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by

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Testing of a New Direction Finding Method in Impulsive Interference Environments[1]

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Abstract

In this report, a new direction finding method is introduced for optimal performance over a wide range of non-Gaussian additive noise environments. The maximum likelihood approach is used for the bearing estimation of multiple sources from a set of snapshots when the interference is impulsive in nature. The analysis is based on the assumption that the additive noise can be modeled as a *complex isotropic Cauchy* process. It is shown that the Cauchy beamformer greatly outperforms the Gaussian beamformer in a wide variety of non-Gaussian noise environments, and performs comparably to the Gaussian beamformer when the additive noise is Gaussian. The robustness of the new method in a wide range of impulsive interference environments is demonstrated via extensive simulation experiments. The testing employs two sonar signal waveforms (an acoustic emission, and a single pulse of square CW) in a realistic setting.

1 Introduction

One of the most interesting problems in the context of signal parameter estimation from sensor array data is that of estimating the direction of arrival (DOA) of narrow-band sources of the same known center frequency. In the past, the problem has been studied extensively under the assumption of Gaussian distributed signals and/or noise, and a variety of methods for its solution have been proposed. As a result of the Gaussianity assumption, most methods are based on the second or higher order statistics of the signals.

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The Maximum Likelihood (ML) method was one of the first to be investigated [1]. When applying the ML technique to the source localization problem, two different assumptions for the signal waveforms result into two main methods. According to the *Stochastic ML* (SML) the signals are modeled as Gaussian random processes. This is often motivated by the Central Limit Theorem and results in mathematically convenient expressions. On the other hand, in the *Deterministic ML* (DML) the signals are considered as unknown, deterministic quantities that need to be estimated together with the direction of arrival. This is a natural model for digital communication applications where the signals are far from being normal random variables, and where estimation of the signal is of equal interest.

However, due to the high computational load of the multivariate nonlinear maximization problem involved with the ML estimator, suboptimal methods have also been developed. The better known ones are cited here: Minimum Variance method of Capon [2], and the so called eigenvector–based methods including the MUSIC method of Schmidt [3], and the related Minimum Norm method of Redi [4] and Kumaresan and Tufts [5], and the ESPRIT method of Roy et al. [6]. The performance of the aforementioned methods is inferior to that of the ML method especially for low signal–to–noise ratio (SNR) values or when the number of observation snapshots is small.

The MUSIC method, a generalization of Pisarenko's harmonic retrieval method, has received the most attention and triggered the development of a large number of algorithms referred to as *eigenvector* or *subspace* techniques. Apart from offering a new geometric interpretation of the array processing problem, MUSIC used concepts from complex vector spaces and well known tools from linear algebra, such as the singular value decomposition (SVD), in order to achieve high resolution while keeping the computational complexity relatively low compared to the ML methods.

Recently, special interest has been shown in relaxing some of the assumptions about the statistical nature of the noise in the bearing estimation problem. For example, the bispectrum beamformer was introduced and it was demonstrated that, for the case of spatially correlated Gaussian additive noise with unknown cross–spectral matrix (CSM), it provides better bearing estimates than the stochastic ML method with known CSM [7].

Many times in the real world the Gaussian noise assumption can be inadequate, and systems designed under this assumption exhibit a significant performance degradation. There exist physical processes generating interferences containing noise components that are impulsive in nature. These processes can be natural, as well as man–made, and include underwater acoustic signals, lightning in the atmosphere, and transients in power lines and car ignitions. In modeling this type of signals the *stable distribution law* provides a very attractive theoretical tool. It was proven that under broad conditions, a general class of impulsive noise follows the stable law [8]. As a result, considerable research interest has been shown in designing robust signal processing algorithms for detection, direction finding, and equalization in the presence of impulsive noise environments [9, 10, 11].

This report is devoted to the testing of a new robust array signal processing algorithm for optimal performance in impulsive noise and interference environments which can no longer be modeled according to the Gaussian model. We concentrate in the Direction of Arrival (DOA) estimation problem from an array of sensors in a realistic sonar environment. The maximum likelihood estimator of the DOA's of multiple sources in the presence of interference modeled as a complex isotropic Cauchy process is derived. The new algorithm is compared with the ML estimator based on the Gaussian noise assumption, and with the MUSIC method in a wide range of additive noise environments, via simulation experiments.

In Section 2 we proceed with the problem formulation. We present the deterministic ML (DML) estimator based on the additive Gaussian noise assumption in Section 3. The MUSIC method, a large sample approximation to the ML estimator, is also discussed in Section 3. In Section 4 we develop the ML estimator based on the complex Cauchy noise model. Simulation results employing sonar signals and testing the performance of the discussed methods in different noise environments are presented in Section 5. Finally, conclusions are drawn in Section 6.

2 Problem Formulation

Consider an array of p sensors with arbitrary locations and arbitrary directional characteristics, that receive signals generated by q narrow–band sources with known center frequency ω and locations $\theta_1, \theta_2, \ldots, \theta_q$. Since the signals are narrow–band, the propagation delay across the array is much smaller than the reciprocal of the signal bandwidth, and it follows that, by using a complex envelop representation, the array output can be expressed as

$$\mathbf{x}(t) = \sum_{k=1}^{q} \mathbf{a}(\theta_k)s_k(t) + \mathbf{n}(t) \tag{1}$$

where

- $\mathbf{x}(t) = [x_1(t), \ldots, x_p(t)]^T$ is the vector of the signals received by the array sensors
- $s_k(t)$ is the signal emitted by the kth source as received at the reference sensor 1 of the array
- $\mathbf{a}(\theta_k) = [1, e^{-j\omega\tau_2(\theta_k)}, \ldots, e^{-j\omega\tau_p(\theta_k)}]^T$ is the steering vector of the array toward direction θ_k
- $\tau_i(\theta_k)$ is the propagation delay between the first and the ith sensor for a waveform coming from direction θ_k
- $\mathbf{n}(t) = [n_1(t), \ldots, n_p(t)]^T$ is the noise vector

(1) can be expressed in a compact form as

$$\mathbf{x}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where $\mathbf{A}(\theta)$ is the $p \times q$ matrix of the array steering vectors

$$\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_q)] \tag{3}$$

and $\mathbf{s}(t)$ is the $q \times 1$ vector of the signals

$$\mathbf{s}(t) = [s_1(t), \ldots, s_q(t)]^T. \tag{4}$$

Assuming that M snapshots are taken at time instants $t_1, \ldots, t_M$, the data can be expressed as

$$\mathbf{X} = \mathbf{A}(\theta)\mathbf{S} + \mathbf{N} \tag{5}$$

where $\mathbf{X}$ and $\mathbf{N}$ are the $p \times M$ matrices

$$\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_M)] \tag{6}$$

$$\mathbf{N} = [\mathbf{n}(t_1), \ldots, \mathbf{n}(t_M)] \tag{7}$$

and $\mathbf{S}$ is the $q \times M$ matrix

$$\mathbf{S} = [\mathbf{s}(t_1), \ldots, \mathbf{s}(t_M)]. \tag{8}$$

Our objective is to estimate the directions of arrival $\theta_1, \ldots, \theta_q$ of the sources from the M snapshots of the array $\mathbf{x}(t_1), \ldots, \mathbf{x}(t_M)$.

Toward this target we are going to make the following assumptions regarding the array, the signals, and the noise.

A.1 The number of signals is known and is smaller than the number of sensors, i.e., $q < p$

A.2 The steering vectors are linearly independent among themselves

A.3 The noise samples $n_i(t_j)$; $i = 1, \ldots, p$; $j = 1, \ldots, M$, come from a complex (bivariate) distribution

A.4 The noise samples $n_i(t_j)$ are statistically independent from each other both along the array sensors, namely, along the index i, and along time, namely, along the index j

Assumptions A.1 and A.2 guarantee the uniqueness of the solution. The conventional assumptions for A.3 and A.4 consider the noise as a stationary complex valued Gaussian process. The new method that we introduce here, assumes that the noise is modeled as a complex isotropic Cauchy process. This new assumption is well suited for modeling impulsive noise, and results in a beamformer that is robust over a wide range of noise and interference environments.

3 DOA Estimators Based on the Gaussian Noise Assumption

In this section we consider the case of additive Gaussian noise of zero mean and variance matrix $\sigma^2\mathbf{I}$, where σ^2 is the unknown noise power, and $\mathbf{I}$ is the identity matrix. This problem has been studied extensively in the past. Here, we give the derivation of the ML estimator, followed by a brief description of the MUSIC algorithm.

