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On Blind Identification of A Band-Limited Nonminimum-Phase Channel from Its Output Autocorrelation

by

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On Blind Identification of A Band-Limited Nonminimum-Phase Channel from Its Output Autocorrelation[1]

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Abstract: It has been well established in the literature that when the input to a linear time invariant (LTI) channel (system) is a continuous-time white noise, the identification of the phase of the channel from its output autocorrelation is impossible. To make the channel identification from its output autocorrelation unique, we need to know a priori the whole phase response of the channel. In this paper, we show that when the input is a discrete-time white noise sequence, what is needed to make the channel identification from its output autocorrelation unique is reduced to the phase response in the frequency range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$.

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1 Introduction

In high-speed data communications, there exist several scenarios, such as multipoint network and multipath fading channel, where blind equalization (equalization without the transmission of a training sequence) has to be performed to mitigate the channel distortion caused by intersymbol interference. A closely related problem to blind equalization is blind identification, which is to identify the impulse response of a LTI channel from its output only, when the input to the channel is white. It is well established [1] that when the input to a channel is a continuous-time white noise, no phase information can be extracted from the second order statistics (thus the autocorrelation) of the channel output. However, the situation can be different when the input to the channel is a discrete-time white noise sequence, as in data communications scenarios. The received signal (before sampling) becomes cyclostationary, instead of being stationary. In [2], Gardner proposes a channel identification method by exploiting the cyclostationarity of the received signal. The method is based on the second order statistics of the received signal. However, the method assumes that a pilot signal is superimposed with transmitted data and that we know a priori the shape of the pilot signal. Thus, it is not a true blind identification method. In this paper, we address the problem of blind identification of a band-limited nonminimum-phase channel from its output autocorrelation when the input is a discrete-time white noise sequence. The paper is organized as follows. In Sec. 2, we show that the received signal is cyclostationary when the input is a discrete-time white noise sequence. In Sec. 3, we address the blind identifiability of a LTI channel from its output autocorrelation only.

2 Cyclostationarity of The Received Signal

Let us consider a synchronous double-sideband quadrature amplitude modulated (QAM) communication system as shown in Fig. 1. Discrete-time data, a_n, are transmitted every T interval through a LTI channel, which models the effect of the shaping filter, modulation, transmission media, demodulation, and matched filter on the transmitted data, a_n. The channel output is the continuous-time received signal $y(t)$ given by[1]

$$y(t) = a_n * h(t) = \sum_n a_n h(t - nT) \tag{1}$$

where $*$ denotes convolution.

[1]In practice, the received signal is contaminated by additive noise. However, in data communications scenarios where channel equalization is used, intersymbol interference usually has much greater effect than noise. Therefore, the noise is assumed to be negligible and omitted in the analysis.

Now we show that the received signal is cyclostationary with its period equal to T. The mean of $y(t)$ is given by

$$\begin{aligned} E\{y(t)\} &= E\{\sum_n a_n h(t-nT)\} \\ &= \sum_n E(a_n)h(t-nT) \\ &= m_a \sum_n h(t-nT) \end{aligned} \tag{2}$$

which is periodic with period T. The autocorrelation of $y(t)$ is given by

$$\begin{aligned} R_y(t_1,t_2) &= E\{\sum_n\sum_m a_n h(t_1-nT)a_m^* h^*(t_2-mT)\} \\ &= \sum_n\sum_m E(a_n a_m^*)h(t_1-nT)h^*(t_2-mT) \\ &= \sum_n\sum_m R_a(n,m)h(t_1-nT)h^*(t_2-mT) \end{aligned} \tag{3}$$

which is also periodic with period T. That is,

$$R_y(t_1+kT, t_2+kT) = R_y(t_1,t_2) \tag{4}$$

Therefore, the received signal (before sampling) is cyclostationary with its period equal to the transmitting interval T.

3 Blind Identifiability of A Channel from Its Output Autocorrelation

We consider the synchronous double-sideband QAM communication system shown in Fig. 1. A discrete-time white data sequence a_n is transmitted through a LTI channel $h(t)$. The channel output is the continuous-time received signal $y(t)$. In practice, the continuous-time received signal is usually first sampled and the obtained samples are subsequently used for blind identification. However, information may get lost in the sampling process, for example, the cyclostationarity of the received signal is lost in symbol-rate sampling. That is why we consider the received signal before sampling. The problem is whether the autocorrelation of the continuous-time received signal $y(t)$ contains sufficient information to uniquely determine the channel impulse response $h(t)$.

