Equilibrium in Local Marijuana Games

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ABSTRACT

We examine black market marijuana agriculture with the tools of game theory, population biology, and marginal analysis. A local marijuana game (LMG) is a nonlinear dynamical system, the totality of optimizing behavior of Growers, Ripoffs, and Narcs in an environment where agent behavior does not affect the constant marijuana price. Growers grow, Ripoffs steal, and Narcs eradicate and arrest. Growers resemble K-strategists, Ripoffs resemble r-strategists, and Narcs resemble predators. We study two types of LMG equilibria: n-person game-theoretic equilibria and ecological steady-state carrying capacities. The population ratio of Growers to Ripoffs drives the game-theoretic equilibria. We show that Narc increase induces Ripoff increase and that optimal Grower planting strategies resemble the optimal nesting strategies of many species. A minimal mathematical model describes the LMG carrying capacity as the maximal sustainable proportion of planted marijuana patchland given any agent mix. The carrying capacity defines a unique fixed-point equilibrium of the LMG dynamical system, and the LMG system quickly converges to it exponentially. A simple testable relationship describes this equilibrium patch proportion P_e: $P_e = 1 - (r + n)/g$. The equilibrium analysis applies with change of coefficients to black market poppy and coca shrub agriculture. We discuss extensions to similar games, including the "border game" played by Aliens, Bandits, and Patrols.

The Marijuana Black Market

Government and press analysts have estimated that marijuana is at least the fourth largest cash crop in the United States after corn, wheat, and soybeans. It may be the third or second largest cash crop, maybe even the largest. The *Los Angeles Times* (Jehl, 1989) reports that the United States produced 3,500 metric tons of marijuana in 1987 and 4,500 metric tons in 1988, largely grown on government lands. U.S. Forest Service land covers 191 million acres divided into 156 areas.

U.S. federal and state laws prohibit marijuana cultivation, distribution, and consumption. Prohibition produces an elaborate black market that resembles the black market for prohibited alcoholic beverages in the late 1920s in the United States. Nadelmann (1989) describes the U.S. black market in marijuana as unique among international drug black markets:

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> Because marijuana is far bulkier per dollar of value than either cocaine or heroin, it is harder to conceal and easier to detect. Stepped-up interdiction efforts in recent years appear to have reduced the flow of marijuana into the United States and to have increased its price to the American consumer. The unintended consequences of this success are twofold: the United States has emerged as one of the world's leading producers of marijuana; indeed, U.S. producers are now believed to produce among the finest strains in the world; and many international drug traffickers appear to have redirected their efforts from marijuana to cocaine.

We can largely explain the macro-structure of the marijuana black market with the standard microeconomic tools of supply-demand equilibria, risk analysis, and information lags. But we cannot explain as easily the micro-structure of the marijuana black market. The micro-structure analysis belongs more properly in the domain of ecology and population biology, even sociobiology in the broad sense of Wilson (1975), than in the domain of microeconomics, except in the limiting case.

The agents who grow, steal, and eradicate marijuana produce a peculiar tug-of-war. This conflict, this "grass game" in every sense of the term, depends on the local environmental parameters that affect the growing, hiding, protecting, harvesting, and transporting of illegal marijuana patches. We show that the techniques of game theory and carrying capacities yield intuitive, predictive insight into this $10-billion-a-year game (*Newsweek*, 1982).

Some properties of these marijuana games appear structurally similar to more general patterns and laws of ecological behavior. We predict that similar equilibration takes place in "outlaw" games. Standard black-market analyses may be too coarse-grained to detect or describe these transient and steady-state properties.

Analytical Methods

We cannot easily measure grass-game behavior. Relevant government statistics are hard to obtain, politically filtered, and inaccurate.

The optimum procedure would be direct measurement in the field. We did not attempt this and do not advise it. Marijuana growers have become sophisticated and vicious in how they protect their patches. News sources report that growers protect their plant sites with guns or booby trap them with bear traps, trip-wired hand grenades and zip guns, rattlesnakes, poison-tipped nail boards, homemade land mines, and the like.

Analytical methods provide an alternative approach. The analytical methods of game theory and population biology provide insight into the behavior of marijuana production-related agents. They also provide testable propositions about local aggregate behavior. Our motivation for applying game theory stems from the sociobiological analyses of Maynard-Smith (1974, 1982) and his concept of an "evolutionarily stable strategy" (ESS), an uninvadable gene strategy. The related notions of a "culturally stable strategy" (CSS) of Dawkins (1980) and a "collectively stable strategy" (CSS) of Axelrod (1984) have also motivated this analysis, since these researchers have applied these game-theoretic equilibria to human agents who maximize utility.

But the ESS and CSS's, and the many other equilibria based on the underlying concept of a Nash (1950) equilibrium, are noncooperative equilibria. They do not suffice for the analysis of gaming behavior that is exclusively human. Humans cooperate as well as compete. They form coalitions, make side payments, take bribes, issue threats, collude, and bargain.