3.1 Deterministic Maximum Likelihood Estimator Based on the Gaussian Assumption

Under assumption A.4, it follows from (2) that the density function of the sampled data is given by

$$f(\mathbf{X}) = \prod_{t=1}^{M} \frac{1}{\pi \sigma^{2p}} \exp\left(-\frac{1}{\sigma^2} |\mathbf{x}(t) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)|^2\right). \tag{9}$$

Hence the log likelihood function $L(\mathbf{X}; \sigma^2, \mathbf{S}, \boldsymbol{\theta})$, ignoring constant terms, is expressed as:

$$L(\mathbf{X}; \sigma^2, \mathbf{S}, \boldsymbol{\theta}) = -Mp \log(\sigma^2) - \frac{1}{\sigma^2} \sum_{t=1}^{M} |\mathbf{x}(t) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)|^2. \tag{10}$$

The ML estimator is obtained by maximizing $L(\mathbf{X}; \sigma^2, \mathbf{S}, \boldsymbol{\theta})$ with respect to σ^2, S, and $\boldsymbol{\theta}$. Fixing S and $\boldsymbol{\theta}$, and maximizing with respect to σ^2 we get

$$\hat{\sigma}^2 = \frac{1}{Mp} \sum_{t=1}^{M} |\mathbf{x}(t) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)|^2. \tag{11}$$

Substituting (11) back into (10) and maximizing with respect to S we get a Least–Squares estimate for the signal

$$\hat{\mathbf{s}}(t) = (\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta}))^{-1}\mathbf{A}^H(\boldsymbol{\theta})\mathbf{x}(t). \tag{12}$$

Thus, the dimension of the required optimization is reduced, and the maximum likelihood estimator for $\boldsymbol{\theta}$ is given by the following minimization problem

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \mathrm{Tr}\left\{\mathbf{P}^{\perp}_{\mathbf{A}(\boldsymbol{\theta})}\hat{\mathbf{R}}\right\}, \tag{13}$$

where $\mathrm{Tr}\{\cdot\}$ is the trace operator,

$$\hat{\mathbf{R}} = \frac{1}{M} \sum_{t=1}^{M} \mathbf{x}(t)\mathbf{x}^H(t) \tag{14}$$

is the sample covariance matrix, and

$$\mathbf{P}^{\perp}_{\mathbf{A}(\boldsymbol{\theta})} = \mathbf{I} - \mathbf{A}(\boldsymbol{\theta})(\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta}))^{-1}\mathbf{A}^H(\boldsymbol{\theta}) \tag{15}$$

is the orthogonal projector onto the null space of $\mathbf{A}^H(\boldsymbol{\theta})$.

The solution of (13) requires a q–dimensional optimization for computing the directions of arrival. Analytical solutions for this problem are not available in general, and one has to resort to numerical methods, such as Newton–type algorithms.

3.2 THE MUSIC ALGORITHM

A standard assumption in the derivation of the MUSIC algorithm is that the signal waveforms are noncoherent. Means of reducing the susceptibility of the method to coherent signals have been introduced in the literature for special array structures [12].

The covariance matrix of the observation vector $\mathbf{x}(t)$ is given by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\Sigma\mathbf{A}^H + \sigma^2\mathbf{I}, \tag{16}$$

where we write $\mathbf{A}$ instead of $\mathbf{A}(\theta)$ for notational convenience. $\Sigma = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the signal covariance matrix. The $q \times q$ matrix Σ has full rank since we assumed noncoherence of the q incoming plane waves. Assumption A.3 implies that matrix $\mathbf{A}$ has full rank q.

MUSIC belongs to the class of spatial spectral estimation techniques which are based on the eigenvalue decomposition of the covariance matrix $\mathbf{R}$. The rationale behind this approach is that one wants to emphasize the choices for the steering vector $\mathbf{a}(\theta)$, which correspond to signal directions. The method exploits the property that the directions of arrival determine the eigenstructure of the matrix.

The analysis widely found in the literature (see, for example, [13]) leads to the following conclusions. If the acoustic field contains q distinct noncoherent propagating signals in a spatially white noise environment, then the eigenvalue decomposition of the spatial covariance matrix $\mathbf{R}$ results in the formation of two disjoint subspaces that are the orthogonal complement of each other. The first one, called the *signal plus noise subspace,* is spanned by the eigenvectors corresponding to the q largest eigenvalues of $\mathbf{R}$. The second, called the *noise subspace* is spanned by the eigenvectors corresponding to the $p-q$ smallest eigenvalues of $\mathbf{R}$. Given the eigenvectors of $\mathbf{R}$ we may determine the signal directions of arrival by searching for those steering vectors $\mathbf{a}(\theta)$ that are orthogonal to the noise subspace.

In practice, $\mathbf{R}$ is unknown, but can be consistently estimated from the available data as

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{t=1}^{M}\mathbf{x}(t)\mathbf{x}(t)^H, \tag{17}$$

Because of the uncertainty in the eigenvector estimates $\{\hat{\mathbf{v}}_{q+1}, \hat{\mathbf{v}}_{q+2}, \ldots, \hat{\mathbf{v}}_p\}$ introduced by the way we estimate the matrix $\mathbf{R}$, we can only search for the steering vectors that are most closely orthogonal to the noise subspace. The MUSIC algorithm estimates the signal directions as the peaks of the MUSIC spatial spectrum estimate given by

$$P_{MUSIC}(\theta) = \frac{1}{\sum_{i=q+1}^{p} |\mathbf{a}(\theta)^H \hat{\mathbf{v}}_i|^2}. \tag{18}$$

4 DOA Estimator Based on the Cauchy Noise Assumption

In this section we develop the Maximum Likelihood (ML) estimator of the source locations in the presence of noise modeled as a **complex isotropic Cauchy process.** There are two reasons for choosing the Cauchy model in our analysis: First, the Cauchy distribution belongs to the class of heavy–tailed *bivariate symmetric α–stable (SαS) distributions* which are well suited for describing noise processes that are impulsive in nature. But only the Cauchy from all the non-Gaussian $S\alpha S$ distributions does have a closed form expression for its density function. This results to a straightforward implementation of the maximum likelihood estimation with closed form expressions for the Cramér–Rao bound. Secondly, it is shown through simulations that the Cauchy beamformer is very robust in different impulsive noise environments, i.e., its performance does not change significantly when the parameter α of the $S\alpha S$ noise varies in the interval $(0, 2]$.

Assume that the noise present at the array sensors is modeled as a complex isotropic Cauchy process with probability density function given by

$$f_\gamma(x_1, x_2) = \chi_\gamma(r) = \frac{\gamma}{2\pi(r^2+\gamma^2)^{3/2}}, \tag{19}$$

where γ ($\gamma > 0$) is the *dispersion* of the distribution, and determines the spread of the distribution around the origin similarly to the variance of the Gaussian distribution.

Under assumption A.4, it follows from (1) and (19) that the joint density function of the sampled data is given by

$$f(\mathbf{X}) = \prod_{t=1}^{M}\prod_{i=1}^{p} \chi_{1,\gamma}\left(\left|x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)\right|\right) \tag{20}$$

or

$$f(\mathbf{X}) = \prod_{t=1}^{M}\prod_{i=1}^{p} \frac{1}{2\pi}\frac{\gamma}{\left(\gamma^2 + |x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)|^2\right)^{3/2}}, \tag{21}$$

where $a_1(\theta_k) = 1$ and $a_i(\theta_k) = e^{-\jmath\omega\tau_i(\theta_k)}$; $i = 2, \ldots, p$. Hence the log likelihood function $L(\mathbf{X}; \gamma, \mathbf{S}, \theta)$, ignoring constant terms, is expressed as:

$$L(\mathbf{X}; \gamma, \mathbf{S}, \theta) = Mp\log(\gamma) - \frac{3}{2}\sum_{t=1}^{M}\sum_{i=1}^{p}\log\left(\gamma^2 + \left|x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)\right|^2\right). \tag{22}$$

The ML estimator is obtained by maximizing $L(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ with respect to γ, $\mathbf{S}$, and $\boldsymbol{\theta}$, i.e.,

$$\max_{\gamma,\mathbf{S},\boldsymbol{\theta}} L(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta}) \tag{23}$$

To reduce the dimension of this optimization problem, we first fix γ and $\boldsymbol{\theta}$, and maximize $L(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ with respect to the signal $\mathbf{S}$. For fixed t we take the derivative of $L(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ with respect to $s_k(t)$:

$$\frac{\partial L}{\partial s_k(t)} = -\sum_{i=1}^{p} \frac{a_i(\theta_k)\,[x_i(t) - a_i(\theta_k)s_k(t)]^*}{\gamma^2 + |x_i(t) - a_i(\theta_k)s_k(t)|^2}. \tag{24}$$

Unfortunately, no explicit solution of (24) is possible, and an approximation method must be applied. Returning to (23), we observe that maximizing $L(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ with respect to the signal $\mathbf{S}$ is equivalent to the following minimization problem:

$$\min_{\mathbf{S}} \mathcal{L}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta}) = \min_{\mathbf{S}} \left\{ \sum_{t=1}^{M}\sum_{i=1}^{p} \log(1 + \frac{|x_i(t) - a_i(\theta)s(t)|^2}{\gamma^2}) \right\}. \tag{25}$$