From (3) $R_y(t_1,t_2)$ is related to $R_a(n_1,n_2)$ by the following equation.

$$R_y(t_1,t_2) = \sum_{n_1}\sum_{n_2} R_a(n_1,n_2)h(t_1-n_1T)h^*(t_2-n_2T) \tag{5}$$

By taking two-dimensional Fourier transform on both sides of (5), we obtain

$$S_y(f_1,f_2) = S_a(f_1,f_2)H(f_1)H^*(-f_2) \tag{6}$$

where

$$\begin{aligned} S_y(f_1,f_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_y(t_1,t_2)e^{-j2\pi(f_1t_1+f_2t_2)}dt_1dt_2 &(7)\\ S_a(f_1,f_2) &= \sum_{n_1}\sum_{n_2} R_a(n_1,n_2)e^{-j2\pi(f_1n_1T+f_2n_2T)} &(8)\\ H(f) &= \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}dt &(9)\end{aligned}$$

The input a_n is white, $R_a(n_1,n_2) = \sigma_a^2\delta(n_1-n_2)$. By substituting it into (8) and (6), we obtain

$$\begin{aligned} S_a(f_1,f_2) &= \sum_k \sigma_a^2\delta(f_1+f_2+\frac{k}{T}) &(10)\\ S_y(f_1,f_2) &= \sum_k \sigma_a^2\delta(f_1+f_2+\frac{k}{T})H(f_1)H^*(-f_2) &(11)\end{aligned}$$

Thus, $S_y(f_1,f_2)$ is nonzero only on those lines of $f_1+f_2+\frac{k}{T}=0$ where k is an integer (Fig. 2). Note that $S_y(-f_2,-f_1) = S_y^*(f_1,f_2)$. Therefore, the values of $S_y(f_1,f_2)$ on the lines of $f_1+f_2+\frac{k}{T} = 0$ for $k \geq 0$ completely specifies the autocorrelation $R_y(t_1,t_2)$. By writing $S_y(f_1,f_2) = |S_y(f_1,f_2)|\ exp(j\psi(f_1,f_2))$ and $H(f) = |H(f)|\ exp(j\phi(f))$, we have on those lines of $f_1+f_2+\frac{k}{T}=0,\ k\geq 0$

$$\psi(f_1,f_2) = \phi(f_1) - \phi(-f_2) = \phi(f_1) - \phi(f_1+\frac{k}{T}) \tag{12}$$

Now we determine the blind identifiability of a channel from its output autocorrelation, or equivalently, the determinacy of $H(f)$ from the values of $S_y(f_1,f_2)$ on the lines of $f_1+f_2+\frac{k}{T}=0$, for $k\geq 0$.

On the line of $f_1+f_2=0$,

$$S_y(f_1,f_2) = \sigma_a^2\delta(0)H(f_1)H^*(-f_2) = \sigma_a^2\delta(0)|H(f_1)|^2 \tag{13}$$

Thus, $|H(f)|$ can be determined from the values of $S_y(f_1,f_2)$ on this line. What remains to be determined from $S_y(f_1,f_2)$ is the channel phase response $\phi(f)$.

The phase response $\phi(f)$ is related to $S_y(f_1,f_2)$ by (12). From (12), we see that $\phi(f)$ is related to $\phi(f+\frac{k}{T})$ only. Thus, the phase identification problem can be decomposed into solving infinitely many sets of equations. For each f in the range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$, the set of equations of (12) which relates directly or indirectly to $\phi(f)$ can be written in the following form:

$$\phi(f+\frac{i}{T}) - \phi(f+\frac{j}{T}) = \psi(f+\frac{i}{T}, -(f+\frac{j}{T})) \text{ for } i < j \tag{14}$$

