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A Unified Approach for Filtering and Edge Detection in Noisy Images [1]

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ABSTRACT

We consider the problem of enhancement and edge detection on noisy, real images.A unified framework for smoothing and edge detection based on an autoregressive(AR) random field model is presented.An edge is detected, if the first and second directional derivatives and a local estimate of the variance at each point satisfy certain criteria.When noise is present we would like to estimate the directional derivatives from a restored version of the noisy image.We propose a Reduced Update Kalman Filter(RUKF) to perform the restoration.Then we can perform edge detection recursively using a small (4×4) window and still be fairly robust in the presence of noise.Since the edge detector operates on the restored image,it follows the RUKF by a fixed lag.A min-max replacement technique is introduced inbetween the RUKF and the edge detector to improve edge strength.The results compare favorably with those of other edge detectors.

1 Introduction

Edge detection is an important topic to most researchers in the area of image understanding.A number of different techniques exist in the literature.A summary of the earlier approaches to edge detection can be found in Davis('75) [1].Since then,several new techniques have appeared.Among them are the ones that perform statistical tests, [2],[3],[4], and [5], Modestino and Fries's recursive filter/edge detector [6], Shanmugam et.al.'s optimum filter [7], Marr and Hildreth's zero crossing edge detector [8], the Nevatia-Babu line finder [9], Canny's edge detector [10], the regularization approach [11], the facet model based approach [12] and the optimum matched, detection and localization filters [13].

When noise is present in an image, one usually does some form of preliminary image smoothing using standard presmoothing techniques such as median filtering etc.Presmoothing is then followed by a standard edge detection algorithm.Oftentimes, the choice of a specific edge detector has no relation whatsoever to the presmoothing algorithm. In this paper,we propose a unified framework for edge detection and smoothing. The image is assumed to be adequately represented by a 2-D AR space-variant model.An edge is detected if the first and second directional derivatives and a local estimate of the variance at each point satisfy certain criteria.Due to the modelling assumptions, the directional derivatives become functions of the model parameters and the neighboring pixels in a (3×3) window.In the absence of noise,the model parameters can be adaptively estimated at each point from the underlying image itself.When noise is present , restoration helps to improve the SNR and consequently edge detection itself as the directional derivatives are also dependent on the neighboring pixel values.This problem of simultaneous estimation of model parameters and pixel values can be formulated as an Extended Kalman Filter(EKF).Since this is computationally too intensive we have proposed a Reduced Update Kalman Filter (RUKF) to perform image restoration and an adaptive least-squares(LS) technique for parameter estimation.The RUKF implemented along the lines suggested by Woods [14] nevertheless differs from it in that the parameters of the AR model are estimated adaptively.A reduced update region implies the existence of a larger update region.In connection with these two update regions,we (following [14])use the local and global state vectors to characterize them. In general the global state vector is composed of M previous lines for a nonsymmetric half plane representation of the noise free image using a $M \times M$ model. When a finite order model is used, a subset of the global state called the local state can be used.This region defined as the local state is chosen to correspond with the support of the model itself for the sake of convenience..When this is the case, the parameter estimates obtained in the restoration process can be directly used for the edge detector's model parameters.Edge detection could then be performed simultaneously with the RUKF.However, this did not provide adequate filtering and the support was consequently increased to four pixels. Min-max replacement is then performed recursively to enhance edge strength. This forces edge detection to follow restoration by a fixed lag.

The AR model results in an oriented edge detector and may not detect well edges whose orientations are markedly different from the edge detector itself.This problem is partially overcome by running four quarter plane(QP) edge detectors as explained in [15] on rotated versions of the restored image.Since the detection process follows that of restoration, the four QP detectors are run on rotated versions of a (4×4) window.The final output is taken as the union of the outputs of the four QP edge detectors.The synthesis of the four QP edge detectors is called the Full-Plane(FP) edge detector.