The grass game exhibits all these behaviors. Unfortunately, cooperative game theory (Luce & Raiffa, 1957; Owen, 1981; Shapley & Shubik, 1972) is far less tractable and far more qualitative than uncooperative game theory. The qualitative nature of our game-theoretic results reflects this. Unlike our dynamical-system results, these results are pattern predictive, not numerically predictive.

Research Results

We first define and analyze a local marijuana game. Coalition behavior among marijuana growers destabilizes an initial equilibrium reached with marijuana thieves. A new stable equilibrium results that resembles standard agricultural behavior. Then we introduce law-enforcement agents. They behave as predators in an ecosystem of competing prey.

The results include a property similar to the optimal nesting strategies of some birds and reptiles: As overall predation increases, a grower's optimal patch size decreases and his or her optimal number of patches increases. This suggests a more general principle of population biology and ecology. The harsher the environment, the more selection favors competitive r-strategic behavior over cooperative K-strategic behavior—the more it pays to steal, scavenge, plunder, brood parasite, and so forth.

The final and principal result characterizes quantitatively the carrying capacity, or equilibrium, of a local-marijuana-game nonlinear dynamical system. The system converges exponentially quickly to this unique equilibrium from all initial conditions. We should tend to observe this steady-state global behavior rather than transient behavior.

Local Marijuana Games

We study the local behavior of agents involved with marijuana production in some geographical area. We assume that the price, or shadow price, of the marijuana does not vary with agent behavior. Only revenue and profit vary with marijuana quantity. The assumption of a constant price formally distinguishes our analysis from the standard microeconomic analysis of black markets and provides the essential component of local behavior.

A "local marijuana game" (LMG) is a nonlinear dynamical system with three elements: (1) some connected geographical region, such as a county, with disconnected patches of growing marijuana, (2) a constant marijuana price throughout the region, insensitive to all agent behavior, and (3) three types of optimizing agents: Growers, Ripoffs, and Narcs.

Growers and Ripoffs maximize expected profit. They behave so as to maximize the their expected quantity of marijuana minus their operating and opportunity costs. (The continuity assumption includes continuous planting, tending, harvesting, and raiding of patches. This assumption is more realistic in warm climates than in cool climates and, in general, has become more realistic as the use of greenhouses and fast-maturing marijuana hybrids spreads.)

Growers grow the marijuana on the patchland. Ripoffs try to steal it. Growers tend the patches, protect them, and harvest them as best they can.

Ripoffs behave as hit-and-run opportunists, marauders. They search the patchland randomly. They hazard grower defenses, penetrate camouflage, and avoid previously searched areas. If Ripoffs independently search according to a uniform probability distribution, then, according to the Law of Search established by Koopman (1980), the probability that Ripoffs locate a unit patch in subregion A at time t is $1 - e^{-[ct/\mathrm{Area}(A)]}$ for some scaling constant $c > 0$. The success probability rapidly increases as the search area shrinks and decreases as it expands.

Roughly speaking, we can identity Growers can with K-strategists and Ripoffs with r-strategists, in the terminology of MacArthur and Wilson (1967). Growers resemble a stable species. They are simply farmers. The K refers to the LMG Grower carrying capacity, the optimal Grower population size. Aggregate Grower behavior tends to maintain this steady-state carrying capacity. Ripoffs play a shorter-term strategy. They adapt best to harsh, unpredictable environments. The "r" in r-strategist historically represents the Malthusian factor $b - d$, the difference of the population birth and death rates. Perhaps b and d correspond to Ripoff entry and departure rates. Or perhaps b and d correspond to the switching rates of agents who switch their strategy mix from some Grower behavior to less or more Grower behavior. This assumes that mixed Grower–Ripoff strategies occur, as they surely do in practice. Wilson's (1975, p. 99) characterization of r-strategists, humanly interpreted, neatly summarizes Ripoffs:

> Such a species will succeed best if it can do three things well: (1) discover the habitat quickly, (2) reproduce rapidly to use up the resources before other competing species exploit the habitat, or the habitat disappears altogether, and (3) disperse in search of other new habitats as the existing one becomes inhospitable.

Grower–Ripoff Game-Theoretic Equilibria

We now seek a stable LMG equilibrium. We seek an equilibrium where no agent has an incentive to change his or her strategy mix of growing and stealing or to quit playing the grass game altogether (or to join if he or she does not play).

Suppose there are no Narcs in the LMG. Suppose there are many more Growers than Ripoffs. Then it pays to steal, marginally speaking. For there are more patches to raid, and the average Grower has less chance of being raided and will tend to behave less cautiously. If, in the extreme case, everyone were a Grower, it would be maximally advantageous to steal.

Now suppose there are more, or nearly more, Ripoffs than Growers. Then it pays to grow. For, even if a Grower stands to be ripped off, he or she has a fighting chance of harvesting something. Ripoffs have fewer patches to raid, and those patches will tend to be more carefully protected. If a Ripoff raids a patch, this very Ripoff stands to be ripped off by other Ripoffs. If, in the other extreme case, everyone were a Ripoff, patches would disappear, and it would be maximally advantageous to grow.

