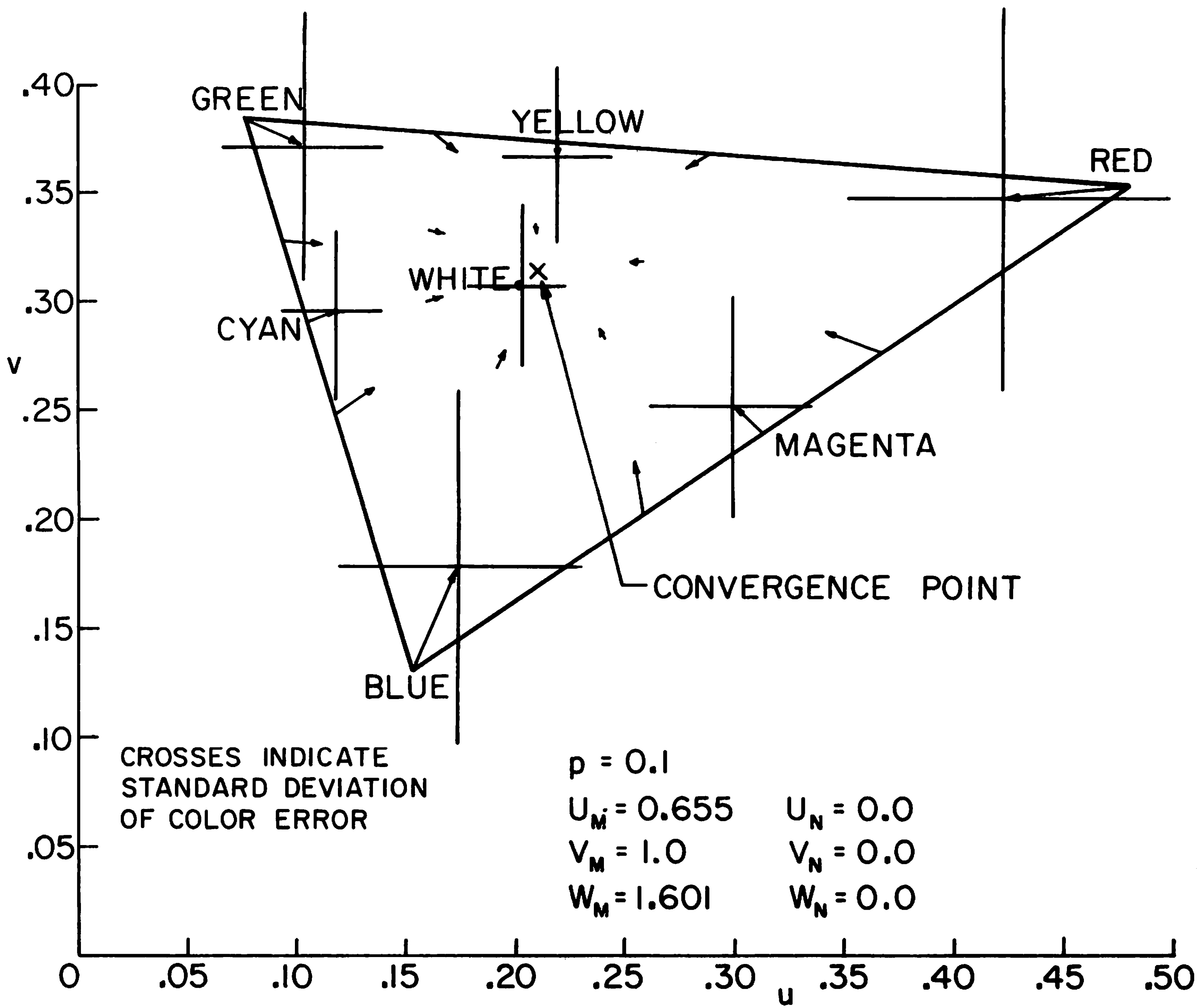


Figure 4-5. Chromaticity Shift for U,V,W Transmission

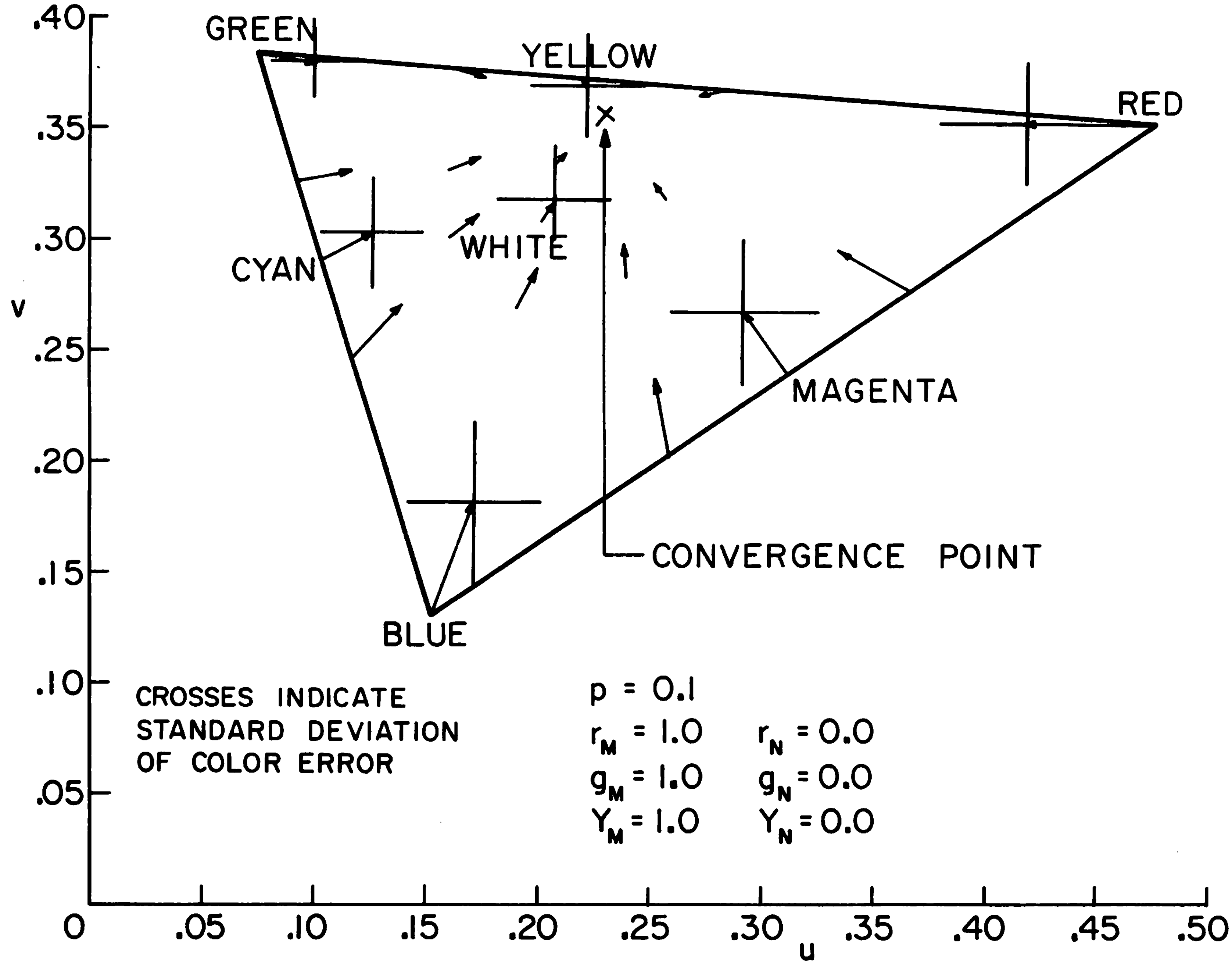


Figure 4-6. Chromaticity Diagram For Y,r,g Transmission

$$a_{12} = (2^N - 1)(\beta_7 R_N + \beta_8 G_N + \beta_9 B_N)$$
$$a_{13} = m_{21}(R_M - R_N)/(2^N - 1)$$
$$a_{14} = m_{22}(G_M - G_N)/(2^N - 1)$$
$$a_{15} = m_{23}(B_M - B_N)/(2^N - 1)$$