For a band-limited channel ($H(f) = 0$ for $|f| \geq W$), let

$$\begin{aligned} m &= \lceil -(W+f)T \rceil \\ n &= \lfloor (W-f)T \rfloor \\ f_k &= f + \frac{k}{T} \end{aligned}$$

where $\lceil a \rceil and \lfloor a \rfloor$ denote the smallest integer larger than a and the largest integer smaller than a, respectively. Then all the possible equations of (14) are as follows:

$$\begin{aligned} \phi(f_m) - \phi(f_{m+1}) &= \psi(f_m, -f_{m+1}) \\ \phi(f_m) - \phi(f_{m+2}) &= \psi(f_m, -f_{m+2}) \\ &\vdots \\ \phi(f_m) - \phi(f_n) &= \psi(f_m, -f_n) \\ \phi(f_{m+1}) - \phi(f_{m+2}) &= \psi(f_{m+1}, -f_{m+2}) \\ \phi(f_{m+1}) - \phi(f_{m+3}) &= \psi(f_{m+1}, -f_{m+3}) \\ &\vdots \\ \phi(f_{n-1}) - \phi(f_n) &= \psi(f_{n-1}, -f_n) \end{aligned}$$

Combining all these equations into matrix form we obtain

$$\boldsymbol{A\phi = \psi} \tag{15}$$

where

$$\begin{aligned} \boldsymbol{\phi} &= (\phi(f_m), \phi(f_{m+1}), \cdots, \phi(f_n))^T \\ \boldsymbol{\psi} &= (\psi(f_m, -f_{m+1}), \psi(f_m, -f_{m+2}), \cdots, \psi(f_m, -f_n), \\ &\quad \psi(f_{m+1}, -f_{m+2}), \psi(f_{m+1}, -f_{m+3}), \cdots, \psi(f_{n-1}, -f_n))^T \end{aligned}$$

and $\boldsymbol{A}$ is a sparse coefficient matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 0 & 0 & . & . & & 0 \\ 1 & 0 & -1 & 0 & . & . & & 0 \\ . & . & . & . & . & . & & . \\ . & . & . & . & . & . & & . \\ 1 & 0 & 0 & 0 & . & . & & -1 \\ 0 & 1 & -1 & 0 & . & . & & 0 \\ 0 & 1 & 0 & -1 & . & . & & 0 \\ . & . & . & . & . & . & & . \\ . & . & . & . & . & . & & . \\ 0 & 0 & 0 & 0 & . & . & 1 & -1 \end{bmatrix} \tag{16}$$

In Equations (15), there are $n - m + 1$ unknown variables $\phi(f_m), \phi(f_{m+1}), \cdots, \phi(f_n)$, but the rank of $\boldsymbol{A}$ is $n - m$ (Appendix I), thus **the equations are underdetermined.** Moreover, the matrix formed by deleting any one column of $\boldsymbol{A}$ still has the rank $n - m$ (Appendix I). Therefore, **if we know any one of $\phi(f_k)$, we can determine the other unknown phase variables through (15).** Several remarks are provided below.

1. Because the above arguments are true for arbitrary f in the range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$, what is needed to make the channel identification from its output autocorrelation unique is reduced to the phase response in the frequency range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$. That is, if we know a priori the phase response $\phi(f)$ for $-\frac{1}{2T} \leq f < \frac{1}{2T}$, the channel is uniquely determined by its output autocorrelation.

2. For a real channel, $S_y(f_1, f_2)$ has the additional symmetry with respect to the line of $f_1 = f_2$, and what is needed to make the channel identification from its output autocorrelation unique is further reduced to the phase response in the frequency range of $0 \leq f \leq \frac{1}{2T}$ because of the odd symmetry of the phase response.

3. In data communications, the channels are band-limited. When data are transmitted at Nyquist rate, that is, when $T = \frac{1}{2W}$, the phase response in the frequency range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$ is the whole phase response (Fig. 3). Therefore, nothing can be gained by employing the cyclostationarity of the received signal. This is not the case with sub-Nyquist transmission (Fig. 4). In that case, the phase response in the frequency range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$ is only a portion of the whole phase response.

4 Conclusions

We show in this paper that when the input to a LTI channel is a discrete-time white noise sequence, the identification of the channel from its output autocorrelation is still not unique, although what is needed to make the channel identification from its output autocorrelation unique is reduced to the phase response in the frequency range of $-\frac{1}{2T} \leq f < \frac{1}{2T}$.