The organization of the paper is as follows.Section II deals with the FP edge detector and its details,while Section III presents the implementation of the RUKF and the adaptive parameter estimator.Section IV discusses the experimental results and Section V presents the conclusions.

2 Design of the Full-Plane(FP) edge detector

In this paper, edge detection is performed recursively and follows the restoration process by a fixed lag.This means that our image model for edge detection is assumed to be an adequate representation of the restored image. The FP edge detector then is applied to the restored image. As mentioned in the introduction, the image model is a 2-D AR space variant model.Let $\hat{s}(x_0, y_0)$ be an estimate of the the gray level at position (x_0, y_0).The details of this restoration process are given in Section III.The model is expressed as follows.

$$\hat{s}(x_0, y_0) = c_1\hat{s}(x_0 - x, y_0) + c_2\hat{s}(x_0, y_0 - y) \\ +c_3\hat{s}(x_0 - x, y_0 - y) + n(x_0, y_0) \qquad (1)$$

At this point it is assumed that the parameters c_1, c_2 and c_3 are known.The first directional derivative is written as

$$\frac{\partial \hat{s}}{\partial \alpha} = \frac{\partial \hat{s}}{\partial x} cos\alpha + \frac{\partial \hat{s}}{\partial y} sin\alpha \qquad (2)$$

and because of the modelling assumption, it can be approximated as

$$0.5[c_1[\hat{s}(x_0 - 2, y_0) - \hat{s}(x_0, y_0)] \\ +c_3[\hat{s}(x_0 - 2, y_0 - 1) - \hat{s}(x_0, y_0 - 1)]]cos\alpha \\ + 0.5[c_2[\hat{s}(x_0, y_0 - 2) - \hat{s}(x_0, y_0)] \\ +c_3[\hat{s}(x_0 - 1, y_0 - 2) - \hat{s}(x_0 - 1, y_0)]]sin\alpha$$

where

$$\alpha = arctan(\frac{\frac{\partial \hat{s}}{\partial y}}{\frac{\partial \hat{s}}{\partial x}}) \qquad (3)$$

And the second directional derivative

$$\frac{\partial^2 \hat{s}}{\partial \alpha^2} = \frac{\partial^2 \hat{s}}{\partial x^2} cos^2\alpha + 2\frac{\partial^2 \hat{s}}{\partial x \partial y} cos\alpha sin\alpha + \frac{\partial^2 \hat{s}}{\partial y^2} sin^2\alpha \quad (4)$$

is approximated as

$$[c_1[\hat{s}(x_0 - 2, y_0) - 2\hat{s}(x_0 - 1, y_0) + \hat{s}(x_0, y_0)] \\ +c_3[\hat{s}(x_0 - 2, y_0 - 1) - 2\hat{s}(x_0 - 1, y_0 - 1) \\ +\hat{s}(x_0, y_0 - 1)]]cos^2\alpha \\ +0.5[c_3[\hat{s}(x_0, y_0) - \hat{s}(x_0 - 2, y_0) \\ -\hat{s}(x_0, y_0 - 2) + \hat{s}(x_0 - 2, y_0 - 2)]]cos\alpha sin\alpha \\ +[c_2[\hat{s}(x_0, y_0 - 2) - 2\hat{s}(x_0, y_0 - 1) + \hat{s}(x_0, y_0)] \\ +c_3[\hat{s}(x_0 - 1, y_0 - 2) - 2\hat{s}(x_0 - 1, y_0 - 1) \\ +\hat{s}(x_0 - 1, y_0)]]sin^2\alpha$$

The details of how the first and second directional derivatives are approximated as above may be found in [15].

We detect an edge if a)the second directional derivative in the direction of the estimated gradient is negative b)the magnitude of the first directional derivative is above some threshold c)and the first derivatives along the x and y directions are non-negative.To the above, a final condition is added,viz.,d)the local estimate of the sample variance (4×4 was used) must be greater than a threshold.This threshold is important in real, noisy pictures in order to reduce the effects of noise.

