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Graduated Nonconvexity Algorithm for Image Estimation Using Compound Gauss Markov Field Models

by

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Abstract

This paper is concerned with developing a deterministic algorithm for obtaining the global maximum *aposteriori* probability (MAP) estimate from an image corrupted by additive Gaussian noise. The MAP algorithm requires the probability density function of the original undegraded image and the probability density function of the corrupting noise. By assuming that the original image is represented by a compound model consisting of a 2-D noncausal Gaussian Markov random field (GMRF) to represent the homogeneous regions and a line process model to represent the discontinuities, the MAP algorithm is written in terms of the compound GMRF model parameters. The solution to the MAP equations is then realized by a deterministic relaxation algorithm. The deterministic algorithm which is an extension of the graduated non convexity (GNC) algorithm, is able to find the global MAP estimate. As a by product, the line process configuration determined by the MAP estimate produces an accurate edge map without any additional cost. Unlike the simulated annealing method, the deterministic algorithm converges in

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a small number of iterations.

Due to the modeling assumption the restoration algorithm depends on the GMRF model parameters. We obtain estimates of the compound GMRF model parameters from the original image using a new expectation maximization (EM) estimation technique. The EM algorithm enables estimation of the GMRF model parameters without being affected by the edges present in the image. Experimental results are given to illustrate the usefulness of our method.

1 Introduction

Image estimation of gray level images requires the ability of the estimate to preserve the sharpness of edges, which play an important role in human interpretation of images. Linear filters [1] [2], as well as MAP estimation assuming Gauss Markov random field (GMRF) model [3] for the intensity tend to smear the edges. To avoid this problem a class of models called compound GMRF was suggested in [4]. The model suggested is a GMRF coupled with a line process. In this model the structure of the line process determines the value of the GMRF model coefficients. The GMRF parameters are required to satisfy some symmetry constraint, and the choice of the parameters for a given line configuration remains an open problem. Our simplified compound GMRF model is a member of the class of models proposed in [4]. In our formulation the line process breaks the correlation between neighboring pixels when the gray level jump is above a threshold. The model can be viewed as an extension of Blakes [5] weak membrane model used for surface interpolation and edge detection. The GMRF breaks when edges occur, which in turn creates homogeneous GMRF patches separated by the line process. Qualitative and quantitative values correspond to all the new model parameters, and they can be estimated from the image. As an example, the regularization term in the weak membrane model is replaced by the ratio of the noise variance to the GMRF model variance.

Given the distribution of the corrupting noise we write the conditional density of the degraded image conditioned on the original image. The degradation we consider in this work is additive white Gaussian noise. Thus, the posterior density is written in terms of the GMRF model parameter estimates.

To obtain the global MAP estimate for an image obeying a compound GMRF model and corrupted by additive Gaussian noise, Jeng and Woods [4] use simulated annealing. This stochastic algorithm is slow, and can not in practice obey the theoretical requirement on the initial temperature. We use a modified version of the GNC algorithm [5]. The algorithm is deterministic and good results are obtained in less then 100 iterations. We have used the algorithm on a real airport image and obtained good restoration results. The configuration of the line process is determined by the MAP estimate. It provides an accurate estimate of the image edges without any additional cost.

In this paper we also address the problem of estimating the GMRF model parameters from a noise free realization of the original image. In order to avoid the edges one can limit the estimation domain only to homogeneous regions, which will make maximum likelihood (ML) estimation impractical. Instead we have developed the following procedure. We detect the edges on the image using the GNC algorithm [5] and discard the intensity data in a strip of four pixels centered at each edge. We develop an EM algorithm [6] that calculates the conditional expectation of the intensity given the parameters of the model and the neighboring intensity levels which is used to replace the discarded data, and then calculate the new parameters estimate from the smoothed image using least squares(LS) [2] or ML techniques. The process is repeated till it converges. As initial parameters we used the LS estimates of the image including the edges. The current version includes only results based on LS, and we are implementing the ML version. The parameters we obtain have much lower model variance then the parameters estimated from the image with the edges, although the image variance reduced only slightly. In the future, we would like to extend this process to estimation from the noisy image itself.

The paper is organized as follows. Section 2 presents an extension to compound GMRF model, and the modified GNC algorithm that finds the global MAP solution for the additive noise problem. Section 3 discusses parameter estimation using the EM algorithm. Section 4 presents restoration and edge detection results of an image corrupted by 5dB and 10dB additive noise.

2 GNC algorithm for MAP Estimation of Images Corrupted by Additive Noise

The compound GMRF model is an extension of the weak membrane model defined by Blake [5], and used for surface interpolation as well as edge detection. We first review the weak membrane formulation.

2.1 The Weak Membrane Model

Surface interpolation and edge detection can be viewed as the problem of fitting a weak membrane (that is, an elastic membrane under weak continuity constrains) to a surface [5]. The location of discontinuities in the weak membrane corresponds to discontinuities in the intensity (step edges).