These two countervailing forces in the LMG push and pull according to the size of G/R, the ratio of the number of Growers to the number of Ripoffs.

The two forces equilibrate in a noncooperative game-theoretic equilibrium mix of Growers and Ripoffs, relative to the environment, achieved by marginal agents switching strategies (or joining or leaving the LMG). Some agents will give up the plow for the sword, others the reverse. The harsher the environment—the more mountainous, the more violent rains or droughts, the more "predators"—the more cultural selection favors the strategic behavior of Ripoffs, and hence the smaller the equilibrium Grower–Ripoff population ratio.

This noncooperative equilibrium is unstable. Growers form protective coalitions that destabilize the equilibrium. Growers enjoy the economies of scale of coordinated communication, alarm, and defense systems—economies of scale denied to the marauding and treacherous Ripoffs. A Grower can improve his or her lot if this Grower teams up up with a nearby Grower. Those two can improve their lots if they team up with another Grower, and so on. We assume that the environment is not so prohibitively harsh that it prevents sustained Grower communication and joint action.

Ripoffs respond in two ways. First, some form bands. But bands are hard to maintain for hit-and-run opportunists. Bands increase the probability of Grower detection for each band member. And given the nearly random nature of Ripoff search, a band has little or no more chance of locating patches than has a band member who searches alone. If the band members spread out and search independently, a band member has an incentive to pocket his or her own findings and encourage the other members to evenly share theirs. *R*-strategists are not team players.

Ripoffs respond in a second way as well. To some degree they yield to Grower-coalition pressure and become Growers. They increase their Grower, and decrease their Ripoff, strategy proportions. Or they quit the game.

The new Growers increase the protective strength of all Growers. This new Grower strength marginally increases the number of Growers, relative to the number of Ripoffs. And the increased number of Growers increases their strength. This feedback process continues until diseconomies of scale dominate. If the LMG environment is not excessively harsh, the number of Ripoffs rapidly decays to the zero asymptote. This final mix of agents is stable. The final equilibrium has reduced to the standard agriculture case. Ranchers and Rustlers play a similar game.

LMG Predation Effects

Narcs behave as predators. They prey on Growers and Ripoffs, with a marked preference for Growers, just as foxes prey on rival rabbit species.

Narcs have two objectives. They seek to minimize the total LMG marijuana crop. And they seek to minimize the attendant Growers (and Ripoffs if they catch any).

Consider the dynamics of the fox–rabbit analogy. The more rabbits in an area, the easier for foxes to find and catch them. The more foxes, the fewer rabbits. If the foxes catch all the rabbits, they chase other prey (other law-breakers) or starve (lose their jobs). In either case, new rabbits soon appear and the cycle continues. Narc–Grower interaction approximates this dynamical system.

Narcs destabilize the stable Grower–Ripoff equilibrium reached in the previous section. The game changes qualitatively. Suppose, in the analogy, foxes markedly prefer red rabbits to blue rabbits. Then how does a fox increase affect the blue rabbit population?

The number of surviving blue rabbits will increase. The size of the increase depends on the size of the ratio of red to blue rabbits, on G/R. Eventually the increase passes into decrease as foxes begin to wipe out the red rabbit population, relative to the blue rabbit population, and switch their attention from red to blue rabbits. Reflecting the same ecological principle, when humans have tried to exterminate an insect or other pest species in a competitive ecosystem, some or many competitor pest species have quickly restored the balance, often making the original problem worse.

The fox–rabbit analogy motivates a surprising and robust proposition:

Proposition 1. If Narcs increase, Ripoffs increase—up to a threshold of probability of capture.

A simple argument establishes Proposition 1. Since Narcs prey mainly on Growers, Ripoffs encounter marginally less Grower defense as Growers shift some of their protection time and resources from Ripoffs to Narcs. A few Ripoffs succeed at raiding patches when before they would have failed.

Predator-Prey Model of Agent Interaction

We can use a simple Lotka-Volterra predator–prey dynamical model to approximate the conditions of Proposition 1 and see to what extent model behavior supports it. In general such models are not robust. Different modelers can plausibly cast them in different ways. More important, these dynamical systems model only the rapid fluctuation of short term agent behavior, increasing the model's sensitivity to the modeler's assumptions. In the next section, we develop a robust global model of longer-term LMG behavior, the regional carrying capacity.

Let G, R, and N denote continuously differentiable non-negative functions of time that represent the respective number of Growers, Ripoffs, and Narcs in the LMG. More generally, we can model these population functions as random variables with dynamics described by stochastic differential equations. We shall assume the simpler continuous deterministic model. We also assume no structure on the shape of the predatory function N. It enters the model exogenously as an arbitrary function of time.

A simple Grower equation that describes the interactive dynamics of G, R, and N takes the form

$$\dot{G} = G(T_G - G) - GR - NG \tag{1}$$

The overdot denotes time differentiation. T_G denotes a Grower threshold constant (in general it varies slowly), and $T_G > 0$. If $G > T_G$, Growers inhibit or overcrowd each other. If $G < T_G$, the LMG can sustain more Growers. Scale economies still exist. In steady state,

where $\dot{G} = 0,$

$$G_{\text{Steady-State}} = T_G - R - N \tag{2}$$

So T_G equals the LMG Grower carrying capacity in the standard agriculture case, when $R = N = 0$.