The parameters c_1, c_2 and c_3 are estimated from the restored image.Since real images are non-stationary, it is preferable to estimate these parameters adaptively.The nature of the model allows us to use a recursive Kalman filter approach the details of which can be found in [16].

At the core of this approach is the QP edge detector(as mentioned in the introduction) and it arises out of the underlying causal model.The causal AR model results in an oriented edge detector and may not detect edges whose orientations are significantly different from that of the edge detector.This orientation corresponds to the first quarter plane as depicted in Fig 1.The value of α in equation (3) is restricted to lie between 0° and 90°. The Full-Plane(FP) edge output is obtained from a union of the four QP edge outputs and Fig 1. shows this implementation in relation to the positioning of the RUKF and the min-max replacement process.

3 Restoration using a RUKF

Given the special emphasis on noisy, real images, the restoration process is particularly significant as far as the edge detector is concerned.The particular technique used here is the RUKF and this method and other Kalman Filter approximation methods are fully discussed by Woods in [14]. In his work, Woods assumed a Non-Symmetric-Half-Plane(NSHP) model driven by spatially white noise to represent the original image.

$$s(m,n) = c_1^{(m,n)}s(m-1,n) + c_2^{(m,n)}s(m+1,n-1) \\ +c_3^{(m,n)}s(m,n-1) + c_4^{(m,n)}s(m-1,n-1) + w(m,n) \quad (5)$$

where $w(m,n)$ is the white process noise field.

A particular case of this model is the (1×1) model used in [14].Here the support region of the model consists of the four neighboring pixels($s(m-1,n), s(m-1,n-1), s(m,n-1)$ and $s(m+1,n-1)$).When the support region is reduced to include only three neighbors ($s(m-1,n), s(m-1,n-1)$, and $s(m,n-1)$),(5) is equivalent to the space invariant version of (1).

$s(m,n)$ in equation (5) refers to the original noise-free image.This image is assumed to be corrupted by additive white Gaussian noise ($v(m,n)$) with the variance σ_v^2.In the equations that follow $r(m,n)$ represents the noisy image and is the only data avaiable to the RUKF.

$$r(m,n) = s(m,n) + v(m,n)$$

$$w(m,n) \approx N(0,\sigma_w^2)$$

and

$$v(m,n) \approx N(0,\sigma_v^2)$$

The indices m and n refer to the position of the RUKF .The coefficients of the model are not space invariant as in [14],but are estimated by a recursive LS technique.Ideally, we could estimate both the parameters and states using a EKF but the computational burden would be very intensive.

The RUKF consists of two major processes, prediction and update.Before prediction can be performed, in our modification of the RUKF, we need the space-variant model at that point.This is done by a Least-Squares recursion.The pixel value of the point (m,n) is now predicted based on the model whose parameters have been estimated.Since the model is causal in the raster-scanning sense, it makes use of the restored values of the previous points in the recursion.Along with the predicted value of the point (m,n) denoted by $\hat{s}_b(m,n)$, the prediction error covariance matrix $\mathcal{R}_b^{(m,n)}(i,j;k,l)$ which denotes the prediction error between the pairs of points (i,j) and (k,l) is also estimated.This is carried out over the global state region.

$$\begin{aligned}\hat{s}_b^{(m,n)} &= c_1^{(m,n)}\hat{s}_a^{(m-1,n)}(m-1,n)\\ &+c_2^{(m,n)}\hat{s}_a^{(m-1,n)}(m+1,n-1)\\ &+c_3^{(m,n)}\hat{s}_a^{(m-1,n)}(m,n-1)\\ &+c_4^{(m,n)}\hat{s}_a^{(m-1,n)}(m-1,n-1)\end{aligned} \tag{6}$$

and

$$\begin{aligned}R_b^{(m,n)}(m,n;k,l) &= c_1^{(m,n)}R_a^{(m-1,n)}(m-1,n;k,l)\\ &+c_2^{(m,n)}R_a^{(m-1,n)}(m+1,n-1;k,l)\\ &+c_3^{(m,n)}R_a^{(m-1,n)}(m,n-1;k,l)\\ &+c_4^{(m,n)}R_a^{(m-1,n)}(m-1,n-1;k,l)\end{aligned} \tag{7}$$