The problem has the following mathematical form: Find the configuration that minimizes the following functional corresponding to a weak membrane with a simple line process:

$$F = \int \{(u-d)^2 + \lambda^2(\nabla u)^2\} dA + \alpha \int dl \tag{1}$$

The first integral is evaluated over the area in which the data d is defined, and the second along the length of all discontinuities. α is a penalty per unit length of discontinuity. λ is a characteristic length for smoothing the continuous portions of the data and is also a characteristic distance for interaction between discontinuities. Since data is only available on grid points, and the problem does not have a closed form solution, we need to discretize the problem. Blake [5] suggests minimizing the cost function.

$$F = D + \sum_{ij} h_{\alpha,\lambda}(u_{i,j} - u_{i-1,j}, l_{ij}) + \sum_{ij} h_{\alpha,\lambda}(u_{i,j} - u_{i,j+1}, m_{ij}) \tag{2}$$

where

$$D = \sum_{ij}(u_{i,j} - d_{i,j})^2 \tag{3}$$

l_{ij} activates the line process in the northernly direction,

$$h_{\alpha,\lambda}(t,l) = \lambda^2(t)^2(1-l) + \alpha l$$

and m_{ij} activates the line process in the easternly direction. The problem is thus reduced to the following optimization problem.

$$\min_{\{u_{ij}\}} \left(D + \min_{\{l_{ij}\}} \left(\sum_{ij} h_{\alpha,\lambda}(u_{i,j} - u_{i-1,j}, l_{ij}) \right) + \min_{\{m_{ij}\}} \left(\sum_{ij} h_{\alpha,\lambda}(u_{i,j} - u_{i,j+1}, m_{ij}) \right) \right) \tag{4}$$

As D does not involve l_{ij}, m_{ij}, minimization over l_{ij}, m_{ij} can be performed and one is then left with minimization with respect to u_{ij}:

$$\min_{\{u_{ij}, l_{ij}, m_{ij}\}} F = \min_{\{u_{ij}\}} \left(D + \sum_{ij} g_{\alpha,\lambda}(u_{i,j} - u_{i-1,j}) + \sum_{ij} g_{\alpha,\lambda}(u_{i,j} - u_{i,j+1}) \right)$$

where

$$g_{\alpha,\lambda}(t) = \min_{l \in \{0,1\}} h_{\alpha,\lambda}(t,l) = \min(\lambda^2(t)^2, \alpha)$$

2.2 The Compound GMRF Model

In this section we extend the weak membrane model to a compound GMRF and line process model. Since the parameters corresponding to the compound GMRF model can be estimated from the noisy image using bias compensated least square (BCLS) techniques,[7], this extension enables us to get better reconstruction of weakly continuous surfaces. It also gives a qualitative meaning to λ in the weak membrane algorithm and allows us to estimate it's value from the noise and model variances.

We define the following conditional distribution for the compound GMRF model:

$$p(y(s)|y(s+\tau), y(s-\tau), l(s,\tau), l(s-\tau,\tau)\ ,\ \tau \in N^*) = \frac{e^{-U(y(s)|y(s+\tau),y(s-\tau),l(s,\tau),l(s-\tau,\tau)\ ,\ \tau \in N^*)}}{Z}$$

where N^* is the set of shift vectors corresponding to the neighborhood of the GMRF model. The line process notation is illustrated in Fig 0. For a second order GMRF model $N^* = \{(0,1),(1,0),(1,1),(-1,1)\}$

$$U(y(s)|y(s+\tau), y(s-\tau), l(s,\tau), l(s-\tau,\tau)\ ,\ \tau \in N^*) =$$
$$\frac{1}{2\nu}[\sum_{\tau \in N^*} \Theta_\tau[(y(s)-y(s+\tau))^2(1-l(s,\tau)) + (y(s)-y(s-\tau))^2(1-l(s-\tau,\tau))]$$
$$+(1-\sum_{\tau \in N^*} 2\Theta_\tau)y(s)^2]$$

The line process is now defined on the edges that connect the nodes (the grid points) that are neighbors of a given pixel.