As we can see, (25) involves minimizing a double sum expression of logarithmic functions of the form $\log(1+z)$. In the unit disc $B_1(0) = \{z \in C : |z| < 1\}$, the function $\log(1+z)$ can be expressed as an infinite series:

$$\log(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} z^n = z - \frac{z^2}{2} + - \cdots; \quad |z| < 1 \tag{26}$$

Hence for $|x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)| < \gamma$ the functional $\mathcal{L}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ can be written in the form

$$\mathcal{L}(\mathbf{X};\boldsymbol{\theta},\mathbf{S}) = \sum_{t=1}^{M}\sum_{i=1}^{p}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\gamma^{2n}} |x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)|^{2n}. \tag{27}$$

A first order approximation of the above expression results in the following $\mathcal{L}^{(1)}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ functional:

$$\mathcal{L}^{(1)}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta}) = \frac{1}{\gamma^2}\sum_{t=1}^{M}\sum_{i=1}^{p} |x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)|^2 \tag{28}$$

which, by using (2), can be written in a more compact form as:

$$\mathcal{L}^{(1)}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta}) = \frac{1}{\gamma^2}\sum_{t=1}^{M} |\mathbf{x}(t) - \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t)|^2. \tag{29}$$

Hence the minimization of $\mathcal{L}^{(1)}(\mathbf{X};\gamma,\mathbf{S},\boldsymbol{\theta})$ with respect to $\mathbf{S}$ is equivalent to the Least-Squares (LS) estimation of $\mathbf{S}$. This problem has a well known solution:

$$\hat{\mathbf{s}}(t) = (\mathbf{A}^H(\boldsymbol{\theta})\mathbf{A}(\boldsymbol{\theta}))^{-1}\mathbf{A}^H(\boldsymbol{\theta})\mathbf{x}(t). \tag{30}$$

The dispersion γ can now be estimated by using the measurements of the first sensor and the expression for the fractional lower order moments (FLOM) of the noise given in [14], where $E|X|^p$ is approximated by an average sum:

$$\hat{\gamma} = \frac{\left[\frac{1}{M}\sum_{t=1}^{M}|x_1(t) - \sum_{k=1}^{q}\hat{s}_k(t)|^p\right]^{\frac{1}{p}}}{[C_2(p,1)]^{\frac{1}{p}}}, \tag{31}$$

where $p < 1$, and $C_2(p,1)$ is given by:

$$C_2(p,1) = p2^p \frac{\Gamma(\frac{p}{2})\Gamma(-p)}{\Gamma(-\frac{p}{2})} \tag{32}$$

By using the above estimates for the signal S and the noise dispersion, we obtain the following optimization problem:

$$\max_{\boldsymbol{\theta}} L(\mathbf{X};\hat{\gamma},\hat{\mathbf{S}},\boldsymbol{\theta}) = \max_{\boldsymbol{\theta}}\left\{Mp\log(\hat{\gamma}) - \frac{3}{2}\sum_{t=1}^{M}\sum_{i=1}^{p}\log\left(\hat{\gamma}^2 + \left|x_i(t) - \sum_{k=1}^{q}a_i(\theta_k)\hat{s}_k(t)\right|^2\right)\right\}. \tag{33}$$

Now we can apply an iterative procedure based on the gradient descent principle in order to solve for $\boldsymbol{\theta}$. Concluding this section we point out, for one more time, the two main assumptions made in order to obtain a closed form expression for the signal estimate, $\hat{\mathbf{S}}$:

B.1 Assumption $|x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)| < \gamma$ (holds with probability 0.3) enabled us to express the logarithmic function as an infinite series;

B.2 Assumption $|x_i(t) - \sum_{k=1}^{q} a_i(\theta_k)s_k(t)| \ll \gamma$ enabled us to take a first order approximation of the infinite series, and thus to obtain a Least–Squares closed form estimate for the signal.

Obviously, if the above assumptions are not satisfied the performance of the estimator will not be "optimal".

5 Simulation Experiments

To demonstrate the performance of the methods described in the previous sections we conducted a set of simulation experiments. We compare the ML estimator based on the Cauchy noise assumption (MLC) with the ML estimator based on the Gaussian noise assumption (MLG), and with the MUSIC estimator (MUS).

The array is linear with five sensors spaced half wavelength apart. We use two sonar signal waveforms

Power (dB)	**Frequency (Hz)**
0.00	0.0
-1.75	0.6
-3.50	1.2
-5.50	1.8
-8.00	2.4
-10.75	3.0
-14.75	3.6
-19.75	4.2
-25.00	4.8
-31.50	5.4
-36.00	6.0

Table 1: Power spectrum of the acoustic emission

- **Signal S1:** An acoustic emission centered a 990 Hz and defined as shown in Table 1. Beyond 6 Hz the roll off is 18 dB/octave. This can be considered as a hostile or friendly unintentional acoustic emission.
- **Signal S2:** Single pulse of square CW, 0.1 second long, centered at 1 Khz. This can be considered as an intentional hostile sonar transmission.

At first, the signal is assumed to be known at the receiver and the ML method is applied to estimate the direction of arrival. Also, the pseudo ML method is applied using a least squares estimate of the signal. The signal impinges to the array from direction 5°. The noise is assumed to follow the bivariate isotropic stable distribution. We change the impulsive nature of the noise by changing the characteristic exponent α of the stable distribution. We conducted three experiments in order to study the influence of different parameters to the performance of the algorithms. In every experiment we performed 100 Monte–Carlo runs and computed the mean and the mean square error (MSE) of the direction of arrival estimates.

Experiment #1

In the first experiment, we study the influence of the number of snapshots, M, to the performance of the algorithm. The noise follows the Cauchy distribution with dispersion $\gamma = 1$ (c.f. Eq. 19). Although it is known that the the Cauchy distribution has infinite variance, the "effective" SNR in these finite sample realizations is approximately -15dB. Figure 2(a) shows the resulting MSE of the estimated DOA as a function of the number of snapshots when the signal S1 is known at the receiver. The CRB is also plotted. As expected from the theoretical analysis, the MLC estimator has the best performance.

Figure 2(b) shows the results obtained for the MSE when a Least Squares estimate is used for the signal S1. The MLC estimator gives again the least MSE. Comparing these curves with the analogous curves obtained assuming exact signal knowledge, we observe a larger MSE for the pseudo ML estimates, as expected. Also, it seems that the MLC estimator is more negatively influenced by the LS estimate of the signal than the MLG estimator.

Since the MUSIC algorithm does not use an explicit estimate for the signal, its MSE curve is the same in both figures. The fact that MUSIC is highly sensitive to deviations from the mathematical model (additive white Gaussian noise) assumed for its derivation can be seen from the figures, as the performance of the algorithm degrades when more snapshots are made available to the receiver. Also, the MUSIC exhibits significantly higher bias in the estimation of DOA than the other two methods (Figure 1.)

Similar conclusions can be drawn from Figures 3 and 4 for the case of the other sonar signal S2.

Experiment #2

The importance of this experiment is that it studies the robustness of the algorithms in different noise environments. Of course, by design, the MLG estimator is optimal for additive Gaussian noise, and the introduced MLC estimator is optimal for additive Cauchy noise. An important property of any receiver is to be able to perform reasonably well in a wide range of noise environments. Here, we test the performance of the estimators when the characteristic exponent, α, of the noise stable law is changing.

Figures 5–6 show the resulting MSE of the estimated DOA as a function of the characteristic exponent α. As we can clearly see, for exact signal knowledge (Figures 5(a) and 6(a)), the Cauchy beamformer is practically insensitive to the changes of α. On the other hand, both the MLG and the MUSIC algorithms exhibit very large mean square estimation errors for non–Gaussian noise environments. Note that, when $\alpha = 2$, i.e., for the Gaussian noise case, the MLG beamformer has the least MSE as expected.

Experiment #3

Here, we give some indicative results about the performance of the maximum likelihood estimators for the case of two incoming signals and for different noise environments. The two sonar emissions S1 and S2 impinge from directions -5^o and 5^o. In Figures 7-12 we plot representative MLC and MLG likelihood functions for three different cases of additive noise: Gaussian ($\alpha = 2$), Cauchy ($\alpha = 1$), and α–stable with $\alpha = 0.5$. The noise environment is becoming more impulsive as α decreases. From the plots, it appears that the Cauchy receiver is very robust to noise fields of different characteristic exponent α. On the other hand, as α decreases, the MLG likelihood function becomes smoother and achieves its maximum at the wrong point (for this particular realization, when $\alpha = 0.5$, Figure 12.)

6 CONCLUDING REMARKS

We have presented a testing of a new method for the bearing estimation problem of two sonar source signals in the presence of impulsive additive noise. The method is based on the maximum likelihood estimation technique where the noise is modeled as a complex isotropic $S\alpha S$ process. The Cauchy beamformer has been shown to give better DOA estimates than the Gaussian beamformer in a wide range of impulsive noise environments. The technique inherits the computational complexity of the ML family of methods but avoids the eigendecomposition operations of the eigenvector–based methods, and appears to have superior performance for low SNR values and when the number of observation samples is small. Future work will focus on the performance evaluation of the algorithm with real sonar data.