where $(k,l) \neq (m,n)$ and $(k,l) \in \mathcal{L}_{\oplus+}^{(m,n)}$ the global state. Details about the global state and local state can be found in [14]

$$\begin{aligned}R_b^{(m,n)}(m,n;m,n) &= c_1^{(m,n)}R_b^{(m,n)}(m,n;m-1,n)\\ &+c_2^{(m,n)}R_b^{(m,n)}(m,n;m+1,n-1)\\ &+c_3^{(m,n)}R_b^{(m,n)}(m,n;m,n-1)\\ &+c_4^{(m,n)}R_b^{(m,n)}(m,n;m-1,n-1)+\sigma_w^2\end{aligned} \tag{8}$$

Here a and b refer to *after* and *before* respectively.This is identical to Update and Prediction.Equation(6) obtains the predicted value of $\hat{s}_b$ using the NSHP model. These model parameters,the coefficients $[\underline{c}^{(m,n)}]$ are estimated at each point as described previously, and this is indicated by their superscipts. The major modification here is that the update is restricted to the local state.This is because updates are significant only in a local neighborhood of the point (m,n).In this case the local state is used as the update region.The Kalman gain vector and the update of the prediction state vector($\underline{\hat{s}}_b^{(m,n)}$) is restricted to the local state region.

$$k^{(m,n)} = \frac{R_b^{(m,n)}(m,n;i,j)}{[R_b^{(m,n)}(m,n;m,n)+\sigma_v^2]} \quad (i,j) \in \mathcal{R}_{\oplus+}^{(m,n)} \tag{9}$$

and

$$\begin{aligned}\hat{s}_a^{(m,n)}(i,j) &= \hat{s}_b^{(m,n)}(i,j)\\ +k^{(m,n)}(i,j)&[r(m,n)-\hat{s}_b^{(m,n)}(m,n)]\\ R_a^{(m,n)}(i,j;k,l) &= R_b^{(m,n)}(i,j;k,l)\\ &-k^{(m,n)}(i,j)R_b^{(m,n)}(m,n;k,l)\end{aligned} \tag{10}$$

for

$$(i,j) \in \mathcal{R}_{\oplus+}^{(m,n)}$$

the local state and

$$(k,l) \in \mathcal{L}_{\oplus+}^{(m,n)}$$

the global state.

The model used in the RUKF is not identical with the one used in the design of the FP edge detector.In this case, the support is four pixels,and this means that four and not three parameters have to be estimated. The same form of the adaptive least-squares technique described in the design of the FP detector can be used for estimation of the model parameters.Since the initial parameters are to be estimated from the noisy image, a bias-compensated LS estimate[14] is used .Initial conditions have to be prescribed for the filtering error covariance matrix(R_a).The initial error covariance was chosen to correspond with that of the white noise process(σ_v^2) which has to be estimated.

4 Results and Discussions

The RUKF filter/FP edge detector was run on two noisy versions of the airport image.The airport image after preprocessing(median filtering followed by min-max replacement)is shown in Fig.3.The FP edge detector was run on this image. The noisy images(SNR's of 5 and 0 dB) are shown in Figs.4 and 5 respectively.The edge outputs of the original and noisy images are shown in Figs.6.7 and 8 respectively.

Real images lack an objective criterion on whether an edge exists or not. This implies that our 'best' choice of thresholds is based on critical comparison of the edge outputs from the restored and original images. However the performance of the edge detector can be judged by a group of intersubjective interpreters.In our unified approach,there is a strong match between restoration and edge detection.The same form of the random field model is used in both..One important feature of this approach is that there is no postprocessing operation like thinning involved.Despite this, there is a relative absence of multiple responses to a single edge.This is achieved by localizing the domain of each QP edge detector to its appropriate quadrant and summing up the responses.At a phenomenological level,this technique compares favorably with other techniques aimed at detecting edges in real,noisy images and further details and comparisons can be found in [17].