$$\tau_1 = (0,1)$$
$$\tau_2 = (1,0)$$
$$\tau_3 = (1,1)$$
$$\tau_4 = (-1,1)$$

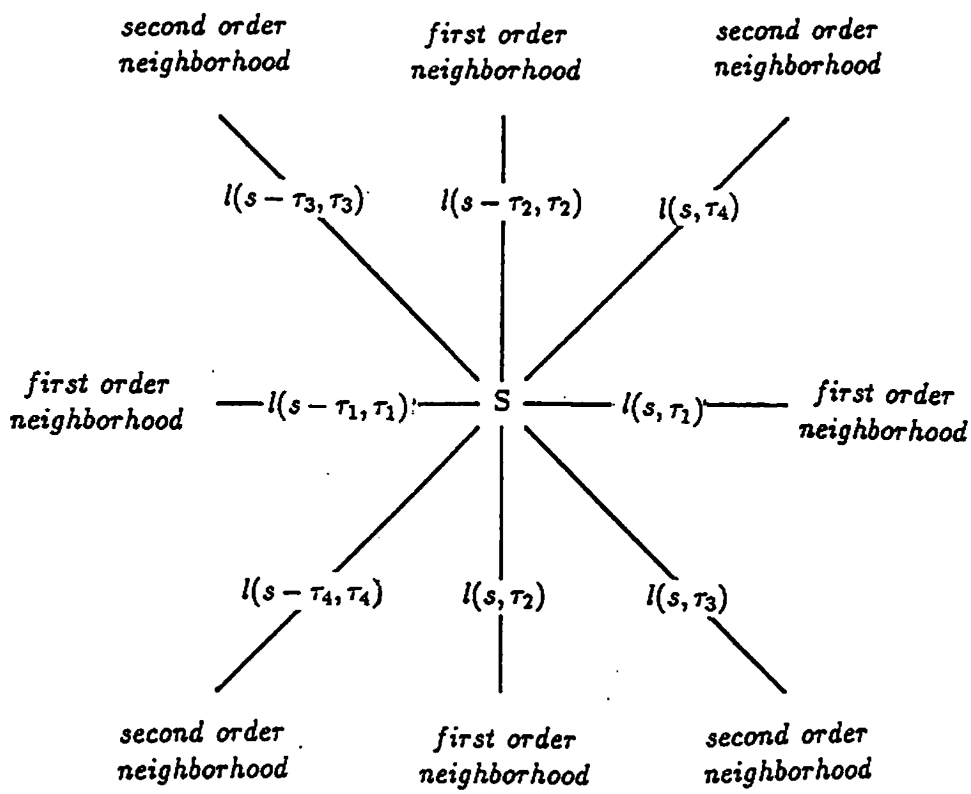

Fig 0: The Line Process Notation

The prior probability of the line process is

$$p(l(s,\tau_i)|l(r,\tau)\ ,\ r\in\Omega\ ,\ \tau\in N^*\ ,\ (r,\tau)\neq(s,\tau_i)) = \frac{e^{-U(l(s,\tau_i)|l(r,\tau)\ ,\ r\in\Omega\ ,\ \tau\in N^*\ ,\ (r,\tau)\neq(s,\tau_i))}}{Z}$$

where

$$U(l(s,\tau_i)|l(r,\tau)\ ,\ r\in\Omega\ ,\ \tau\in N^*\ ,\ (r,\tau)\neq(s,\tau_i)) = \beta l(s,\tau_i)$$

Recall that for a zero mean GMRF model [8]

$$U(y(s)|y(s+\tau),y(s-\tau)\ ,\ \tau\in N^*) = \frac{1}{2\nu}(y(s)-\sum_{\tau\in N^*}\Theta_\tau[y(s+\tau)+y(s-\tau)])^2$$

It is obvious that the conditional distribution of the compound GMRF model is equal to the GMRF model when $l(s,\tau)=0, l(s-\tau,\tau)=0\ ,\ \forall\tau\in N^*$. We shall assume that all $\Theta_\tau>0$ which is almost always the case for the first order neighborhoods.

The corresponding joint distribution of the compound GMRF is

$$p(\mathbf{y},\mathbf{l}) = \frac{e^{-U(\mathbf{y},\mathbf{l})}}{Z}$$

where

$$\begin{aligned} U(\mathbf{y},\mathbf{l}) &= \sum_{s\in\Omega}\Big\{\sum_{\tau\in N^*}\Theta_\tau[\frac{1}{2\nu}(y(s)-y(s+\tau))^2(1-l(s,\tau))+\beta l(s,\tau)] \\ &+ \frac{1}{2\nu}(1-\sum_{\tau\in N^*}2\Theta_\tau)y(s)^2\Big\} \end{aligned}$$

One can see that if $(y(s)-y(s+\tau))$ is bigger then a threshold, it is cheaper to break the connection and introduce an edge in the same way it is done in the weak membrane formulation. The global energy corresponding to the compound GMRF model with additive noise is:

$$
\begin{aligned}
U(\mathbf{y},\mathbf{l}|\mathbf{x}) &= \sum_{s\in\Omega}\left\{\frac{(x(s)-y(s))^2}{2\sigma^2}+\frac{(1-\sum_{\tau\in N^*}2\Theta_\tau)y(s)^2}{2\nu}\right.\\
&+ \left.\sum_{\tau\in N^*}\Theta_\tau[\frac{1}{2\nu}(y(s)-y(s+\tau))^2(1-l(s,\tau))+\beta l(s,\tau)]\right\}\\
&= \frac{1}{2\sigma^2}\sum_{s\in\Omega}\left\{(x(s)-y(s))^2+\lambda^2(1-\sum_{\tau\in N^*}2\Theta_\tau)y(s)^2\right.\\
&+ \left.\sum_{\tau\in N^*}\Theta_\tau[\lambda^2(y(s)-y(s+\tau))^2(1-l(s,\tau))+\alpha l(s,\tau)]\right\} \qquad (5)
\end{aligned}
$$

where $\lambda^2 = \frac{\sigma^2}{\nu}$, $\alpha = 2\beta\sigma^2$. The regularization parameter λ which did not have a quantitative value in the weak membrane model, has both qualitative value and quantitative meaning in the new model. It reflects the confidence we have in the data, as it is the ratio of the measurement noise variance to the GMRF model variance.