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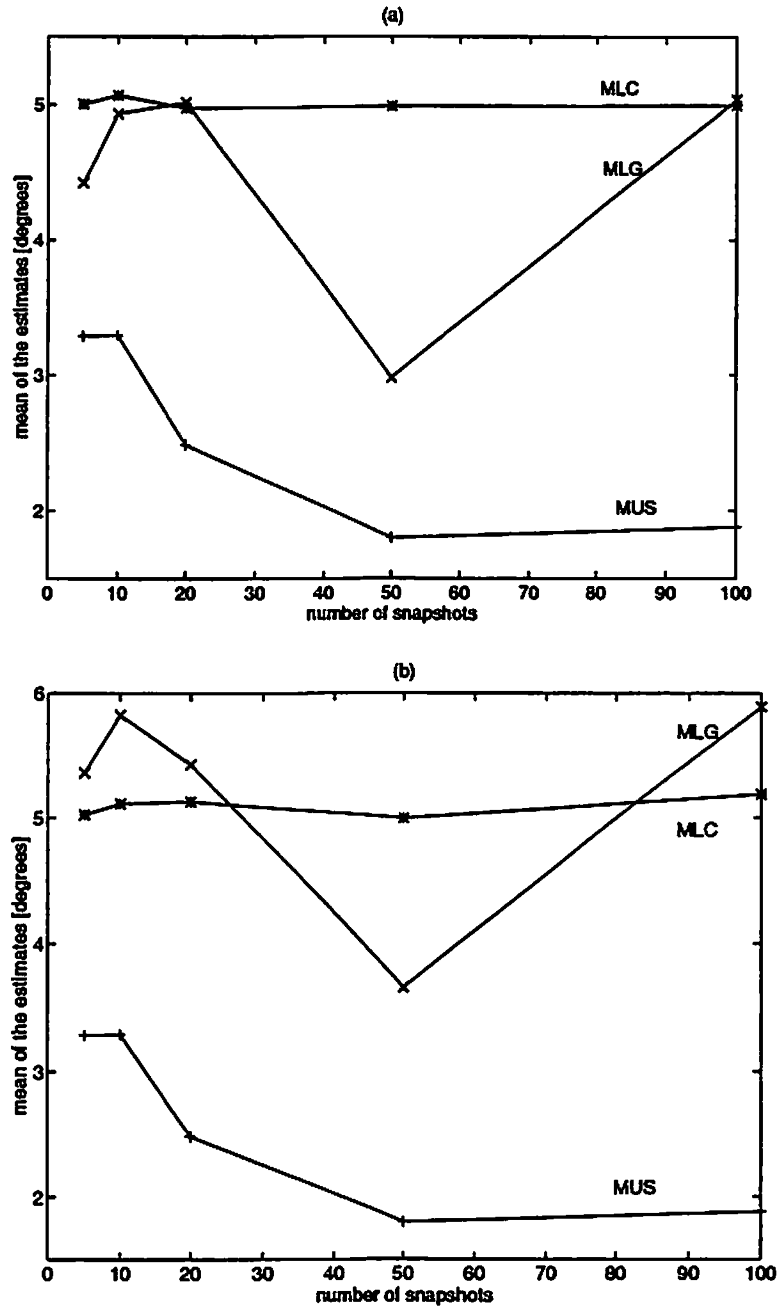

Figure 1: Mean of the estimates of DOA as a function of the number of snapshots M. Experiment #1, signal S1. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

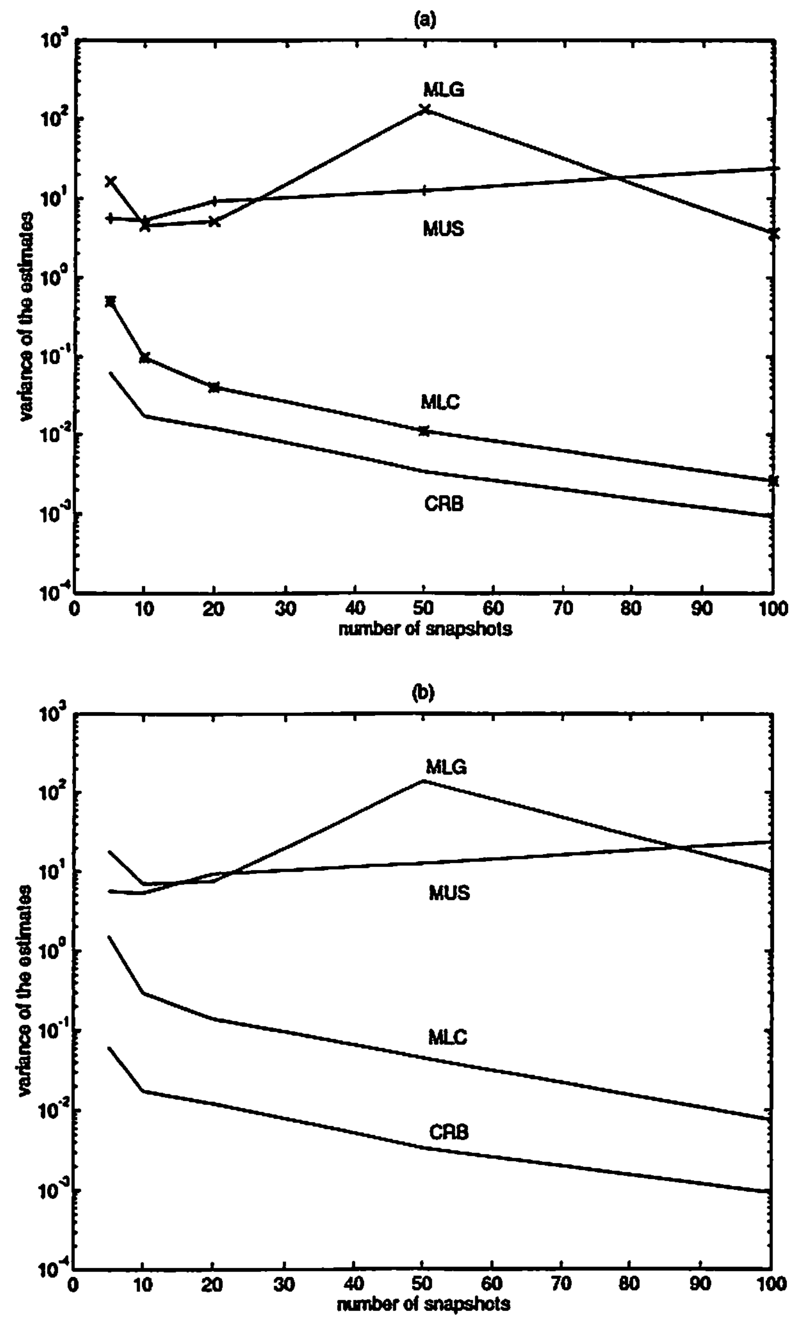

Figure 2: MSE of the estimates of DOA and CRB as a function of the number of snapshots M. Experiment #1, signal S1. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