A similar differential model approximates Ripoff dynamics:

$$\dot{R} = G\,(T_R - R) - R - RN \tag{3}$$

T_R denotes the threshold number of Ripoffs in the LMG. T_R regulates Ripoff competition just as T_G regulates Grower competition.

Ripoffs approach their saturation level when the number of Ripoff–Grower interactions exceeds GT_R. Until that point the marginal gains from stealing remain positive. The term $-R$ in equation (3) describes Ripoff passive decay. If $G = N = 0$, then

then $R(t) = R(0)e^{-t} \longrightarrow 0$. In steady state,

$$R_{\text{Steady-State}} = \frac{T_R G}{1 + N + G} \quad (4)$$

So the steady-state number of Ripoffs defines an increasing function of G and a decreasing function of N. For a fixed endowment of Narcs in the LMG, equation (4) implies that the steady-state number of Ripoffs in the LMG approximates T_R for large G.

The population ratio G/R modulates LMG behavior. To see this, we rewrite equation (3) as

$$\dot{R} = G/R \ \ [(RT_R - R^2) - R/(G/R) - RN/(G/R)].$$

R^2 measures Ripoff interactions. Ripoff population growth amplifies if $R < G$, and damps if $R > G$.

We can now examine Proposition 1 in light of the equilibrium outcomes equations (2) and (4). If we substitute the equilibrium number of Growers G in equation (2) into equation (4) and rearrange, we get $R(1 + T_G + T_R) - R^2 = T_R T_G - T_R N$.

Then to find the marginal quantity or partial derivative $\partial R/\partial N$, the rate of change of R at equilibrium due to a small change in the number N of Narcs, we differentiate implicitly:

$$\frac{\partial R}{\partial N}(1 + T_G + T_R) - 2\frac{\partial R}{\partial N}R = -T_R \quad (5)$$

Then the partial derivative has the hyperbolic form

$$\frac{\partial R}{\partial N} = \frac{T_R}{2R - 1 - T_G - T_r} \quad (6)$$

This ratio has an infinite singularity at $R = (1 + T_G + T_R)/2$. The marginal change $\partial R/\partial N$ decreases or increases according as R is less than or greater than this quantity. In this sense the equilibrium model partly supports Proposition 1 and partly fails to support it. We can ignore the number 1 in the sum since thresholds T_G and T_R are much larger. The model fails to support the proposition only when the number of Ripoffs is less than the average threshold numbers of Growers and Ripoffs, $(T_G + T_R)/2$, a reasonable requirement. Below their thresholds, Growers and Ripoffs have non-Narc incentives to increase in number.

When the model supports the proposition, when $R > (1 + T_G + T_R)/2$, the model behaves as it should. The rate of increase in Ripoffs caused by an increase in Narcs decreases monotonically as the number R of Ripoffs increases.

Predation Effects on Patch Structure

Next we examine how LMG predation affects marijuana patch structure. Predation affects the shape and distribution of patches. Predation affects the average patch size, the number of plants per patch in a typical Grower patch. And it affects the average patch number, the typical number of patches per Grower.

Consider the Grower's optimal strategy in a standard-agriculture equilibrium. The Grower plants a single large patch (ignoring possible diseconomies due to environmental harshness), and puts all eggs in one nest. The Grower can more easily tend one patch than two, two more easily than three, and so on.

Now suppose a small number of Narcs, relative to Growers, enter the game. Then Growers marginally shift their time and resources from Ripoff protection to Narc protection. Growers marginally restructure their patches to shield the patches from helicopter and airplane surveillance. They marginally restructure their alarm systems and transport to allow early detection and escape. They marginally restructure their budgets to account for bonds, legal fees, fines, and income lost while in jail.

Growers suffer a decline in expected profit. Revenue (harvested patches) decreases, and operating costs increase. To break even, Growers must plant more marijuana. But planting takes more effort and resources than before. Grower risk—expected loss—has increased.

Patch structure changes. If a Grower simply increases the size of his or her single patch, the Grower stands to lose everything in a Narc or Ripoff raid. A better strategy hedges the risk by planting two or more smaller patches—put the eggs in different nests, as some reptiles do in the face of increasing predation and environmental harshness. A Grower may find patches harder to tend and protect this way, but more patches can be planted. The Grower can also afford to lose a few patches without facing economic ruin.

So as Narcs increase, patch size tends to decrease and patch number tends to increase. Since Growers plant more but smaller patches, Ripoffs tend to find patches more easily. The probability decreases that Growers protect a given patch. Ripoffs, and Growers, find stealing marginally easier and more profitable than before. This process enhances the behavior described in Proposition 1, speeding the Ripoffs' rise to saturation.