2.3 The GNC Algorithm

One can see the similarity between the global energy function corresponding to the posterior density in equation (5) and the global energy Blake suggested to minimize in the weak membrane formulation (4). The weak membrane model is a special case of the compound GMRF model with a first order neighborhood system and isotropic parameters $\Theta_x = \Theta_y = 1$. Blake has developed the GNC algorithm to find the global minimum of (4). We modify the GNC algorithm to obtain an algorithm which is able to find the global MAP solution for the compound GMRF model. For simplicity in the formulation we restrict our attention to a first order compound GMRF model although there are no difficulties in deriving the algorithm for higher order neighborhood systems.

For the first order compound GMRF model we can write:

$$
\begin{aligned}
U(\mathbf{y},\mathbf{l},\mathbf{m}|\mathbf{x}) &= \sum_{i,j\in\Omega} \frac{1}{2\sigma^2}\Big\{(x_{i,j}-y_{i,j})^2 \\
&+ \lambda^2(1-2(\Theta_x+\Theta_y))y_{i,j}^2 \\
&+ \Theta_x[\lambda^2(y_{i,j}-y_{i+1,j})^2(1-l_{i,j})+\alpha l_{i,j}] \\
&+ \Theta_y[\lambda^2(y_{i,j}-y_{i,j+1})^2(1-m_{i,j})+\alpha m_{i,j}]\Big\}
\end{aligned}
$$

where $l_{i,j}, m_{i,j}$ activate the line process in the x and y directions respectively. We can write the posterior energy function in the form:

$$
U = \frac{1}{2\sigma^2}\left\{D + \sum_{ij}\Theta_x h_{\alpha,\lambda}(y_{i,j}-y_{i-1,j}, l_{ij}) + \sum_{ij}\Theta_y h_{\alpha,\lambda}(y_{i,j}-y_{i,j+1}, m_{ij})\right\} \tag{6}
$$

where

$$
D = \sum_{ij}(y_{i,j}-x_{i,j})^2 + \lambda^2(1-2(\Theta_x+\Theta_y))y_{i,j}^2 \tag{7}
$$

where

$$
h_{\alpha,\lambda}(t,l) = \lambda^2(t)^2(1-l)+\alpha l
$$

Note that $(1-2(\Theta_x+\Theta_y)) > 0$ because of the positivity requirement of the spectral density of the GMRF model. The problem is thus reduced to the following optimization problem.

$$
\min_{\{y_{ij}\}}\left(\frac{1}{2\sigma^2}\left[D + \min_{\{l_{ij}\}}\left(\sum_{ij}\Theta_x h_{\alpha,\lambda}(y_{i,j}-y_{i-1,j}, l_{ij})\right) + \min_{\{m_{ij}\}}\left(\sum_{ij}\Theta_y h_{\alpha,\lambda}(y_{i,j}-y_{i,j+1}, m_{ij})\right)\right]\right)
$$

As D does not involve l_{ij}, m_{ij}, minimization over l_{ij}, m_{ij} can be performed and one is then left with minimization with respect to u_{ij}:

$$
\min_{\{y_{ij},l_{ij},m_{ij}\}} U = \min_{\{y_{ij}\}}\left\{\frac{1}{2\sigma^2}\left[D + \sum_{ij}\Theta_x g_{\alpha,\lambda}(y_{i,j}-y_{i-1,j}) + \sum_{ij}\Theta_y g_{\alpha,\lambda}(y_{i,j}-y_{i,j+1})\right]\right\}
$$

where

$$g_{\alpha,\lambda}(t) = \min_{l\in\{0,1\}} h_{\alpha,\lambda}(t,l) = \min(\lambda^2(t)^2, \alpha)$$

Following Blake we look for a convex approximation to U. The convexity of U^* is guaranteed by requiring that it has a positive definite Hessian matrix $H = \frac{\partial^2 U^*}{\partial y_{i,j}\partial y_{k,l}}$. Suppose g^* is designed to satisfy

$$\forall t \;\; g^{*''}(t) \geq -c^*$$

where $c^* > 0$. Then the "worst case" of H occurs when

$$\forall i,j \;\; g^{*''}(y_{i,j} - y_{i,j+1}) = -c^* \text{ and } g^{*''}(y_{i,j} - y_{i-1,j}) = -c^*$$

so that

$$H = [2 + 2\lambda^2(1 - 2(\Theta_x + \Theta_y))]I - c^* R$$

The matrix R is a symmetric tri diagonal block Toeplitz matrix:

$$R = \begin{bmatrix} B & -\Theta_x I & & \\ -\Theta_x I & B & -\Theta_x I & \\ & -\Theta_x I & B & -\Theta_x I \\ & & -\Theta_x I & B \end{bmatrix} \quad B = \begin{bmatrix} 2\Theta_x + 2\Theta_y & -\Theta_y & & \\ -\Theta_y & 2\Theta_x + 2\Theta_y & -\Theta_y & \\ & -\Theta_y & 2\Theta_x + 2\Theta_y & -\Theta_y \\ & & -\Theta_y & 2\Theta_x + 2\Theta_y \end{bmatrix}$$