Appendix I. Rank of the Coefficient Matrix

From (16), we observe that A is composed of all the possible row vectors of the following form:

$$a_{ij} = (0, \cdots, 0, \overset{i}{1}, 0, \cdots, 0, \overset{j}{-1}, 0, \cdots, 0)$$

where $1 \leq i < j \leq n - m + 1$.

The matrix $\tilde{A}$ which consists of the row vectors $\{a_{ij} : j = i + 1, i = 1, 2, \cdots, n - m\}$

$$\tilde{A} = \begin{array}{cc} \begin{bmatrix} 1 & -1 & 0 & 0 & . & . & & 0 \\ 0 & 1 & -1 & 0 & . & . & & 0 \\ . & . & . & . & . & . & & . \\ . & . & . & . & . & . & & . \\ 0 & 0 & 0 & 0 & . & . & 1 & -1 \end{bmatrix} & \begin{matrix} a_{12} \\ a_{23} \\ . \\ . \\ a_{n-m,n-m+1} \end{matrix} \end{array}$$

is a submatrix of A. And the matrix $\tilde{A}_l$ formed by deleting the lth column of $\tilde{A}$ is given by

$$\tilde{A}_l = \begin{array}{c} \begin{matrix} & & & & & l & & & & \end{matrix} \\ \begin{bmatrix} 1 & -1 & . & . & . & . & . & . & 0 & 0 \\ 0 & 1 & . & . & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & 1 & 0 & . & . & 0 & 0 \\ 0 & 0 & . & . & 0 & -1 & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & -1 & 0 \\ . & . & . & . & . & . & . & . & 1 & -1 \end{bmatrix} \end{array}$$

$\det(\tilde{A}_l) = \pm 1 \neq 0$. Thus, $\text{rank}(\tilde{A}_l) = n - m$ and consequently $\text{rank}(\tilde{A}) = n - m$.

Since every row vector of A is a linear combination of the row vectors of $\tilde{A}$,

$$a_{ij} = a_{i,i+1} + a_{i+1,i+2} + \cdots + a_{j-1,j}$$

Therefore, $\text{rank}(A) = n - m$ and the matrix formed by deleting any one column of A has the rank $n - m$.

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List of Figures

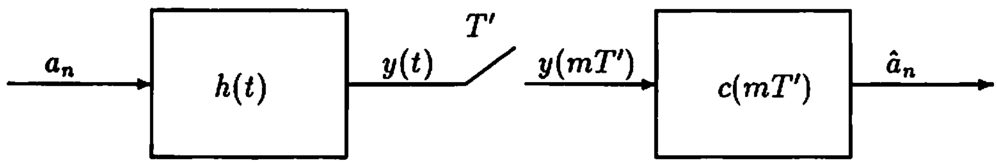

Figure 1: Block diagram of a synchronous data communications system.