Suppose the number of Narcs increases further. Suppose the local attorney general comes up for re-election and doubles the previous number of Narcs. Then Growers marginally adjust their behavior as before. They shift more time and resources from Ripoff protection to Narc protection. They lose more revenue. Their risk increases. They plant more but smaller patches. More Ripoffs cash in on the change.

But now Ripoffs face a slightly greater chance of capture. The inhibitory effects of increased Narc interaction guide the Ripoff population toward saturation. Agents find growing harder, stealing easier. Some Growers switch their LMG strategy to Ripoff. Some quit the game, and new Ripoffs may appear. The new mix of Growers, Ripoffs, and Narcs soon equilibrates the LMG dynamical system to a new carrying capacity, as we show in the next section.

As Narcs continue to increase, the same processes continue to unfold. Growers shift still more time and resources to protection from Narcs, lose more revenue, incur more risk, and as a result plant more but smaller patches. Ripoffs thrive, with saturation pressure mounting. Agents find growing still harder, stealing easier. More Growers convert or quit. The new mix of agents rapidly results in a new but lower carrying capacity.

Further increases in Narcs continue these processes until, finally, the costs to Growers exceed the benefits of defending against Ripoffs and Narcs. The remaining Growers steal or quit. They are wiped out. In turn this wipes out Ripoffs, and perhaps Narcs. Eventually the LMG carrying capacity equals zero. Proposition 2 summarizes LMG predation effects on patch structure:

Proposition 2. As predation increases, optimal patch size decreases and optimal patch number increases.

This ecological principle stretches between two extremes. At one extreme, when there are no predators (Narcs), the optimal Grower strategy is to plant a single large patch, as if Growers were "legitimate" farmers planting soybeans or ginseng. At the other extreme, when predation is nearly prohibitive, the optimal Grower strategy is to randomly plant patches with one plant per patch.

Imagine a maximally hostile beach environment and the sad spectacle of a mother sea turtle randomly crawling about the beach, planting individual eggs in the sand as she crawls. With the current "War on Drugs" in the United States (Goldstein & Kalant, 1990), no doubt many Growers, at least in notorious regions like Mendocino County in California and the Hawaiian mountains and the Daniel Boone National Forest in Kentucky (Jehl, 1989), have shared the grief of the mother turtle. A *Newsweek* (1982) cover story article, compiled in part in the field, finds that in response to increased Drug Enforcement Agency (DEA) marijuana eradication efforts, "In northern California these days, pot patches of more than 100 plants are fast becoming a rarity, and many veteran growers plant their crops in tiny clumps intermixed with foliage."

LMG Carrying Capacity

What is the LMG carrying capacity? In population biology the carrying capacity of a species refers to its steady-state population. The carrying capacity is the maximum number of the species that an environment can sustain indefinitely in steady state. In this sense, the Grower and Ripoff thresholds T_G and T_R define carrying capacities. But they give only steady-state information about one type of LMG agent.

We seek a global LMG carrying capacity that involves the interaction of the environment and all the agents. Which LMG parameter best summarizes LMG behavior?

A natural candidate is the total LMG marijuana crop at a given time. But the total crop is an absolute quantity. It may fluctuate while the LMG multi-agent game structure remains fixed. This happens in effect when we compare the LMG behavior of different geographical regions.

We avoid this with a relative measure, the ratio of actual to potential planted area. We assume the total patchland has only finite extent. Define $P(t)$ as the *proportion of planted patchland* at time t:

$$P(t) = \frac{\text{Planted Patchland at } t}{\text{Total Patchland}}$$

Figure 1 illustrates the proportion $P(t)$. Since $P(t)$ takes values in [0, 1] for all t, we can interpret $P(t)$ as the probability that a "random" unit patch contains marijuana plants at time t.

$P(t)$ can equally measure the proportion of land planted in poppies or coca shrubs. Nadelmann (1989) reports that in South America approximately 2,500,000 square miles of surface area could support coca cultivation but only 700 square miles actually do.

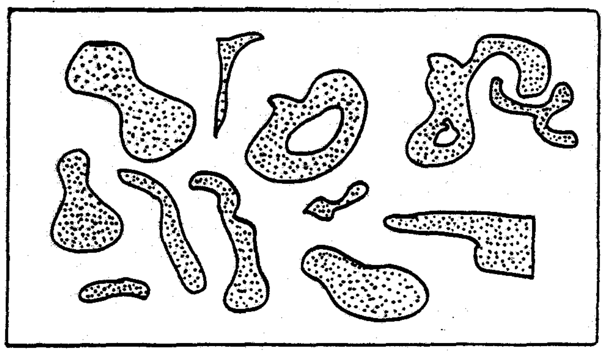

Figure 1. Graphical display of the LMG patchland or sample space. The rectangular region represents the patchland: all the marijuana-plantable land in the LMG region. The speckled subregions represent the spatial distribution of planted patches. $P(t)$ measures at time t the proportion of planted patchland, the ratio of the sum of the speckled areas to the total area. This keeps $P(t)$ in $[0, 1]$. We can interpret $P(t)$ as the probability that a random unit patch is planted at time t.