To prove that H is positive definite it is sufficient to show that the largest eigenvalue $\mathcal{E}_{max}$ of R satisfies $\mathcal{E}_{max} \leq \frac{2+2\lambda^2(1-2(\Theta_x+\Theta_y))}{c^*}$.

The eigenvalues $\mathcal{E}_{i,j}$, $i,j \in (0 \cdots M-1)$ of R can be found using cosine transform [9].

$$\mathcal{E}_{i,j} = \Theta_x(2 - 2\cos\frac{2\pi i}{M}) + \Theta_y(2 - 2\cos\frac{2\pi j}{M})$$

So that $\max \mathcal{E}_{i,j} = 4(\Theta_x + \Theta_y)$, thus to guarantee convexity c^* must satisfy

$$c^* \leq \frac{1 + \lambda^2(1 - 2(\Theta_x + \Theta_y))}{2(\Theta_x + \Theta_y)}$$

Following Blake we construct the best quadratic approximation g^* with a given bound $-c^*$ on its second derivative, satisfying the extra condition: $\forall t \; g^*(t) \leq g(t)$

$$g^*_{\alpha,\lambda}(t) = \begin{cases} \lambda^2(t)^2, & |t| < q \\ \alpha - c^*(|t| - r)^2/2, & q \leq |t| < r \\ \alpha, & |t| \geq r \end{cases}$$

where

$$r^2 = \alpha\left(\frac{2}{c^*} + \frac{1}{\lambda^2}\right), \quad q = \frac{\alpha}{\lambda^2 r}.$$

Thus we obtain the convex approximation for U:

$$U^* = \frac{1}{2\sigma^2}\left[D + \sum_{ij} \Theta_x g^*_{\alpha,\lambda}(y_{i,j} - y_{i-1,j}) + \sum_{ij} \Theta_y g^*_{\alpha,\lambda}(y_{i,j} - y_{i,j+1})\right] \tag{8}$$

A one parameter family of cost functions $U^{(P)}$ is then defined by replacing g^* in (8) by $g^{(P)}$. $g^{(P)}$ is similar to g^* except that c^* is replaced by a variable c, that varies with P.

$$U^{(P)} = \frac{1}{2\sigma^2}\left[D + \sum_{ij} \Theta_x g^{(P)}_{\alpha,\lambda}(y_{i,j} - y_{i-1,j}) + \sum_{ij} \Theta_y g^{(P)}_{\alpha,\lambda}(y_{i,j} - y_{i,j+1})\right] \tag{9}$$

with

$$g^{(P)}_{\alpha,\lambda}(t) = \begin{cases} \lambda^2(t)^2, & |t| < q \\ \alpha - c(|t| - r)^2/2, & q \leq |t| < r \\ \alpha, & |t| \geq r \end{cases}$$

where $c = \frac{c^*}{P}$, $r^2 = \alpha\left(\frac{2}{c} + \frac{1}{\lambda^2}\right)$, and $q = \frac{\alpha}{\lambda^2 r}$

The GNC algorithm begins by minimizing $U^{(P=1)} = U^*$. Then P is decreased from 1 to 0, which makes $g^{(P)}$ change steadily from g^* to g. For every value of P we minimize $U^{(P)}$ starting with the last configuration corresponding to the previous P (local minimum of $U^{(2P)}$). We suggest that minimization of $U^{(P)}$ can be performed efficiently using the optimal step conjugate gradient algorithm. The conjugate gradient search is not sensitive to the noise in the processed image. The number of iterations increases only by 40% in the presence of 0 dB noise, while the successive over relaxation (SOR) algorithm requires the double number of iterations. The GNC algorithm based on conjugate gradient search can be stated as follows:

1. choose h_0 (the edge sensitivity).

2. set $\alpha = h_0^2\lambda/2$.

3. set $P = 1.0$

4. given the initial configuration u^0 compute $G_0 = \nabla F^P(u^0)$ and set $D_0 = -G_0$.

5. calculate (using a univariate search) the optimal step for the given D_k using the global energy function.

6. perform the descent step

$$u^{(k+1)} = u^{(k)} + \alpha_k D_k$$

7. calculate the gradient vector $G_{k+1} = \nabla U^P(y^{(k+1)})$

8. calculate a new conjugate direction $D_{k+1} = -G_{k+1} + \beta_k D_k$ where

$$\beta_k = \frac{(G_{k+1} - G_k)^T G_{k+1}}{G_k^T G_k}$$

9. repeat steps 5-8 until $\max_{i,j} |y_{i,j}^{(k+1)} - y_{i,j}^{(k)}| < \epsilon$, or the energy reduction is below some level.