The analysis will be performed for the u and Y color components. To an accuracy of second order moments, the conditional means of u' and Y' with respect to the transmitted components, $\{M\} = \{R_T, G_T, B_T\}$ are given by

$$E\{u'|M\} = \frac{a_1 E\{R'_T|M\} + a_2 E\{G'_T|M\} + a_3 E\{B'_T|M\} + a_4}{a_5 E\{R'_T|M\} + a_6 E\{G'_T|M\} + a_7 E\{B'_T|M\} + a_8}$$

$$+ \frac{1}{2}\sigma^2\{R'_T|M\}\frac{\partial^2 u'}{\partial R'^2_T} + \frac{1}{2}\sigma^2\{G'_T|M\}\frac{\partial^2 u'}{\partial G'^2_T} \tag{4-48}$$

$$+ \frac{1}{2}\sigma^2\{B'_T|M\}\frac{\partial^2 u'}{\partial B'^2_T}$$

and

$$E\{Y'|M\} = a_{13}E\{R'_T|M\} + a_{14}E\{G'_T|M\} + a_{15}E\{B'_T|M\} \tag{4-49}$$

The conditional variances of the u chrominance and luminance shifts are

$$\sigma^2\{u'|M\} = \sigma^2\{R'_T|M\}\left[\frac{\partial u'}{\partial R'_T}\right] + \sigma^2\{G'_T|M\}\left[\frac{\partial u'}{\partial G'_T}\right]^2$$

$$+ \sigma^2\{B'_T|M\}\left[\frac{\partial u'}{\partial B'_T}\right]^2 \tag{4-50}$$