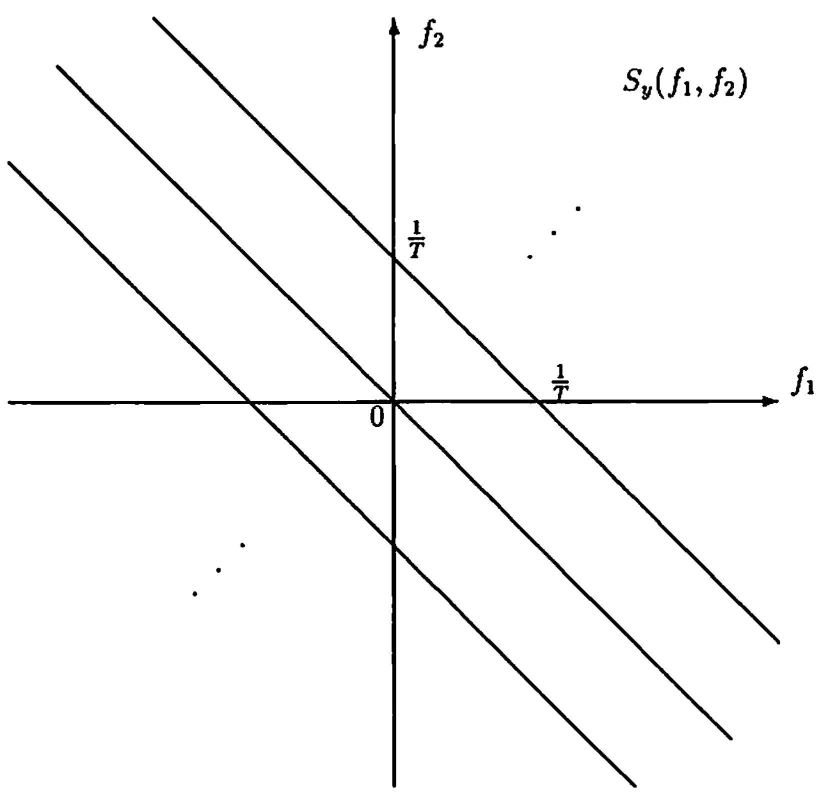

Figure 2: Nonzero lines of $S_y(f_1, f_2)$.

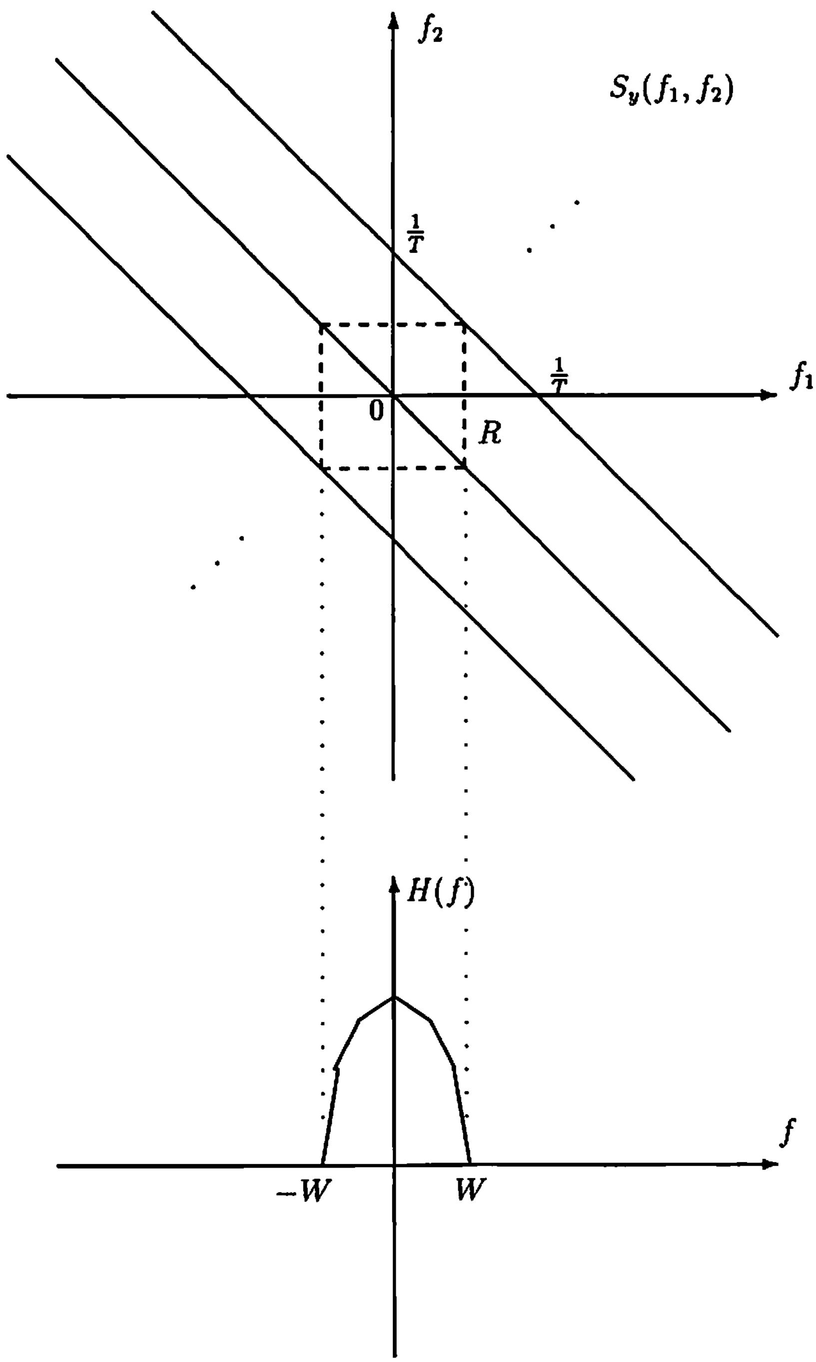

Figure 3: Relation of $S_y(f_1, f_2)$ and $H(f)$ at Nyquist rate.