We define the LMG carrying capacity as a fixed-point attractor of the LMG dynamical system. The LMG carrying capacity is the equilibrium proportion P_e of planted patchland found in the steady state, when

$$\dot{P} = 0.$$

We assume that the agent populations $G(t)$, $R(t)$, and $N(t)$ change faster than $P(t)$ changes. People move faster than plants. Dynamically this means that Growers and Ripoffs, per equations (1) and (3), and Narcs have equilibrated to fixed stable proportions. Denote these respective constant proportions by the positive numbers g, r, and n. (g, r, and n admit different but definite operational interpretations.)

More generally, g, r, and n represent slowly varying functionals of the LMG system behavior, summarizing the recent and distant history of different LMG subsystems. For simplicity we assume that the functionals vary so slowly as to be constant.

How does P change to fit the agent mix g, r, and n? What form should the adaptation equation $\dot{P} = f(g, r, n, P, t)$ take? We develop the dynamical model in minimal steps.

Consider the simplest case, the standard-agriculture case, when $r = n = 0$. Then we expect the Growers to plant all the patchland: $P(t) \rightarrow 1$ as $t \rightarrow$ infinity. The simplest dynamical model has P increase with Grower planting:

$$\dot{P} = gP.$$

But this is too simple. For then $P(t) = P_0 e^{gt} \rightarrow$ infinity, where P_0 denotes the initial patch proportion, $P_0 = P(0)$. Such exponential growth occurs only at first when Growers have planted only little of the patchland and do not plant in each other's way.

Grower overcrowding increases as the proportion of unplanted patchland, $1 - P$, decreases. Planted patchland increases with P and decreases with $1 - P$. This conjunction of patchland pressures leads to the nonlinear dynamical model

$$\dot{P} = gP(1 - P) \tag{6}$$

The nonlinear forcing term $-gP^2$ measures competitive Grower interactions in the expansion $gP - gP^2$.

Equation (8) below implies that equation (6) keeps P in $[0, 1]$ for all t. (We can also show this by solving equation (6) directly.) Equation (9) below and equation (6) imply that $P(t) \rightarrow 1$ as $t \rightarrow$ infinity for *all* initial conditions. Growers rise rapidly in number and then slowly approach the all-planted asymptote, reaching the standard-agriculture case.

Ripoffs and Narcs inhibit marijuana production. They represent parasitism and predation. Their influence increases as the planted patchland P increases. In the simplest case they inhibit planting independently and hence additively. Ripoffs can still inhibit planting if there are no Narcs, and vice versa. The additive inhibition of rP and nP gives the final dynamical model of patchland adaptation:

$$\dot{P} = gP(1 - P) - rP - nP \tag{7}$$

This differential equation has the form of a logistic growth/decay law. With some effort we can solve it exactly to get

$$P(t) = \frac{(g - r - n)\,P_0}{gP_0 + [(g - r - n) - gP_0]e^{-(g-r-n)t}} \tag{8}$$

So if there are no growers, if $g = 0$, then the model obeys intuition: the planted patchland $P(t) = P_0 e^{-(r+n)t}$ decays exponentially to zero for all combinations of Ripoffs and Narcs and all initial conditions P_0.

The stable equilibrium patch proportion P_e, reached exponentially fast, defines a unique fixed-point attractor, a point of global stability. The LMG dynamical system converges to it for any choice of the system parameters g, r, and n (though different choices of the parameters produce different P_e). For any mix of parameters g, r, and n, P_e does not depend on initial conditions, since as time t increases, $e^{-(g-r-n)t} \rightarrow 0$. So for any initial patch distribution P_0,

$$P(t) \rightarrow \frac{(g-r-n)P_0}{gP_0} = \frac{g-r-n}{g} \tag{9}$$

We rewrite equation (9) to at last derive the LMG carrying capacity, or equilibrium patch proportion, P_e:

$$P_e = 1 - \frac{r + n}{g} \tag{10}$$

Again from equation (8) we can assume $g > 0$ in equation (10). Similarly we can assume $g = \max(g, r + n)$. For if $r + n > g$, then the denominator of equation (8) approaches positive infinity exponentially, and thus $P(t)$ decays exponentially to zero.