and

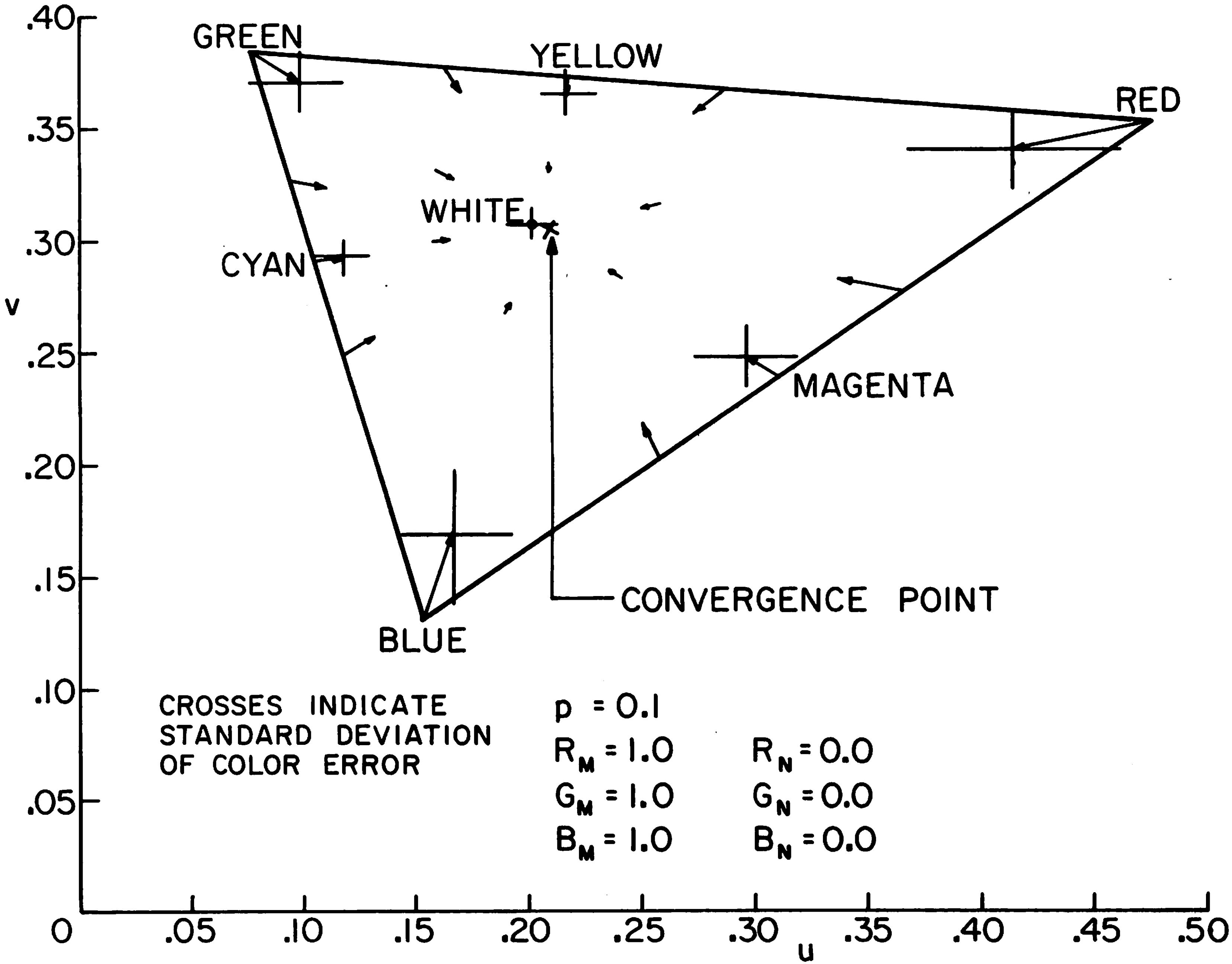


Figure 4-7. Chromaticity Shift For R,G,B Transmission

$$\tilde{t}_1 \equiv \frac{\tilde{T}_1}{\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_3} \quad (6)$$

$$\tilde{t}_2 \equiv \frac{\tilde{T}_2}{\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_3} \quad (7)$$

represent the chromaticity coordinates in the original and new coordinate systems respectively. Then from equations (1) to (3),

$$\boxed{\begin{aligned} \tilde{t}_1 &= \frac{\beta_1 T_1 + \beta_2 T_2 + \beta_3 T_3}{\beta_4 T_1 + \beta_5 T_2 + \beta_6 T_3} \\ \tilde{t}_2 &= \frac{\beta_7 T_1 + \beta_8 T_2 + \beta_9 T_3}{\beta_4 T_1 + \beta_5 T_2 + \beta_6 T_3} \end{aligned}} \quad (8)$$

where

$$\begin{aligned} \beta_1 &\equiv m_{11} \\ \beta_2 &\equiv m_{12} \\ \beta_3 &\equiv m_{13} \\ \beta_4 &\equiv m_{11} + m_{21} + m_{31} \\ \beta_5 &\equiv m_{12} + m_{22} + m_{32} \\ \beta_6 &\equiv m_{13} + m_{23} + m_{33} \\ \beta_7 &\equiv m_{21} \\ \beta_8 &\equiv m_{22} \\ \beta_9 &\equiv m_{23} \end{aligned} \quad (10)$$

and the m_{ij} are conversion matrix elements from the (T_1, T_2, T_3) to the $(\tilde{T}_1, \tilde{T}_2, \tilde{T}_3)$ coordinate systems. The luminance signal is related to the

$$\alpha_7 \equiv m_{21} - m_{23}$$

$$\alpha_8 \equiv m_{22} - m_{23}$$

$$\alpha_9 \equiv m_{23}$$

and the m_{ij} are conversion constants from the (T_1, T_2, T_3) to the $(\tilde{T}_1, \tilde{T}_2, \tilde{T}_3)$ coordinate systems.

Case 4: Luminance/Chrominance to Tristimulus Conversion

In the general situation in which the original chromaticity coordinates are not the C.I.E. x-y coordinates, the conversion is made in a two stage process. From equations (1) to (3)

$$\tilde{T}_1 = n_{11}X + n_{12}Y + n_{13}Z \tag{18}$$

$$\tilde{T}_2 = n_{21}X + n_{22}Y + n_{23}Z \tag{19}$$

$$\tilde{T}_3 = n_{31}X + n_{32}Y + n_{33}Z \tag{20}$$

where the n_{ij} are the constants for a conversion from (X, Y, Z) tristimulus values to $(\tilde{T}_1, \tilde{T}_2, \tilde{T}_3)$ tristimulus values.