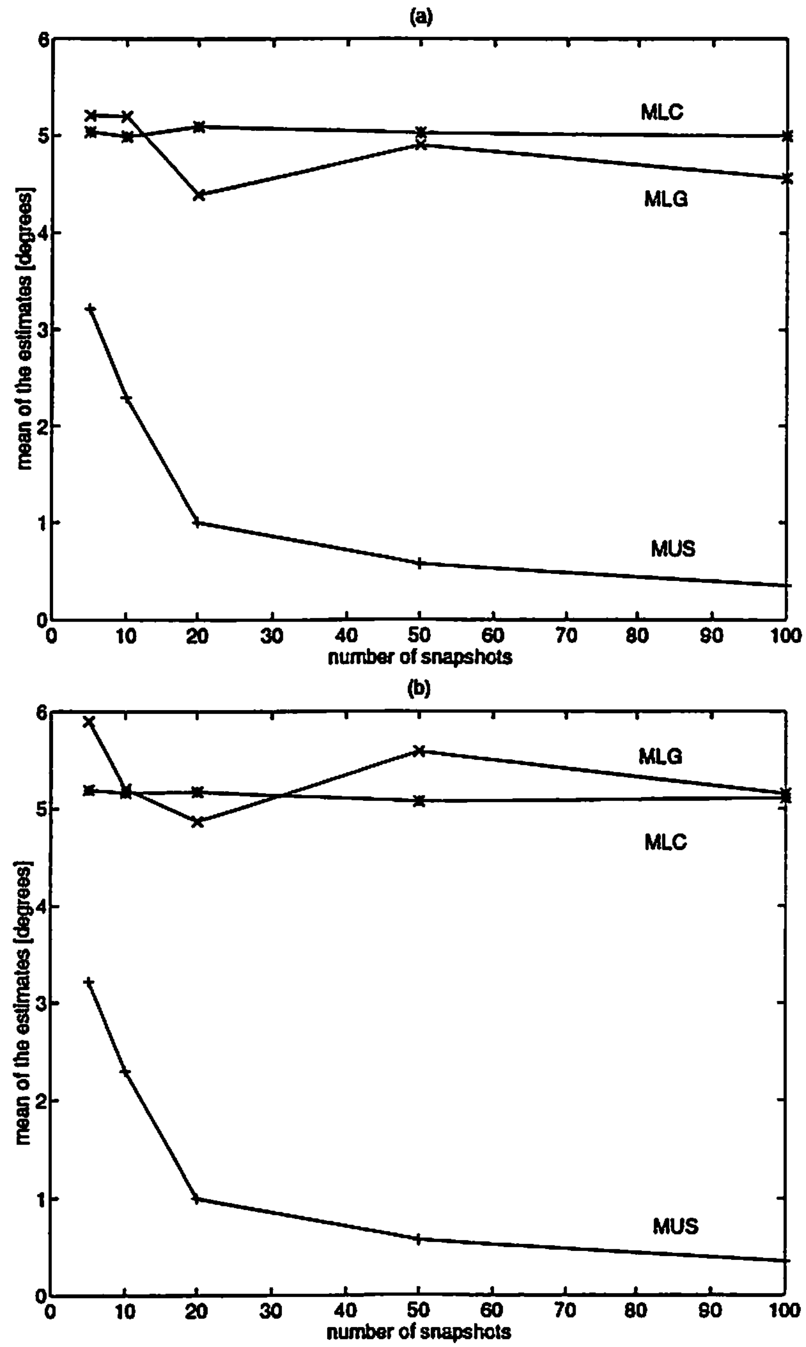

Figure 3: Mean of the estimates of DOA as a function of the number of snapshots M. Experiment #1, signal S2. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

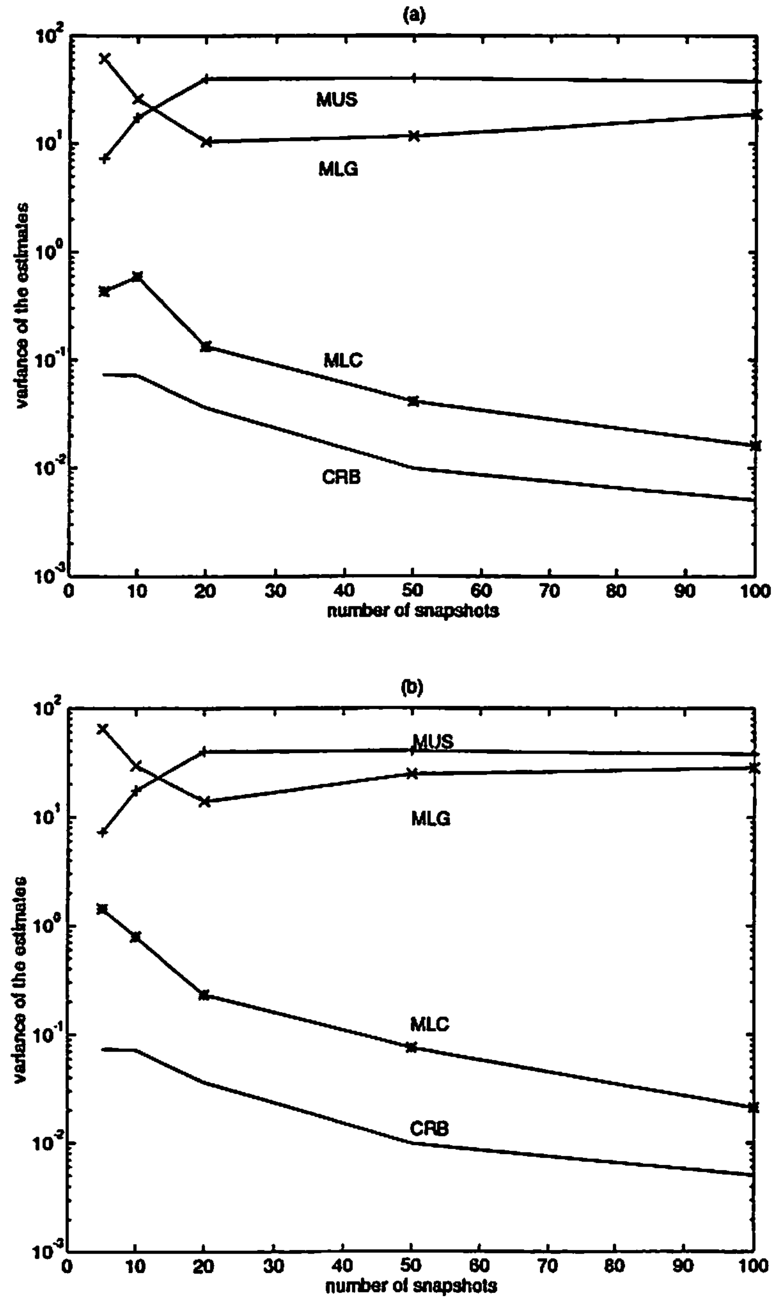

Figure 4: MSE of the estimates of DOA and CRB as a function of the number of snapshots M. Experiment #1, signal S2. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

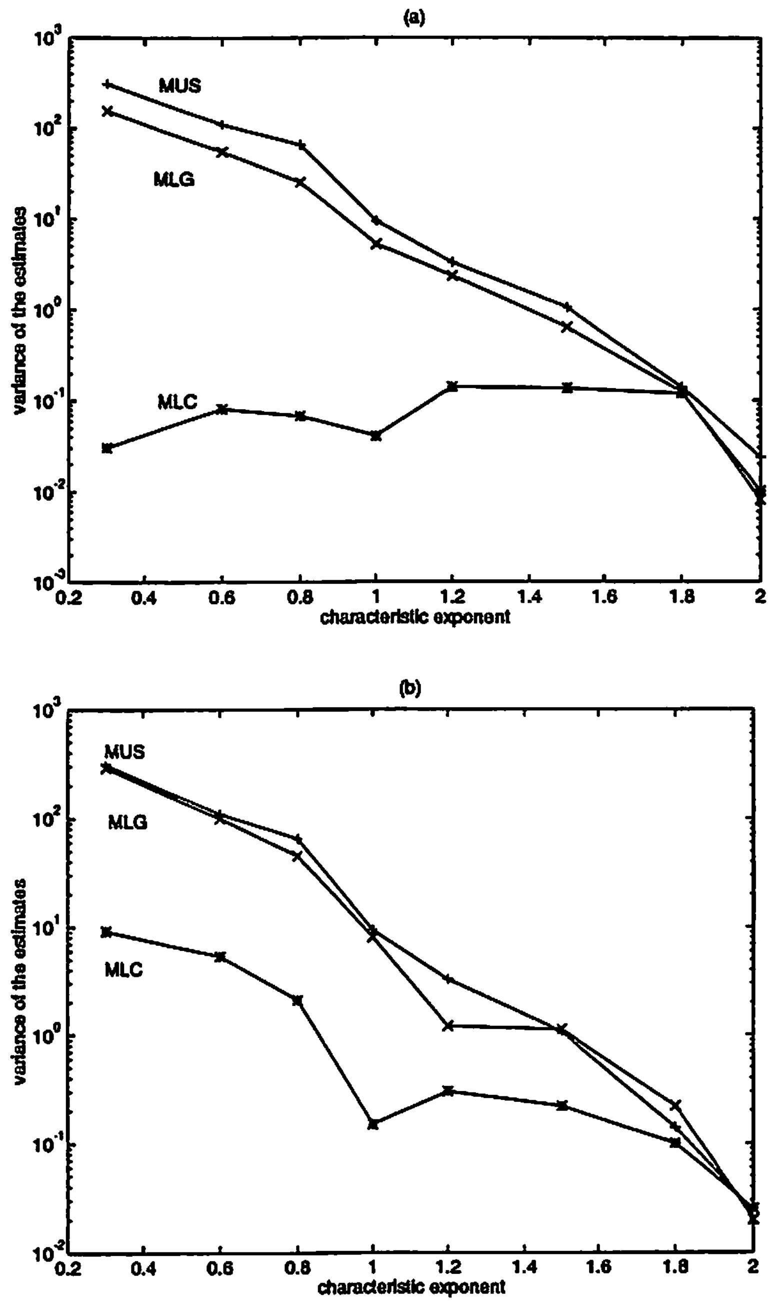

Figure 5: MSE of the estimates of DOA as a function of the characteristic exponent α. Experiment #2, signal S1. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