10. if $P > \frac{c^*}{\lambda}$ then $P = P/2$ $u^0 = u^{(k+1)}$ go back to step 4.

11. calculate the edge location using the following rules:

$$l_{i,j} = \begin{cases} 1 & \text{if } |y_{i,j} - y_{i-1,j}| > r \\ 0 & \text{if } |y_{i,j} - y_{i-1,j}| < q \\ \text{ambiguous} & \text{otherwise} \end{cases}$$

$$m_{i,j} = \begin{cases} 1 & \text{if } |y_{i,j} - y_{i,j+1}| > r \\ 0 & \text{if } |y_{i,j} - y_{i,j+1}| < q \\ \text{ambiguous} & \text{otherwise} \end{cases}$$

$l_{i,j}$ and $m_{i,j}$ are the edge indicator functions in the x and y directions respectively.

3 Parameter Estimation

The compound GMRF model presented in section 2 presents a new problem in parameter estimation. We would like to estimate the GMRF parameters only in homogeneous regions that do not include edges. The problem is that even if the location of the edges is known, removing the adjacent pixels from the rectangular domain we started with, leaves a highly irregular domain, which is cumbersome to work with. Furthermore in the case of ML estimation, the irregularity of the domain makes it impossible to write a close form expression for the likelihood function even if toroidal assumption are made. Instead we suggest a new algorithm based on the EM algorithm. For the time being we present the algorithm for estimating the parameters from the original image . We believe that the method can be extended to estimation techniques such as BCLS [7] which are based on the noisy image.

The idea behind the new algorithm is that we ignore the intensity data in strips centered at the edges and replace this data by the conditional mean of the GMRF model given the model parameters and the neighboring pixels intensity data. The conditional mean is calculated using a relaxation method, i.e. for the first order neighborhood we repeatedly set

$$y^{k+1}(i,j) = \Theta_x(y^k(i+1,j) + y^k(i-1,j)) + \Theta_y(y^k(i,j+1) + y^k(i,j-1)) \qquad (10)$$

Note that we modify the intensity only in the strips centered at the edges. The iterations are bound to converge, because the stability requirement of the GMRF model parameters ensure that the eigenvalues of the iteration matrix are smaller than 1. Once we computed the conditional mean, we perform an estimation step based on LS or ML on the smoothed image. We use the estimated parameters to calculate a new conditional mean for the pixels in the edge vicinity, which in turn are used for a new parameter estimation step. In this paper we present results for the EM algorithm based on LS. We are currently working on the ML version. We use Blakes edge detector [5] to find the location of the edges in the image. In future, we intend to combine the image and parameters estimation. The results we obtain with the new algorithm differ significantly from the results obtained directly from the original image. For example, the estimated model variance is much smaller, although the image variance is reduced only slightly. The results show the importance of suppressing the edges effect in the parameter estimation. The new algorithm is summarized as follows:

- 1. Get an initial estimate for the model parameters, using the LS technique on the original image.
- 2. Find the image edges location using Blakes GNC algorithm.
- 3. By using the current estimate of the model parameters find the conditional mean of the intensity in a strip four pixels wide centered at the edge using the relaxation

equation (10).

- 4. Calculate a new parameter estimate on the smoothed image calculated in step 3.

- 5. Check if the parameters are stable. If not scale them by multiplying the vector Θ by: $\psi = \frac{0.499}{\max_{s\in\Omega}\sum_{\tau_i\in N^*}\Theta_{\tau_i}\Phi_{s,\tau_i}}$ where $\Phi_{s,\tau_i} = \cos(\frac{2\pi s^T\tau_i}{M})$, and $N^* = \{\tau_1, \tau_2\} = \{(0,1),\ (1,0)\}$ are the shift vectors corresponding to the first order GMRF model, to ensure that the scaled parameters $\hat{\Theta}_{\tau_i}$ $\tau_i \in N^*$ satisfy:

$$(1 - 2\sum_{\tau_i\in N^*}\hat{\Theta}_{\tau_i}\Phi_{s,\tau_i}) > 0 \quad \forall s \in \Omega \qquad (11)$$

- 6. If the change in the parameters is small enough stop, else go back to step 3.

Steps 4 and 5 of the algorithm can be replaced by constraint maximization of the likelihood function. We are currently working on the ML version of the algorithm. We experimented the new algorithm on a real airport image and obtained the results summarized in Table 1. Note that the parameter values do not change much after 10 iterations.