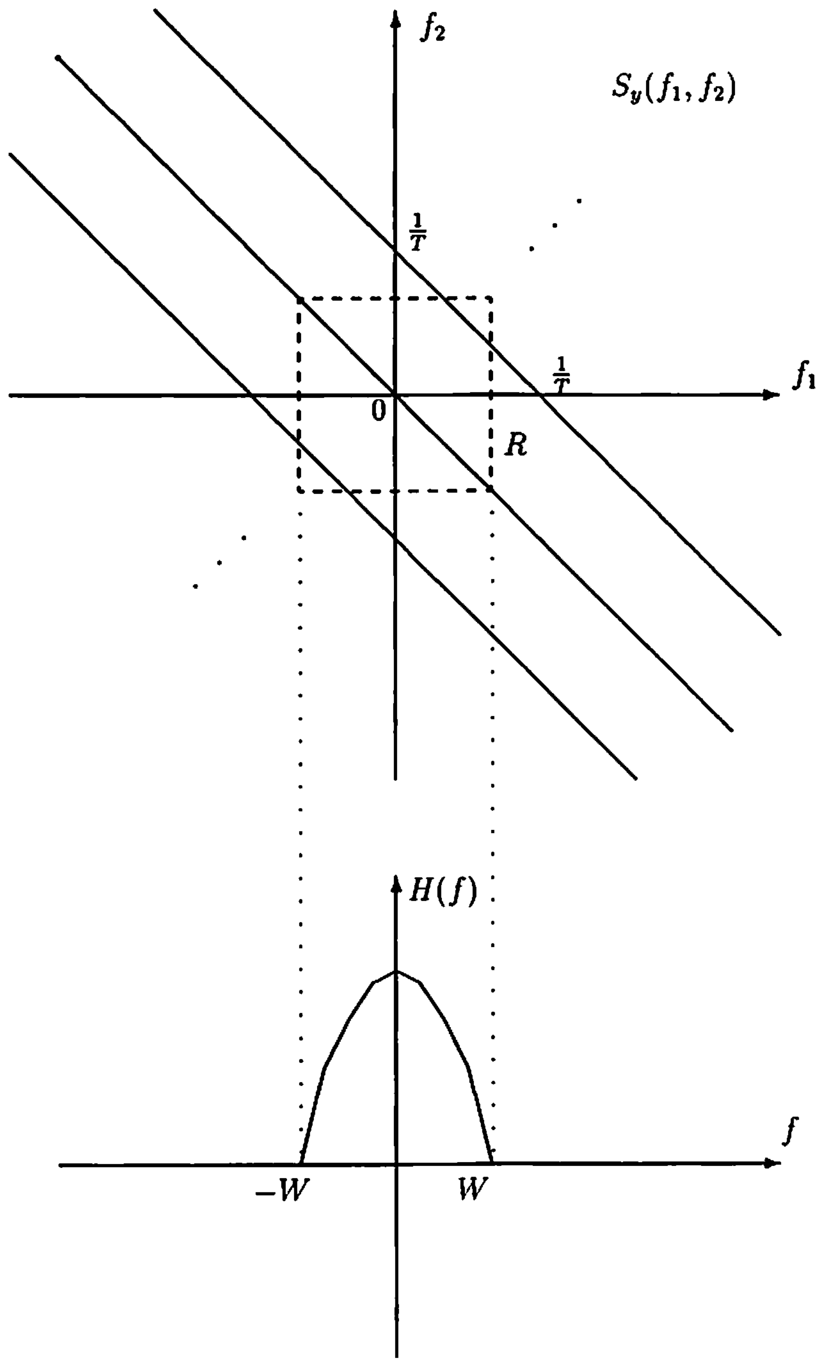

Figure 4: Relation of $S_y(f_1, f_2)$ and $H(f)$ at sub-Nyquist rate.