Thin Plate Spline Registration in the Intrinsic Geometry of the Cortical Surface

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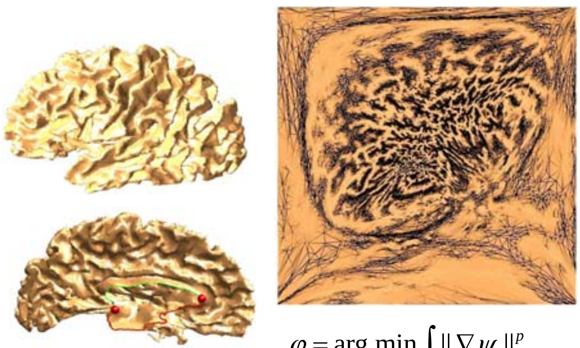
Introduction

We present a method to register individual cortical surfaces to a surface-based brain atlas or canonical template using labelled sulcal curves as landmark constraints. To map one cortex smoothly onto another, we minimize a thin-plate spline energy defined on the surface by solving the associated partial differential equations (PDEs). By using covariant derivatives in solving these PDEs, we compute the bending energy with respect to the intrinsic geometry of the 3D surface rather than evaluating it in the flattened metric of the 2D parameter space. This covariant approach greatly reduces the confounding effects of the surface parameterization on the resulting registration.

Surface extraction and parameterization

We first extract a mask for the cortical surface from an MRI volume using the Brainsuite software [1]. The topology of the mask is corrected automatically using a graph-based approach and tessellated to produce a genus zero surface. We then use a *p*-harmonic functional minimization scheme [2] to map the each cortical hemisphere onto the unit square. The result is a bijective mapping between each hemisphere and the unit square in which the interhemispheric fissure is constrained to map to the boundary of the unit square. This allows us to calculate partial derivatives across the boundary and explicitly model continuity between the two cortical hemispheres.



$$\varphi = \arg\min_{\psi} \int_M \| \nabla \psi \|^p$$

Figure 1: P-Harmonic maps of the cortical surface. The interhemispheric fissure is constrained to lie on the boundary of the unit square.

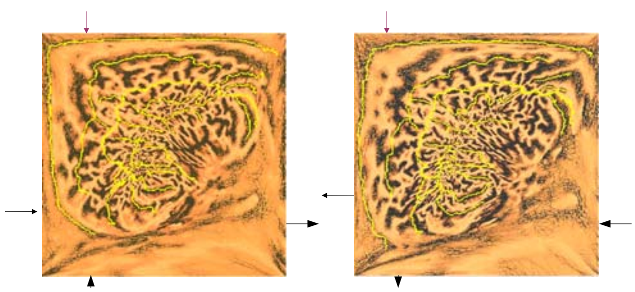


Figure 2: The Coordinate system for the cortical surface. The arrows show connectivity.

$$E_b = \int \left(\left(\frac{\partial^2 \phi}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 \phi}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 \phi}{\partial y^2}\right)^2 \right) dxdy$$

Thin Plate Bending Energy in Euclidean space

$$E_b = \int \left(\left(\psi^j_{,11}\right)^2 + \left(\sqrt{2}\psi^j_{,12}\right)^2 + \left(\psi^j_{,22}\right)^2 \right) g du dv$$

Bending Energy in non-Euclidean space

Calculations of covariant derivatives

$$g_{11} = \left\|\frac{\partial \mathbf{x}}{\partial u}\right\|^2, g_{22} = \left\|\frac{\partial \mathbf{x}}{\partial v}\right\|^2, g_{12} = \left\langle \frac{\partial \mathbf{x}}{\partial u}, \frac{\partial \mathbf{x}}{\partial v} \right\rangle^2, g = g_{11}g_{22} - (g_{12})^2$$

$$\phi^{\beta}_{,\sigma} = \frac{\partial \phi^{\beta}}{\partial u^{\sigma}} + \phi^{\kappa}\Gamma_{\kappa\sigma}{}^{\beta} \quad \text{where } \sigma,\beta,\kappa \in \{1,2\}$$

$$\phi^{\zeta}_{,\beta\sigma} = \frac{\partial \phi^{\zeta}_{,\beta}}{\partial u^{\sigma}} - \phi^{\zeta}_{,\mu}\Gamma_{\beta\sigma} + \phi_{\beta}{}^{\nu}\Gamma_{\nu\sigma}{}^{\zeta} \quad \text{where } \sigma,\beta,\zeta,\mu,\kappa \in \{1,2\}$$

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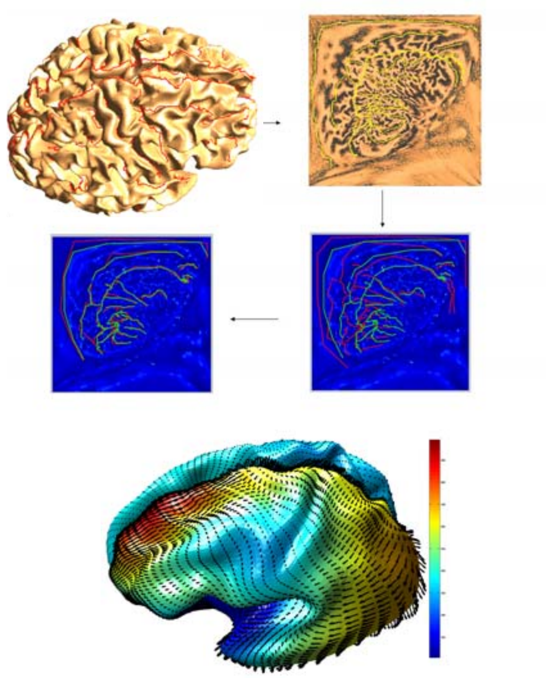

Figure 3: The intrinsic TPS warping procedure. The figure shows extracted cortex, its p-harmonic map And sulci of the subject and atlas mapped to the parameter space before (right) and after (left) warping The figure at the bottom shows warping field computed on the surface. The color indicates magnitude of the deformation.

TPS warping in the intrinsic geometry

Having parameterized the cortical surfaces of the subject and atlas, we align the coordinate systems between the subject and atlas such that a set of hand-labelled sulci are brought into register, i.e. we find a warping field which can be applied in the subject's surface parameter space to align the subject's sulcal features with those of the atlas. The alignment uses a set of interactively labelled sulci, sampled uniformly along their lengths, as a set of point constraints. To compute a smooth warping from one coordinate system to the other we use the thin-plate spline bending energy on the atlas surface as a regularizing function. Since the mapping onto the unit square will inevitably produce metric distortion relative to the original surface, it is attractive to compute the bending energy with respect to the intrinsic geometry of the 3D surface rather than the parameter space itself, using a *covariant* PDE approach [3]. To do this, we solve the biharmonic equation using covariant derivatives to obtain a thin-plate spline warp from subject to atlas coordinates.

Conclusions

Metrically covariant methods for nonlinear surface matching can help compare and integrate cortical data across subjects, by removing anatomical shape and sulcal patterning differences. By taking into account the surface metric in the registration process, covariant PDEs provide surface correspondences that are invariant to the particular parameterization of the surface, and the method to flatten it. This improves consistency and removes this source of registration error from subsequent intersubject comparisons.

References

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