Iteration	Θ_x	$\hat{\Theta}_x$	Θ_y	$\hat{\Theta}_y$	ν	$\hat{\nu}$	Image Variance
0	0.282700	0.266150	0.247330	0.232850	25.6184	51.97603	469.4704
1	0.300411	0.291597	0.213673	0.207403	6.55542	16.98208	359.0647
5	0.310806	0.304306	0.198853	0.194694	5.99739	12.96653	336.8897
14	0.312426	0.306.504	0.196215	0.192496	5.98000	12.55508	350.1088

Table 1: Parameter Estimation Results for Airport Image,Using the EM Algorithm.

4 Restoration Results

The original airport image corrupted by 5dB and 10dB additive white Gaussian noise was reconstructed using the modified GNC algorithm. As an initial condition for the algorithm we used the noisy image. Parameters were estimated from the original image using the new EM algorithm described in section 3. The results were obtained using at most 25 iterations for each value of P. For some values of P the algorithm converged in less then 20 iterations.

In the first experiment we restored an image corrupted by 10dB additive white Gaussian noise. The edge threshold was chosen to be $h = 20$ and the noise variance was assumed to be known. The results after 100 iterations are presented in Figure 1. We then repeated the experiment for the 5dB additive white Gaussian noise case. In this experiment h was set to 25. The results are presented in Figure 2. The results we obtained for 10dB noise are very good. The estimates include all the information in the original image and in addition we obtain a precise edge map. In the 5dB case the results are good also, but a few noisy points were isolated by the line process. This is an inherent problem owing to the simple line process used in this model. It can partially be overcomed by increasing λ. One can also modify the line process to include an energy term that will inhibit the formation of such edges as in [10], but the GNC algorithm can no longer be used for the more sophisticated line process.