5 Conclusions

In this paper, we have described an edge detector which is based on several criteria applied to first and second directional derivatives which in turn are functions of the parameters of an underlying correlated Random Field model.In addition, we have incorporated an adaptive filtering scheme to improve the SNR of the noisy, real images used.The filtering scheme uses the same form of the model as that of edge detection.

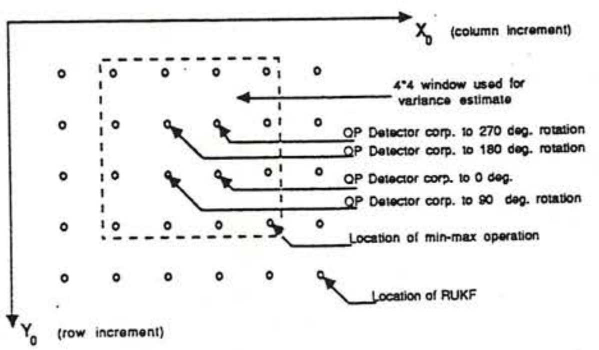

Figure 1: Interleaving of FP edge det.,RUKF and min-max.

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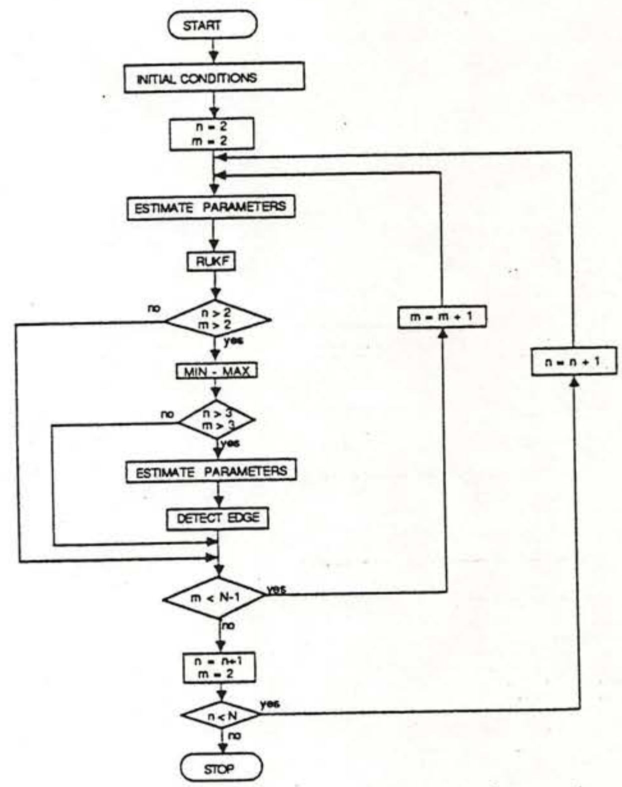

Figure 2: Flow Graph illustrating sequence of computations.

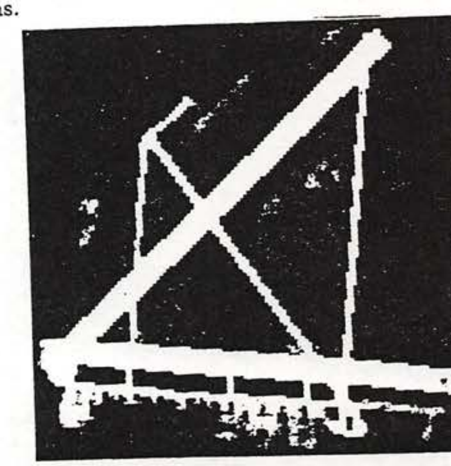

Figure 3: Original airport(Preprocessed).