Then from equations (15) and (16)

$$x = \frac{\alpha_1 t_1 + \alpha_2 t_2 + \alpha_3}{\alpha_4 t_1 + \alpha_5 t_2 + \alpha_6} \tag{21}$$

$$y = \frac{\alpha_5 t_1 + \alpha_6 t_2 + \alpha_7}{\alpha_4 t_1 + \alpha_5 t_2 + \alpha_6} \tag{22}$$

where the α_k are obtained from the m_{ij} of equation (17) for a conversion from a (T_1, T_2, T_3) tristimulus system to an (X, Y, Z) tristimulus system. And since

Exhibit A

Color Coordinate Conversion Constants

1. R_N, G_N, B_N to X, Y, Z [2, p. 1-34]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.607 & 0.174 & 0.201 \\ 0.299 & 0.587 & 0.114 \\ 0.000 & 0.066 & 1.117 \end{bmatrix} \begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix}$$

2. R_C, G_C, B_C to X, Y, Z [2, p. 1-337]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.011 \\ 0.000 & 0.010 & 0.990 \end{bmatrix} \begin{bmatrix} R_C \\ G_C \\ B_C \end{bmatrix}$$

3. U_R, V_R, W_R to X, Y, Z [16]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.331 & -0.092 & 0.265 \\ -0.139 & 0.122 & 0.096 \\ 1.220 & 0.000 & 0.000 \end{bmatrix} \begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix}$$

4. Y, I, Q to X, Y, Z [inverse of 19]

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.967 & 0.318 & 0.594 \\ 1.000 & 0.000 & 0.000 \\ 1.173 & -1.238 & 1.870 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

10. U, V, W to R_C, G_C, B_C [matrix product of No. 7 and No. 5]

$$\begin{bmatrix} R_C \\ G_C \\ B_C \end{bmatrix} = \begin{bmatrix} 2.846 & 0.507 & -0.936 \\ -0.639 & 1.159 & 0.178 \\ 1.521 & -3.041 & 2.018 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

11. R_N, G_N, B_N to U_R, V_R, W_R [matrix product of No. 13 and No. 1]

$$\begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix} = \begin{bmatrix} 0.000 & 0.054 & 0.919 \\ 0.514 & 3.469 & 1.796 \\ 2.474 & 1.800 & 0.237 \end{bmatrix} \begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix}$$

12. R_C, G_C, B_C to U_R, V_R, W_R [matrix product of No. 13 and No. 2]

$$\begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix} = \begin{bmatrix} 0.000 & 0.008 & 0.815 \\ 0.000 & 4.533 & 0.961 \\ 1.852 & 2.742 & 0.072 \end{bmatrix} \begin{bmatrix} R_C \\ G_C \\ B_C \end{bmatrix}$$

13. X, Y, Z to U_R, V_R, W_R [16]

$$\begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix} = \begin{bmatrix} 0.000 & 0.000 & 0.832 \\ -2.329 & 6.446 & 1.369 \\ 2.968 & 2.248 & -0.552 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

14. Y, I, Q to U_R, V_R, W_R [matrix product of No. 13 and No. 4]

$$\begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix} = \begin{bmatrix} 0.963 & -1.018 & 1.537 \\ 5.724 & -2.393 & 1.110 \\ 4.439 & 1.640 & 0.710 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

20. U, V, W to Y, I, Q [3, p. 412]

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.000 & 1.000 & 0.000 \\ 1.403 & 0.534 & -0.907 \\ 1.757 & -1.898 & 0.470 \end{bmatrix} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$

21. X, Y, Z to R_N, G_N, B_N [2, p. 1-34]

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 1.910 & -0.533 & -0.288 \\ -0.985 & 2.000 & -0.028 \\ 0.058 & -0.118 & 0.896 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

22. R_C, G_C, B_C to R_N, G_N, B_N [matrix product of No. 21 and No. 2]

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 0.842 & 0.156 & 0.091 \\ -0.129 & 1.320 & -0.203 \\ 0.008 & -0.069 & 0.897 \end{bmatrix} \begin{bmatrix} R_C \\ G_C \\ B_C \end{bmatrix}$$