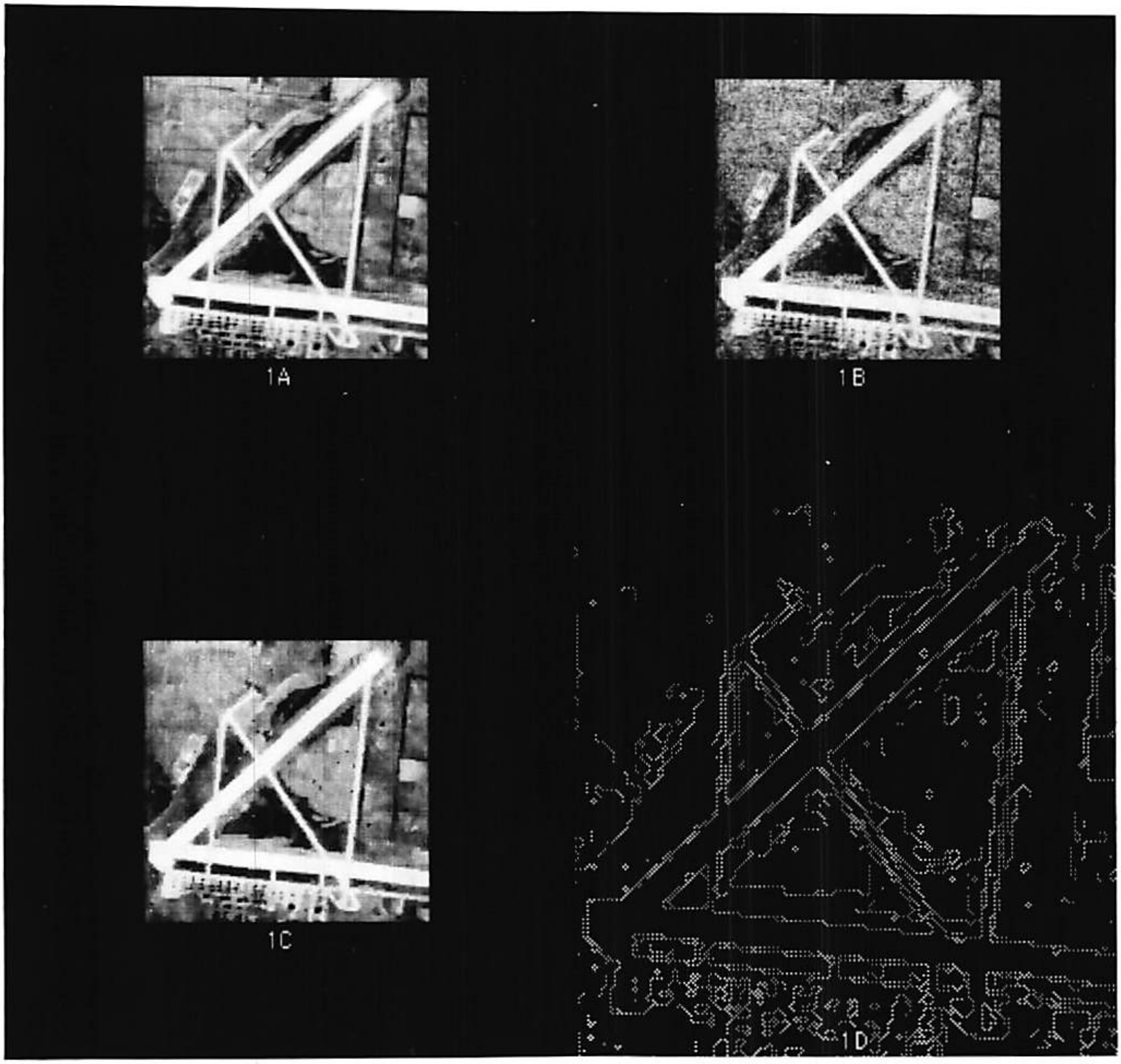

Figure 1: Restoration and Edge Detection on an Airport Image with 10dB Noise

1 A: Original 128 × 128 Airport Image ;

1 B: Image corrupted by 10dB noise ;

1 C: GNC estimate after 100 iterations. ;

1 D: An Edge Map Obtained from the GNC Algorithm ;

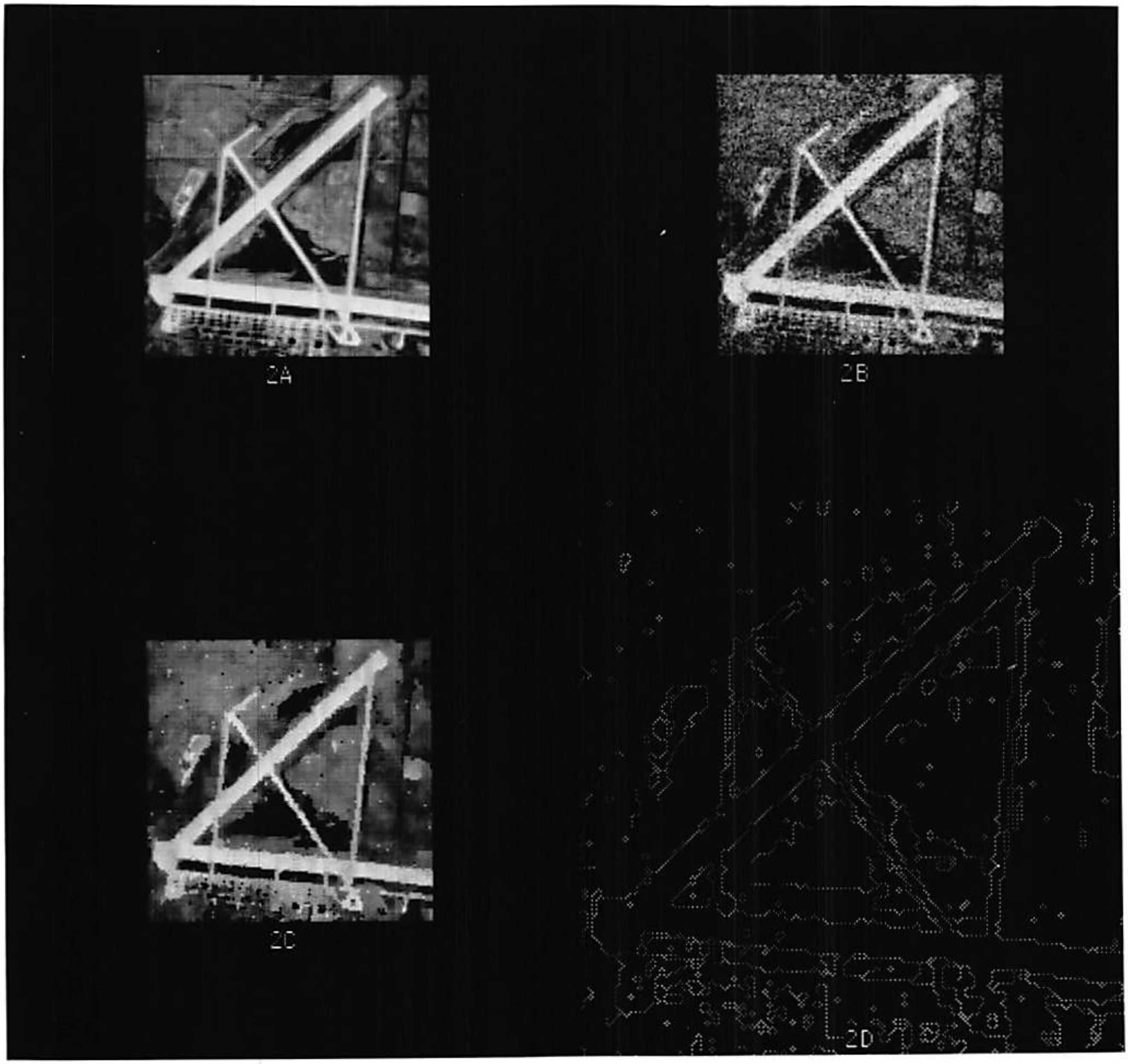

Figure 2: Restoration and Edge Detection on an Airport Image with 5dB Noise

2 A: Original 128 × 128 Airport Image ;

2 B: Image corrupted by 5dB noise ;

2 C: GNC estimate after 100 iterations. ;

2 D: An Edge Map Obtained from the GNC Algorithm ;

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