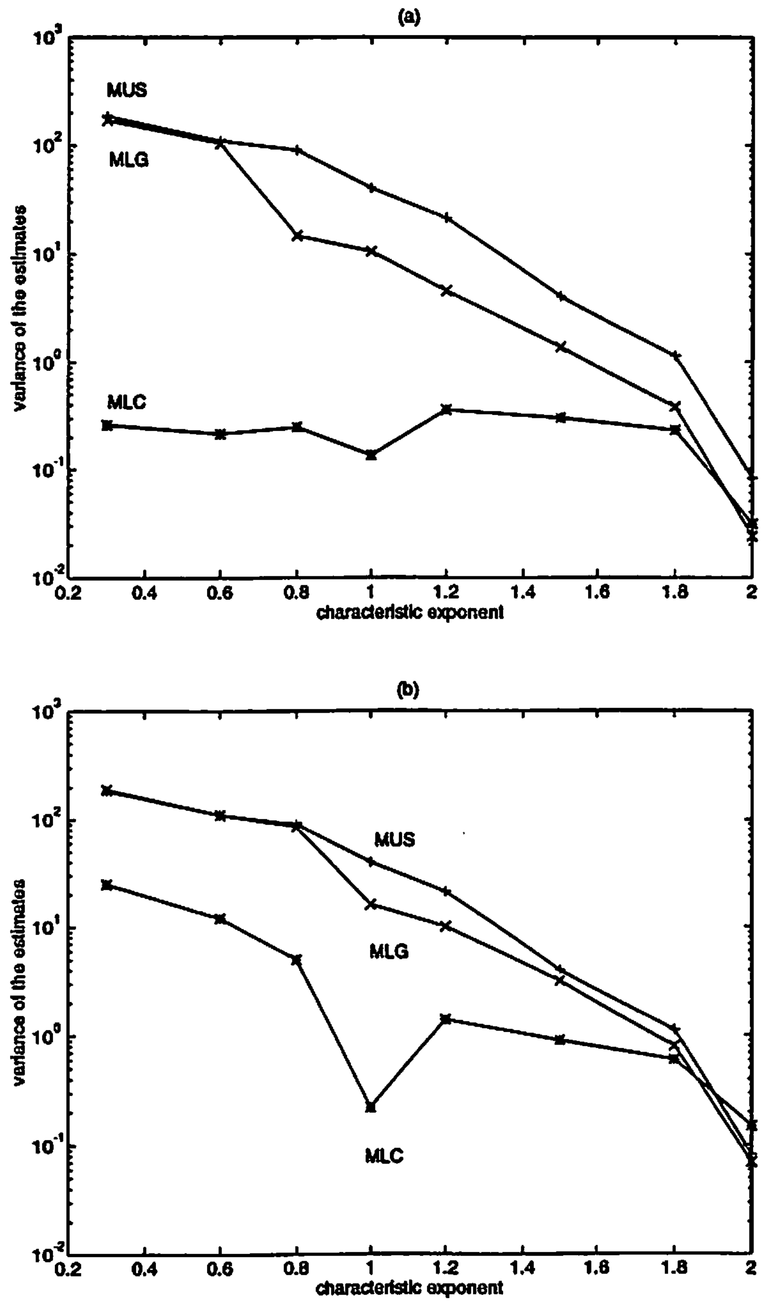

Figure 6: MSE of the estimates of DOA as a function of the characteristic exponent α. Experiment #2, signal S2. (a) Exact signal knowledge, (b) Least–Squares estimate of the signal.

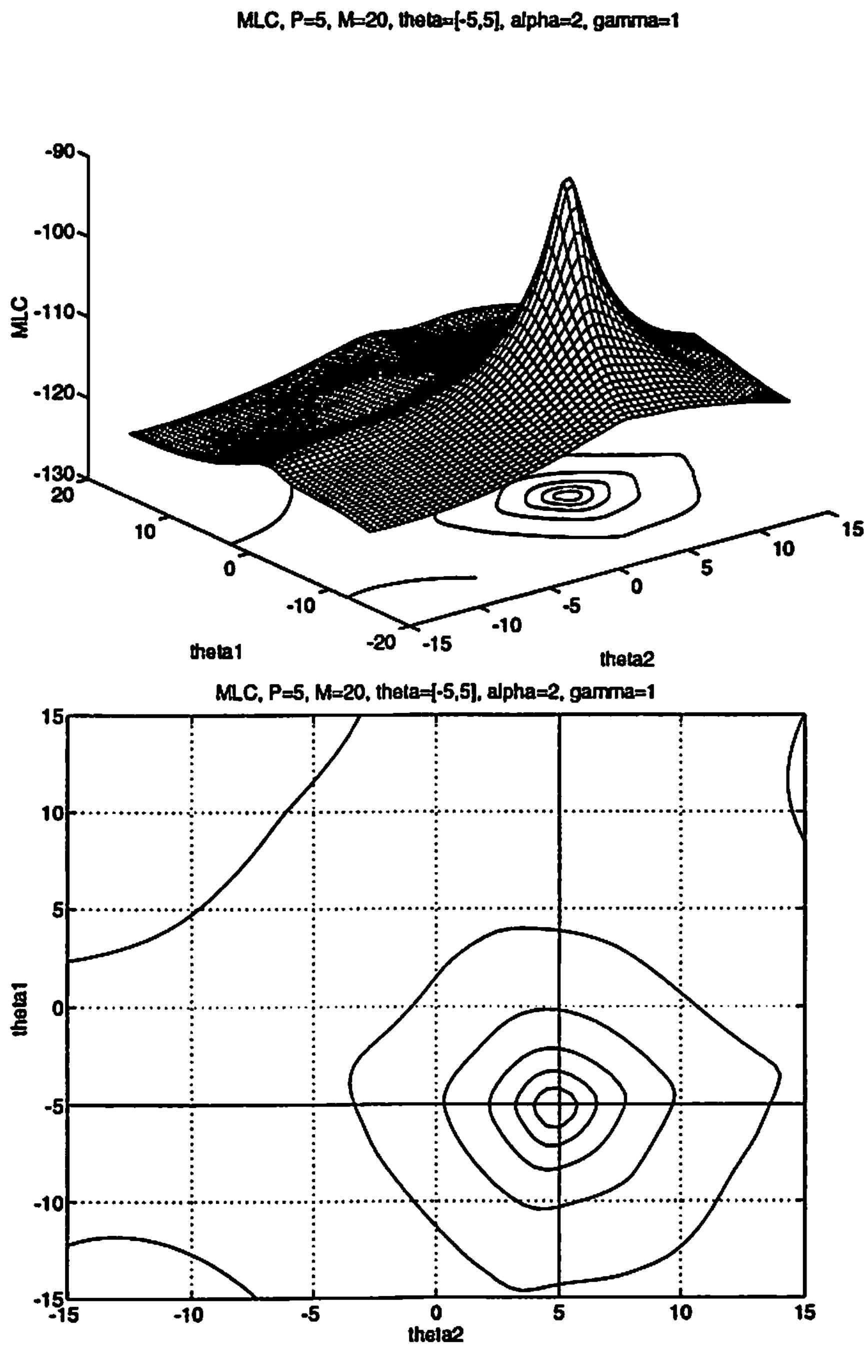

Figure 7: MLC Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5^o, 5^o]$, $\alpha = 2$, $\gamma = 1$. Exact signal knowledge.