23. U_R, V_R, W_R to R_N, G_N, B_N [matrix product of No. 21 and No. 3]

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 0.356 & -0.241 & 0.454 \\ -0.637 & 0.335 & -0.070 \\ 1.124 & -0.020 & 0.004 \end{bmatrix} \begin{bmatrix} U_R \\ V_R \\ W_R \end{bmatrix}$$

24. Y, I, Q to R_N, G_N, B_N [3, p. 413]

$$\begin{bmatrix} R_N \\ G_N \\ B_N \end{bmatrix} = \begin{bmatrix} 1.000 & 0.956 & 0.621 \\ 1.000 & -0.272 & -0.647 \\ 1.000 & -1.106 & 1.703 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

30. Y, I, Q to U, V, W [3, p. 412]

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 0.653 & 0.208 & 0.403 \\ 1.000 & 0.000 & 0.000 \\ 1.599 & -0.780 & 0.623 \end{bmatrix} \begin{bmatrix} Y \\ I \\ Q \end{bmatrix}$$

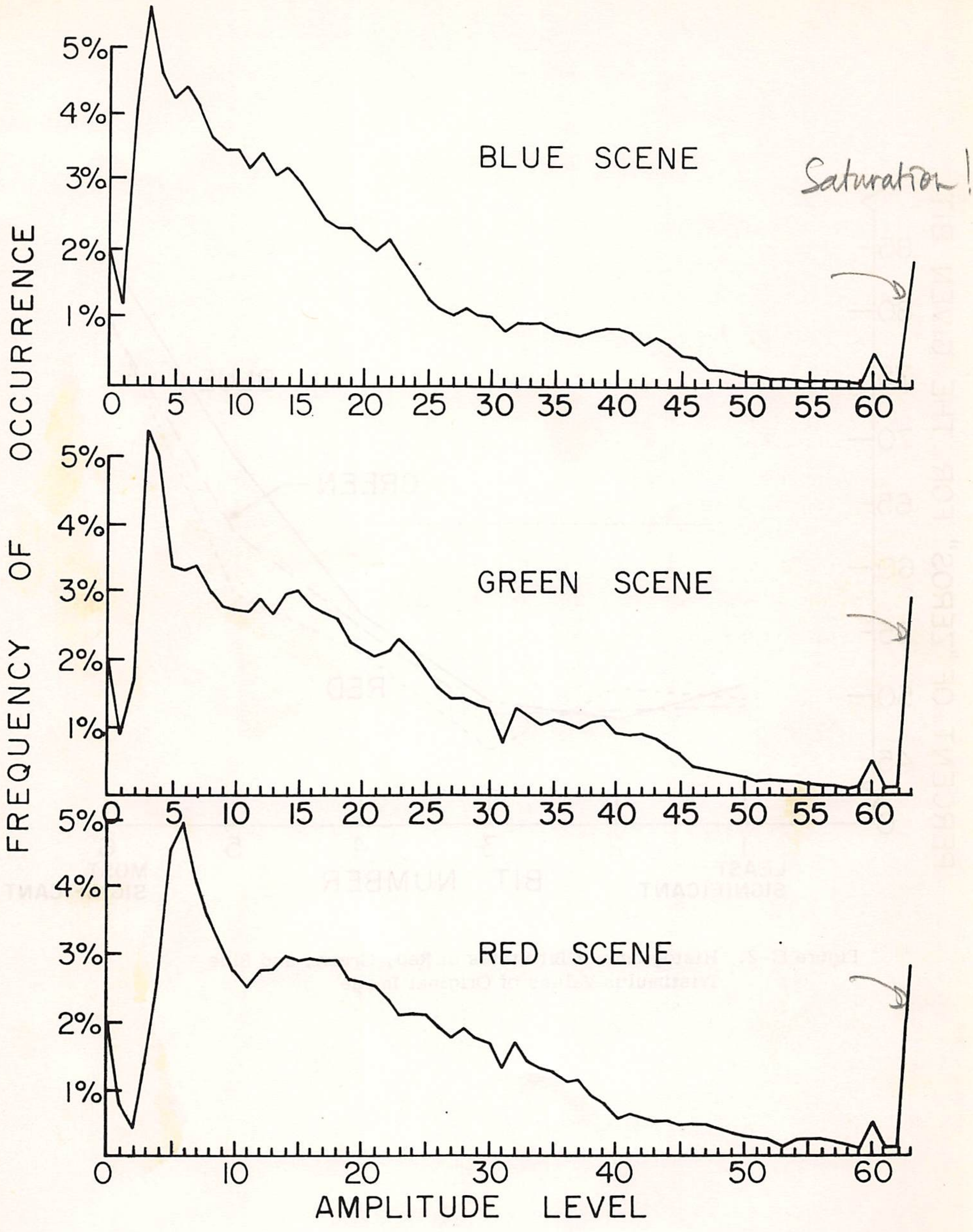


Figure C-1. Histograms of Red, Green, and Blue Tristimulus Values of Original Image

R

G

B

Figure C-3. N.T.S.C. receiver primaries, R,G,B tristimulus color coordinates of original image

Y

I

Q

Figure C-5. N.T.S.C. transmission primaries, Y, I, Q tristimulus color coordinates of original image

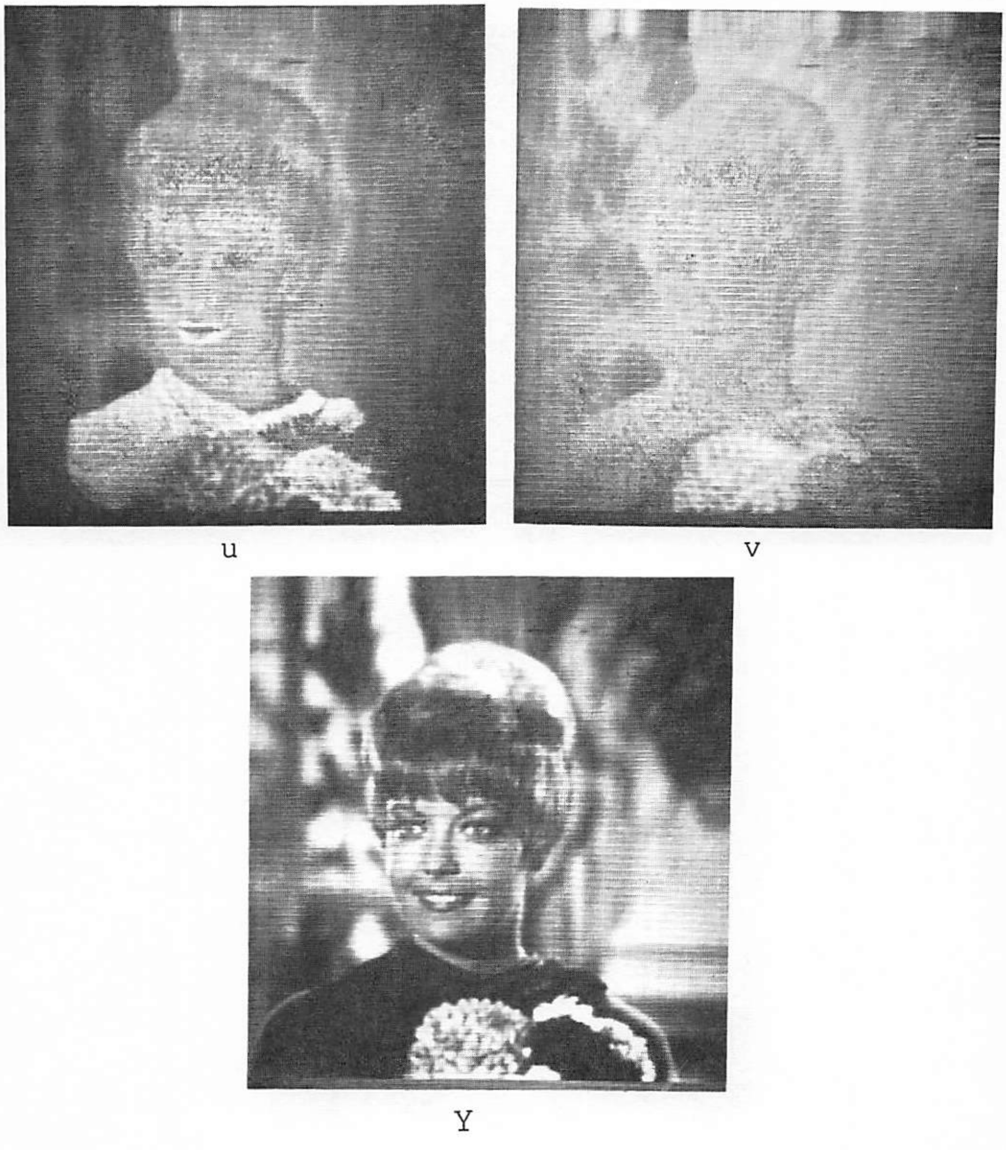


Figure C-7. Y luminance and u,v, chromaticity color coordinates of original image