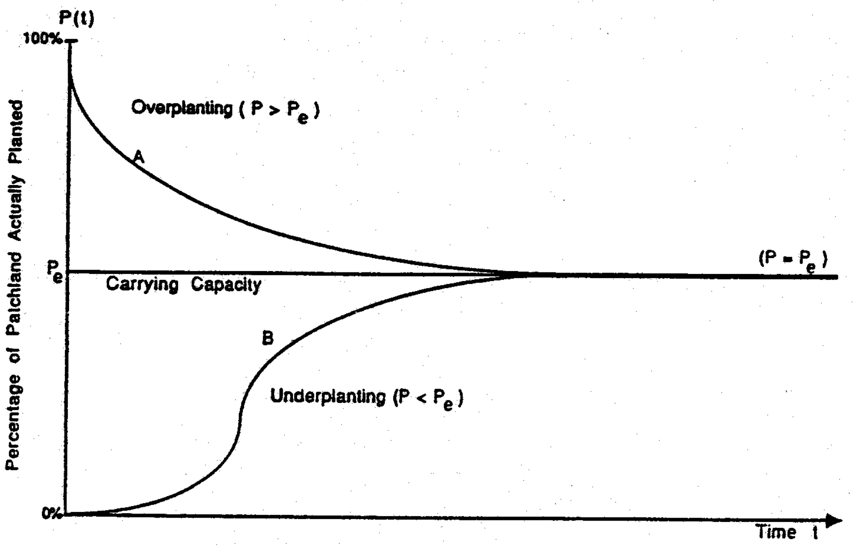

Figure 2. Planted patchland as a function of time. The vertical axis defines the locus of initial conditions. The carrying capacity $P_e = 1 - (r + n)/g$ describes the unique fixed-point equilibrium of the LMG dynamical system. The exponential convergence does not depend on initial patch conditions, only on the agent mix g, r, and n. Curve A shows patch behavior when the Growers overplant the area, when $P(t) > P_e$. Sigmoidal or S-shaped curve B shows patch behavior when Growers underplant, when $P(t) < P_e$.

The unique fixed-point equilibrium in equation (10) grounds and sharpens our intuitions of LMG carrying capacity. Consider the extreme cases. The standard agriculture case results when $r = n = 0$. Then Growers have planted all the patchland: $P_e = 1$.

Suppose the number of Ripoffs and Narcs is prohibitively large relative to Growers. Then $r + n$ equals or exceeds g, and so all patchland goes unplanted: $P_e = 0$. All other mixes of g, r, and n vary smoothly between these two extremes and produce $0 < P_e < 1$. Figure 2 illustrates how underplanting ($P < P_e$) and overplanting ($P > P_e$) equilibrate rapidly to P_e. All trajectories monotonically approach P_e exponentially fast.

We can test the LMG carrying-capacity prediction in equation (10). This simple combination of Grower, Ripoff, and Narc coefficients predicts the gross LMG marijuana crop distribution and how it changes with agent changes.

We can solve the relationship for any of the coefficients to predict aggregate agent behavior. In practice we might estimate the coefficients as linear functions of the mean agent populations. Exponential convergence of P to P_e and the global stability of P_e jointly enhance testability. They suggest that observed LMG behavior tends to resemble equilibrium LMG behavior.

Discussion and Extension to Similar Games

LMGs equilibrate in an agent sense and in an ecological sense. The cooperative-competitive behavior of Growers and Ripoffs produces game-theoretic equilibria. Narc predation shapes these equilibria. More generally the harshness of the environment shapes the equilibria. If we advised Grower–Ripoff agents how to modify their strategy mixes in different environments, we could summarize these environmental effects in a rough heuristic rule: Narcs down, then grow. Narcs up, then steal!

The LMG carrying capacity P_e represents an ecological equilibrium, and more directly admits testing and control than does the game-theoretic equilibrium. The stable agent mix shapes the patch environment, not the other way around. The LMG system reaches both types of equilibria rapidly, and should reach game-theoretic equilibria faster because agents interact on a faster time scale than marijuana cultivation proceeds.

Proposition 1, which predicts a Ripoff increase given a Narc increase, appears a special case of a general population-biological tendency: an increase in environmental harshness favors r-selected behavior over K-selected behavior. (Does the non-warring, communal behavior of early arctic Innuits support or defy this tendency?) So we can expect to see its effect throughout the animal and plant kingdoms. We might expect, for instance, that an increase in temperature variance or sunlight variance favors underbrush over trees.

Instances of Proposition 1 occur in human affairs. I am familiar with two cases in Southern California.

The Lobster Game

Poachers, Ripoffs, and (Game) Wardens play a game off the San Diego Baja coast. In the La Jolla Cove preserve, Poachers poach lobsters in underwater traps and cages. At night Ripoffs in scuba gear raid the traps.

In this lobster game, Wardens try to stop both Poachers and Ripoffs from taking lobsters from the preserve. They catch more Poachers than Ripoffs for obvious reasons. Newspaper anecdotal reports and "scuba street talk" have expressed surprise over the increase in Ripoffs that follows an increase in Wardens or in Warden activities. Time-series data on lobster-population estimates and protection activity could test this tendency.

Similar games may arise in government protected environments where poachers trap endangered species—sea otters along the Pacific coast, exotic birds in tropical forests, small and big game snared in African savannas and forests.

The Grower–Ripoff–Narc game structure extends directly to the deadly games played around the world where entrepreneurs and farmers cultivate poppies and coca shrubs. To a lesser extent it extends to the street environments where other entrepreneurs trade the resultant heroin and cocaine.

The Border Game

The border game provides a second instance of Proposition 1. Agents play this game in Southern California and to a much lesser extent in Arizona, New Mexico, and Texas. On many evenings I have driven along the Mexican side of the Tijuana–San Diego fenced border, and walked the Mexican side of the nearby beach border at Playas, and watched a few dozen United States Border Patrolmen contend with hundreds of Mexican citizens lined up and soon running across the border to California.