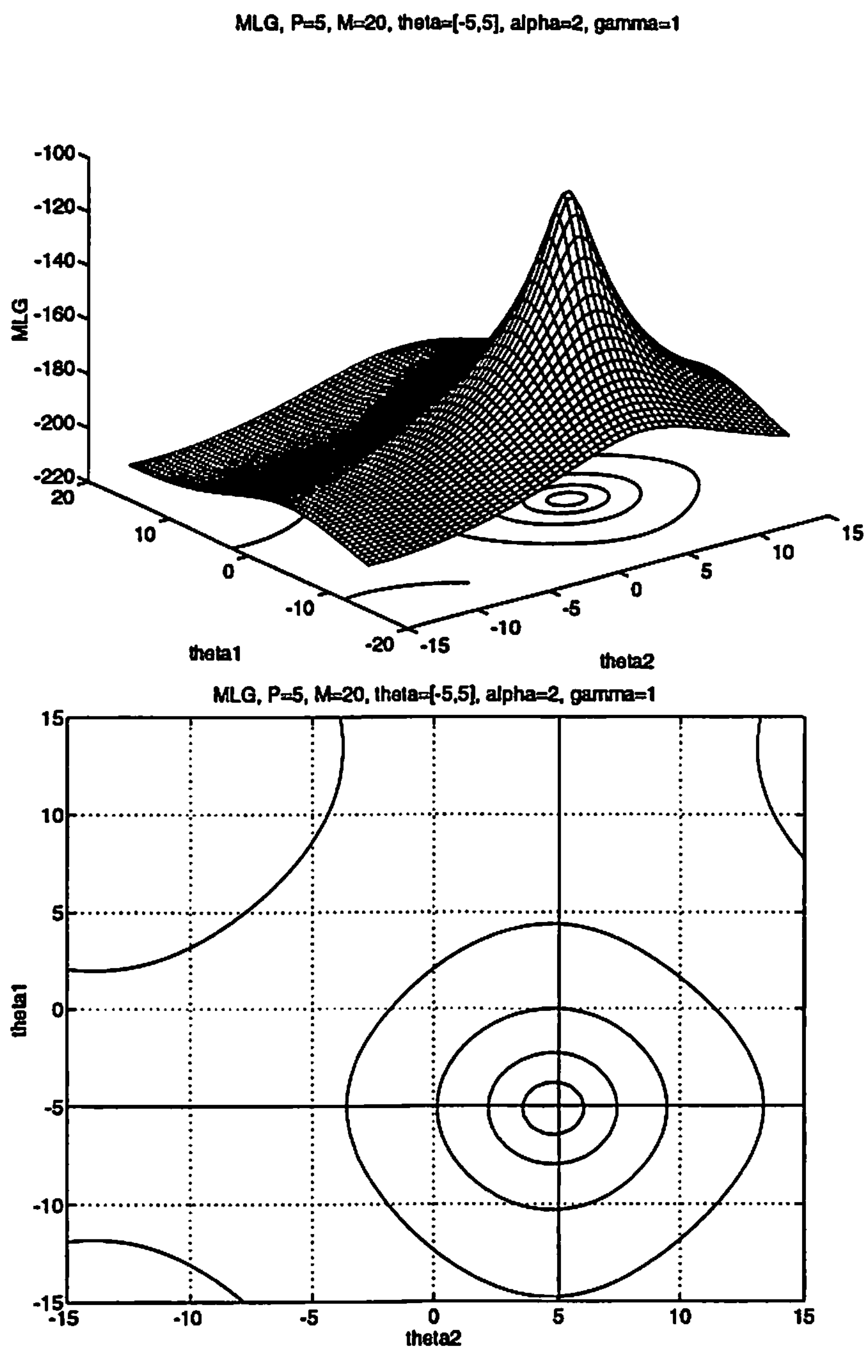

Figure 8: MLG Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5°, 5°]$, $\alpha = 2$, $\gamma = 1$. Exact signal knowledge.

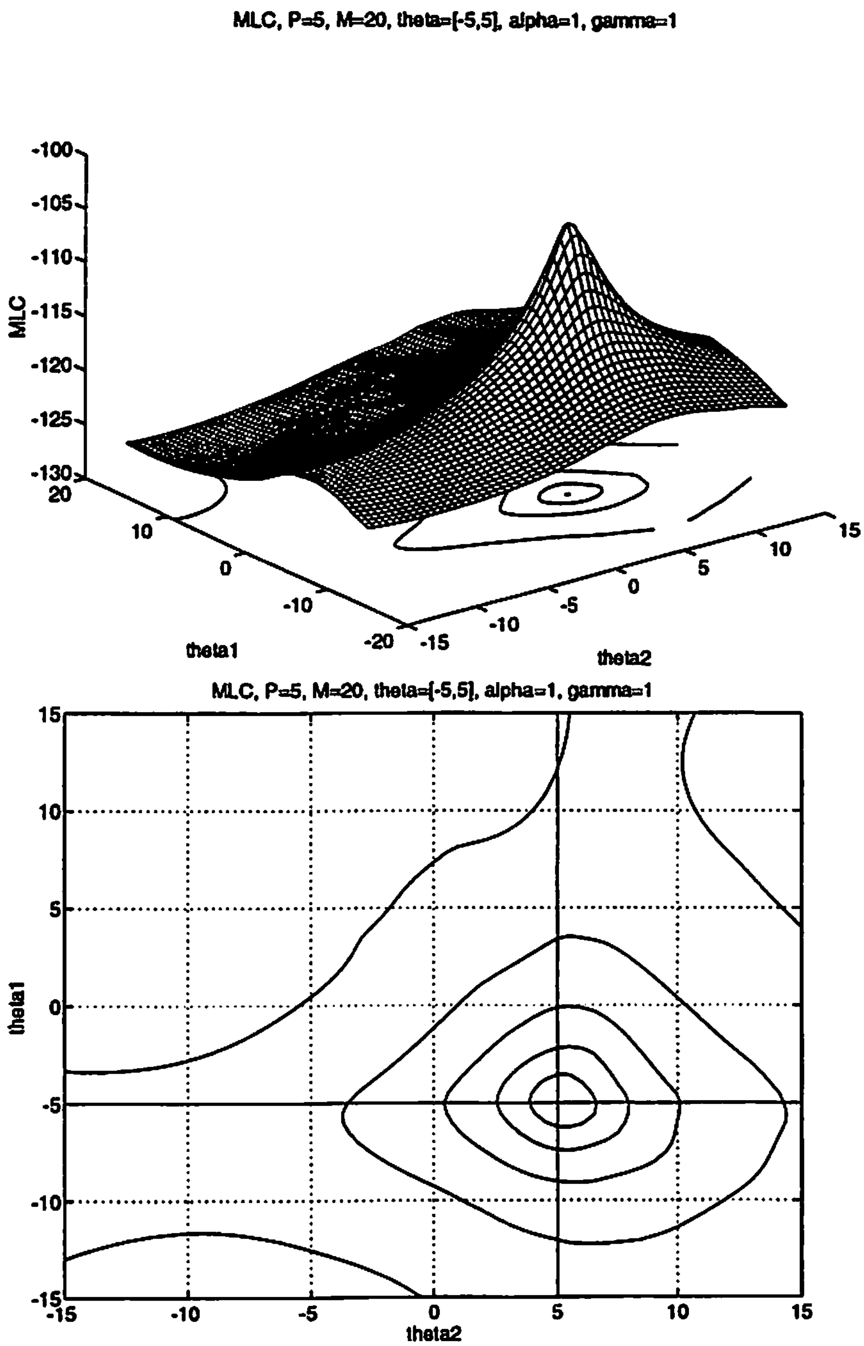

Figure 9: MLC Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5^o, 5^o]$, $\alpha = 1$, $\gamma = 1$. Exact signal knowledge.

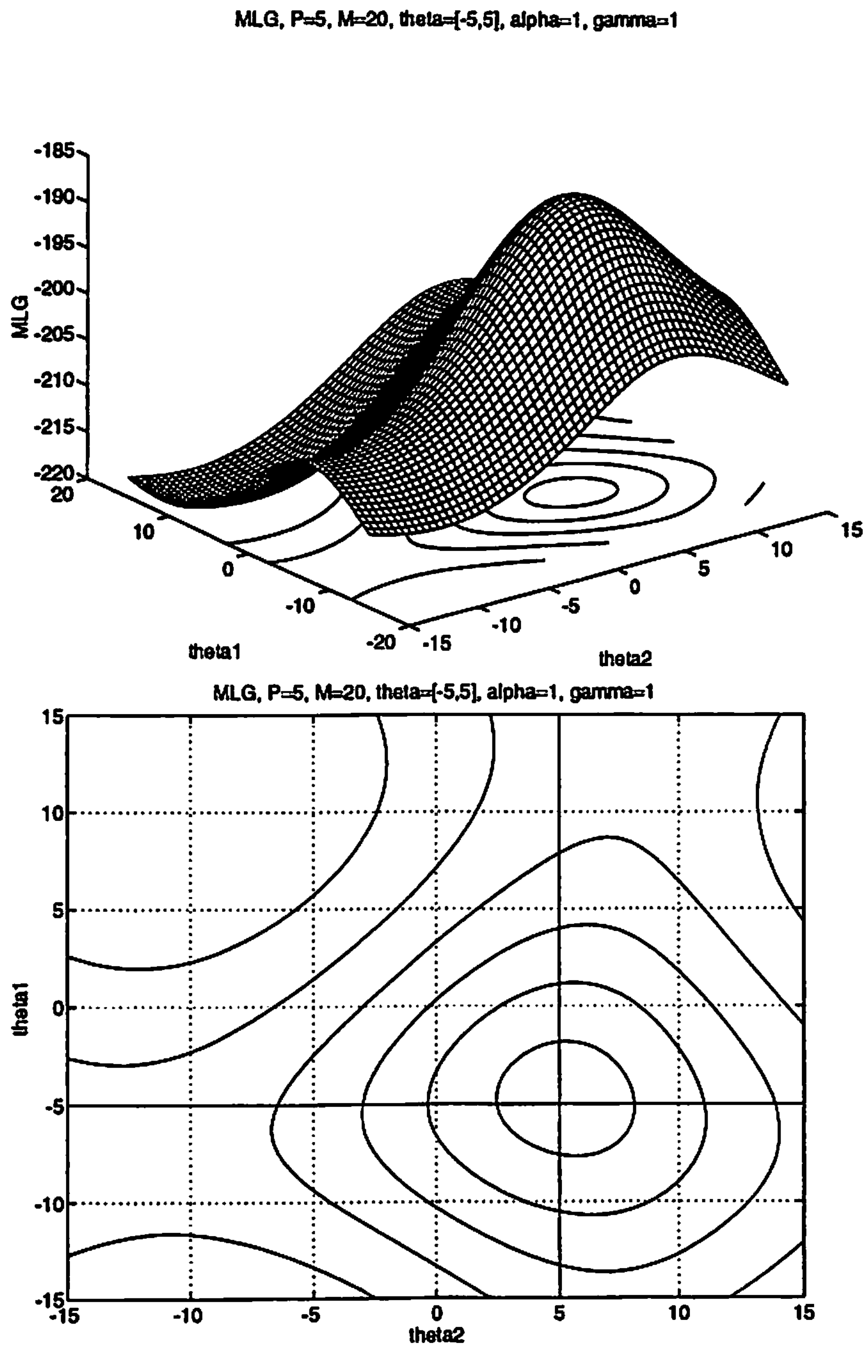

Figure 10: MLG Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5^o, 5^o]$, $\alpha = 1$, $\gamma = 1$. Exact signal knowledge.