References

1. G. Wyszecki and W.S. Stiles, Color Science, John Wiley and Sons, New York (1957).

2. D.G. Fink, editor, Television Engineering Handbook, McGraw Hill, New York (1957).

3. R.W.G. Hunt, The Reproduction of Colour, John Wiley and Sons, New York (1967).

4. W.T. Wintringham, "Color Television and Colorimetry," Proceedings of the IRE, Vol. 39, No. 10, (October,1951), pp. 1135-1172.

5. D.L. MacAdam, "The Theory of the Maximum Visual Efficiency of Colored Materials," Journal Optical Society of America, Vol. 25, No. 8, (August,1935), pp. 249-252.

6. D.L. MacAdam, "Maximum Visual Efficiency of Colored Materials," Journal of the Optical Society of America, Vol. 25, No. 11, (November,1935) pp. 361-367.

7. E.J. Muth and C.G. Persels, "Computation of Geodesics in Color 3-Space by Dynamic Programming," Proceedings of the 4th Hawaii Conference on System Sciences, (January,1971), pp.155-157.

8. E.F. MacNichol Jr., "Three-Pigment Color Vision," Scientific American, (December,1964).

9. The Science of Color, Thomas Y. Crowell Company, New York, (1953).

10. G. Wald, "The Receptors of Human Color Vision," Science, Vol. 145, (1964) p.1007.

11. A. Linksz, An Essay on Color Vision and Clinical Color Vision Tests, Grune and Stratton, New York, (1964).

12. J. Cohen and D. Gordan, "The Prevost-Fechner-Benham Subjective Colors," Psychological Bulletin, Vol. 46, No. 2, (March,1949) pp. 97-136.

24. J.O. Limb, C.B. Rubinstein, and K.A. Walsh, "Coding of Color Pictures by Differential Quantization," Symposium on Picture Coding, North Carolina State University, Raleigh, North Carolina, (September,1970).

25. H. Enomoto and K. Shibata, "Orthogonal Transform Coding System for Television Signals," 1971 Symposium on Application of Walsh Functions, Washington, D.C. (April,1971).

26. W.K. Pratt, "A Comparison of Digital Image Transforms," University of Missouri, Rolla - Mervin J. Kelly Communications Conference, Rola, Missouri, (October,1970).

27. J.J.Y. Huang and P.M. Schultheiss, "Block Quantization of Correlated Gaussian Random Variables," IEEE Trans. on Communication Systems, Vol. CS-11, No. 3 (September, 1963) pp. 289-296.

28. A. Habibi and P.A. Wintz, "Linear Two Dimensional Transformations for Encoding 2-Dimensional Sources," Purdue University, School of Electrical Engineering, Report No. TR-EE 70-2, (March, 1970).

29. A.V. Bedford, "Mixed Highs in Color Television," Proc. IRE, Vol. 38, No. 9, (September,1950) pp.1003-1009.

30. K. McIlwain, "Requisite Color Bandwidth for Simultaneous Color - Television Systems," Proc. IRE, Vol. 40, No. 8, (August,1952) pp. 909-912.

31. E.N. Willmer and W.D. Wright, "Colour Sensitivity of the Fovea Centralis," Nature, Vol. 156, (July 28,1945) pp. 119-121.

32. W.E. Middleton and M.C. Holmes, "The Apparent Colors of Surface of Small Subtance - A Preliminary Report," Journal of the Optical Society of America, Vol. 39, (July,1949) pp. 582-592.

33. I.T. Young and J. Mott-Smith, "On Weighted PCM," IEEE Trans. on Information Theory, (April,1966).