U.S. Border Patrols act to prevent Mexican Aliens from illegally entering the U.S. from Mexico. Bandits accumulate on the Mexican side of the border. The Bandits prey, or rather parasite, on the often destitute Aliens. The Bandits rob them, beat them, double cross them, rape them, and sometimes shoot them. Although Bandits may seem more predatory than parasitic, Patrols actively seek them in the U.S. and sometimes in Mexico. Bandits never seek Patrols.

We predict that an increase in Patrols leads to an increase in Bandits. The increase depends on the size of the Alien/Bandit population ratio. Only scant, anecdotal evidence supports this prediction.

The prediction follows from the argument for Proposition 1. An increase in Patrols forces Aliens to marginally switch some of their attentions, their cautions, from Bandits to Patrols. Since the probability of Patrol capture has increased, this tempts an Alien to stay in Mexico and become a Bandit himself.

Increased Patrols deter border crossings and generate a larger pool of Aliens waiting entry into the U.S. This further encourages banditry. Waiting Aliens, poor by assumption, cannot wait for long. Eventually they must brave the border, return home, or bandit other Aliens. As Patrol predation increases, the risks of illegal entry compound, communications wither, and reliable information becomes scarce.

A countervailing force inhibits Bandit increase that has no parallel in the other games discussed. Aliens captured and deported back to Mexico feed back information about Bandits and Patrols to the pool of waiting Aliens. Deported Aliens may even try to even the score with Bandits. The larger the Alien/Bandit ratio, the less this negative feedback inhibits banditing. The same Bandits are less likely to prey upon Aliens in earshot of the returned Aliens.

Genetic and Economic Arms Races

In the terminology of Dawkins and Krebs (1979, 1982), we have uncovered an arms race between Growers and Ripoffs, Ranchers and Rustlers, Poachers and Ripoffs, etc. An arms race results between species when one species manipulates another for indefinite stretches of evolutionary time. The classic example is cuckoo brood parasitism of warblers.

The exploited species does not lay still for the manipulation, such as egg mimicry. In time the exploited species develops counter strategies. Warblers evolve finer egg discrimination. Over successive generations the two species play a mounting game of countermeasures, counter-countermeasures, counter-counter-countermeasures, and so on up.

The species with most at stake plays harder and manipulates the other species. (Aesop: The rabbit outruns the fox because the fox runs only for its dinner, while the rabbit runs for its life.) The cuckoo's lineage depends on how well it deceives the warbler. The warbler loses only small increments of its total reproductive fitness.

In the LMG grass game, a Ripoff's livelihood depends on his or her nimbleness in Grower–Ripoff interactions. As countermeasures mount, the Ripoff must continually outmaneuver Growers. A Grower may survive economically even if the Grower loses in every bout of the Grower–Ripoff arms race.

References

Axelrod, R. (1984) *The Evolution of Cooperation*. New York: Basic Books.

Dawkins, R. and Krebs, J.R. (1979) "Arms Races between and within Species." *Proceedings of the Royal Society of London, B 205*, 489–511.

Dawkins, R. (1980) "Good Strategy or Evolutionarily Stable Strategy?" in Barlow, G.W. and Silverberg J., eds. *Sociobiology: Beyond Nature/Nurture?* Boulder, CO: Westview, pp. 331–367.

Dawkins, R. (1982) *The Extended Phenotype: The Gene as the Unit of Selection*. Oxford: Freeman.

Goldstein, A. and Kalant, G. (1990) "Drug Policy: Striking the Right Balance." *Science, 249*, 1513–1521.

Jehl, D. (1989) "Forest Service, Marijuana Growers Battle to Standoff." *Los Angeles Times*, 14 October, A22.

Koopman, B.O. (1980) *Search and Screening*. New York: Pergamon.

Luce, R.D. and Raiffa, H. (1957) *Games and Decisions*. New York: Wiley.

MacArthur, R.H. and Wilson, E.O. (1967) *The Theory of Island Biogeography*. Princeton, NJ: Princeton University Press.

Maynard-Smith J. (1974) "The Theory of Games and the Evolution of Animal Conflicts." *Journal of Theoretical Biology, 47*, 209–221.

Maynard-Smith, J. (1982) *Evolution and the Theory of Games*. Cambridge, UK: Cambridge University Press.

Nadelmann, E.A. (1989) "Drug Prohibition in the United States: Costs, Consequences, and Alternatives." *Science, 245*, 939–947.

Nash, J. (1950) "Equilibrium Points in N-Person Games." *Proceedings of the National Academy of Science, USA, 36*, 48–49.

Newsweek (1982) "Guns, Grass, and Money." 25 October, pp. 36–43.

Owen, G. (1982) *Game Theory* (2nd ed.) New York: Academic.

Shapley, L.S. and Shubik, M. (1972) "The Assignment Game I: The Core." *International Journal of Game Theory, 1*, 111–130.

Wilson, E.O. (1975) *Sociobiology: The New Synthesis*. Cambridge, MA: Harvard University Press.

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