

How Many Blonds Mess Up a Nash Equilibrium?

By BART KOSKO

The film "A Beautiful Mind" is the first to deal with a profound social issue that touches everyone but that few people understand—a Nash equilibrium.

The film also focuses on the mental illness of mathematician John Nash, who won the 1994 Nobel Prize in economics for a two-page paper he published in 1950 when he was 20. But without that Nobel Prize, there would have been no film.

Yet the filmmakers failed to correctly explain a Nash equilibrium. And a mere flash of text at the film's start only hints at why the *Journal of Economic Literature* recently said the impact of the Nash equilibrium in the social sciences "is comparable to that of the discovery of the DNA double helix in the biological sciences."

So what is a Nash equilibrium?

A Nash equilibrium shows how selfish competitors should act in relation to those whom they compete against.

Swiss political philosopher Jean-Jacques Rousseau suggested a

hunting example in his 1755 "Discourse on Inequality" that has led to something called the deer game.

Suppose you are one of four starving people in the forest. You have just two options: You can try to catch a rabbit or a deer. But you can catch a deer if—and only if—all four of you work together to catch one. How each person behaves depends on how the others behave.

Suppose the other three people catch rabbits. Then the best you can do is catch a rabbit too because you have no chance to catch a deer. This rabbit hunt is a Nash equilibrium because each person does his selfish best given what the others do and because no one has an incentive to switch hunting strategies.

Thus order arises from competitive struggle.

Now suppose the other three people try to catch a deer. You still eat if you catch a rabbit, but then they can't catch a deer. They will catch a deer if you help them and then you can all have a feast. So it is in your selfish interest to try to catch a deer. The deer hunt is also a Nash equilibrium because each person does the best he can and

has no reason to change his hunting strategy.

The movie gets this backward when it concocts a "blond game" in a bar. Each young man in the bar wants to pick up a blond woman rather than a brunet. Then a blond beauty walks in with several brunets.

The Nash character—Russell Crowe—conceives the Nash equilibrium in this fictitious scene. He claims that no man should pursue the blond because they can't all have her, and the pursuit would insult the brunets. So he claims that the optimal strategy is to pursue only the brunets.

But each man will want to switch from his brunet to the blond if all the other men have brunets. So this is not a Nash equilibrium.

The film's logic says that children will pick up only the pennies on a sidewalk and not the hundred-dollar bill lying next to the pennies because they can't all have the bill. Our own selfishness says otherwise.

Studies of ultimatums have shown that we can be so selfish that we become envious and we don't achieve a Nash equilibrium. Suppose I have \$100 and I offer

you a share of it. The rules let us keep our shares if you accept my offer. But neither of us gets anything if you reject my offer. Then I should offer you as little as possible and you should accept anything I offer. Yet more than half of players reject an offer of less than \$20 even though accepting even \$1 is better than nothing.

Nash equilibrium can also explain the darker side of behavior.

I once wrote a paper about outlaws who grow and steal marijuana plants. It pays to steal if there are many more growers than thieves because growing pot is so risky. But it pays to grow if there are too many thieves because a grower has some chance of harvesting something while thieves find little to steal and other thieves will steal from them.

The outlaws will adjust their strategy mixes of growing and stealing until they reach Nash equilibrium.

John Nash deserved his Nobel Prize—and a more accurate movie.

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