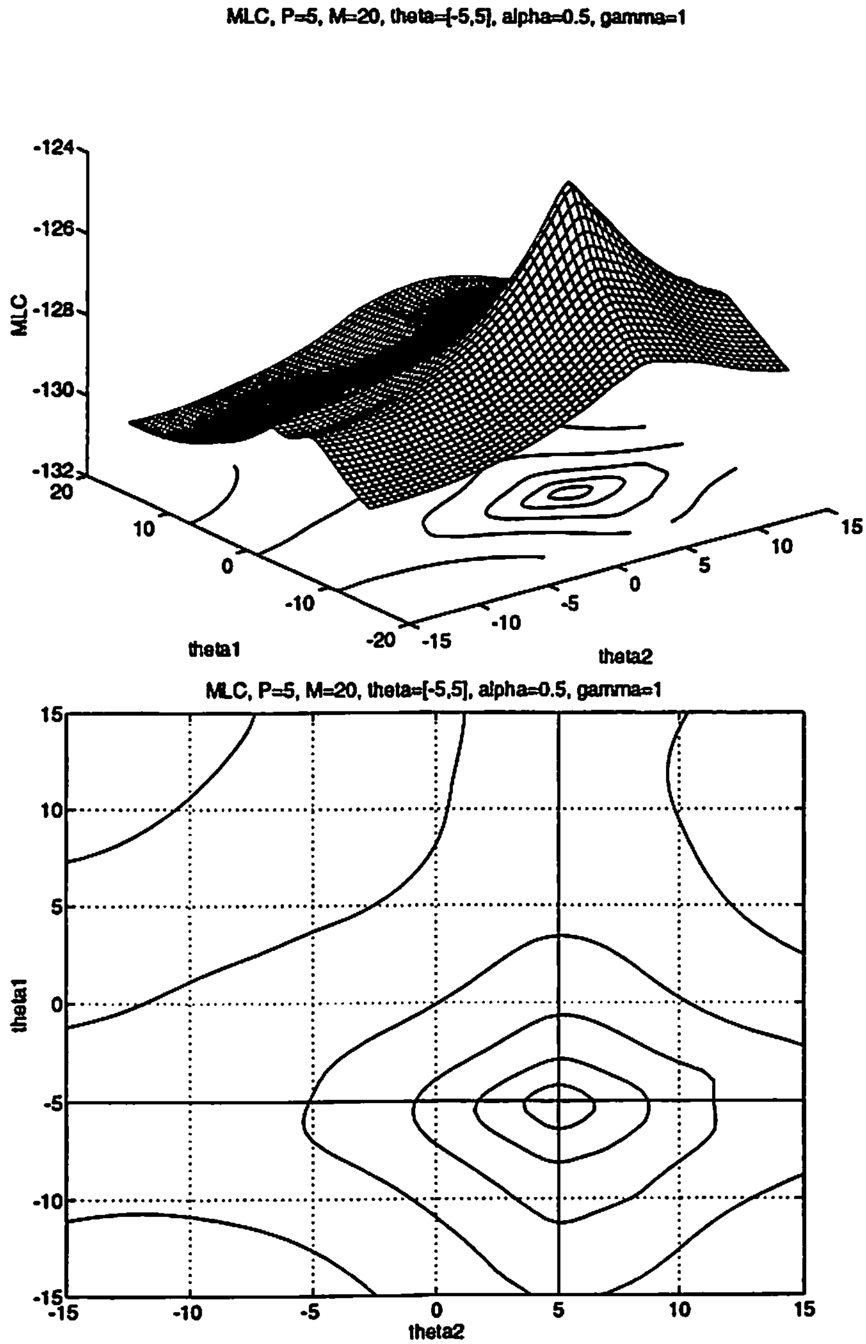

Figure 11: MLC Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5^{\circ}, 5^{\circ}]$, $\alpha = 0.5$, $\gamma = 1$. Exact signal knowledge.

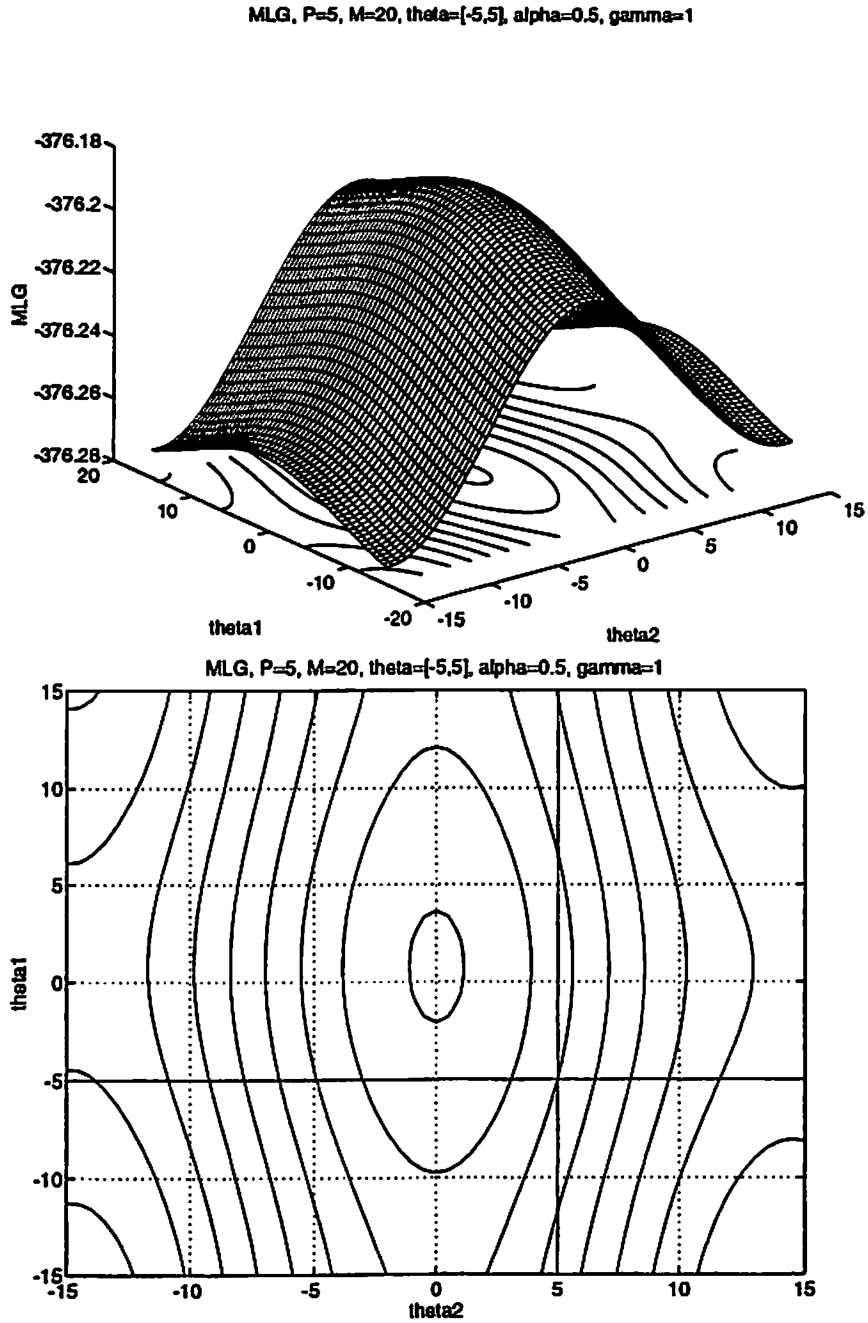

Figure 12: MLG Likelihood function. $P = 5$, $M = 20$, $\Theta = [-5^o, 5^o]$, $\alpha = 0.5$, $\gamma = 1$. Exact signal knowledge.