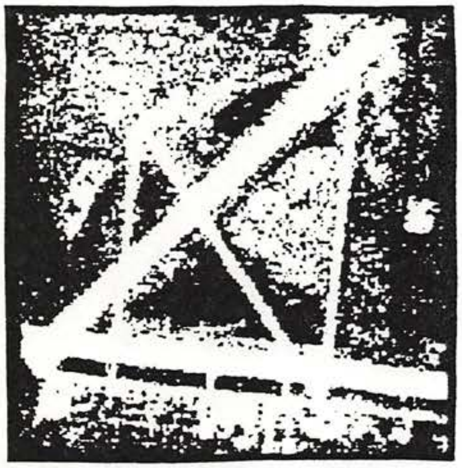

Figure 4: Noisy Airport(5 dB).

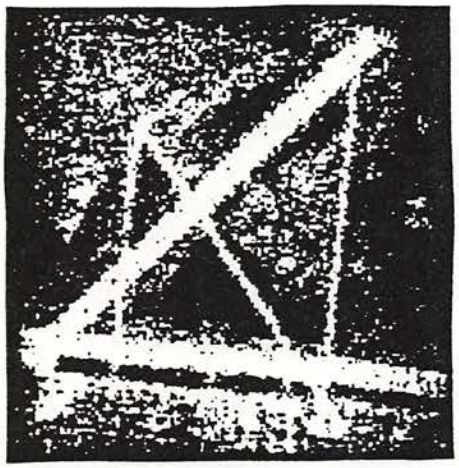

Figure 5: Noisy Airport(0 dB).

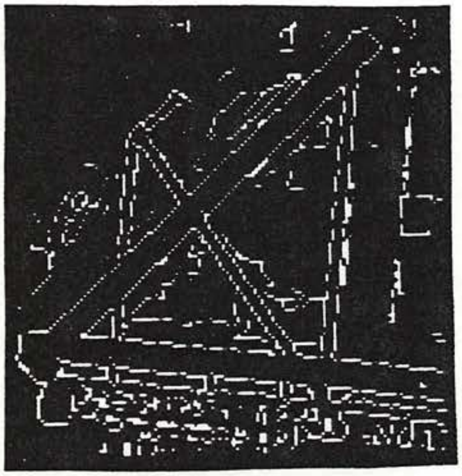

Figure 6: FP detector applied on Original Airport.

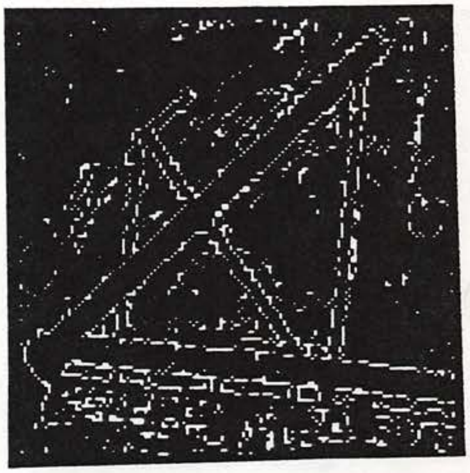

Figure 7: Unified filter/detector on 5dB Noisy Airport.

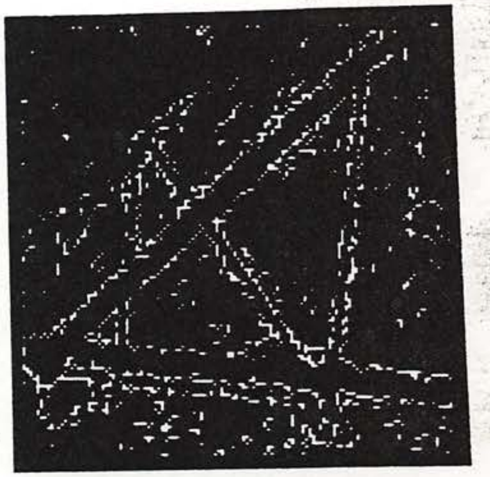

Figure 8: Unified filter/detector on 0dB Noisy Airport.

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