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On a 50% savings in the computation of the centroid of a *symmetrical* interval type-2 fuzzy set

Jerry M. Mendel *

Department of Electrical Engineering, Signal & Image Processing Institute, University of Southern California, 3740 McClintock Avenue EEB 400, Los Angeles, CA 90089-2564, USA

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Abstract

Computing the centroid of a type-2 fuzzy set (T2 FS) is an important operation for such sets. For an *interval* T2 FS, the centroid can be computed by using two iterative procedures that were developed by Karnik and Mendel [2]. In this paper, we prove that if the footprint of uncertainty for an interval T2 FS is symmetrical about the primary variable y at $y = m$, then the centroid is also symmetrical about $y = m$ and its defuzzified value equals m. As a consequence of this, computation of the centroid for such a T2 FS is reduced by 50%, and the importance of obtaining a non-symmetrical interval T2 FS prior to defuzzification is demonstrated.

* Tel.: +1 213 740 4445; fax: +1 213 740 4446.
E-mail address: mendel@sipi.usc.edu

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1. Introduction

Recently, Mendel [6] proposed a fuzzy set (FS) model for words that is based on collecting data from people—*person membership functions (MFs)*—that reflect *intra-* and *inter-levels of uncertainties* about a word, in which a word FS is the union of all such person FSs. The *intra-uncertainty* about a word is modeled using interval type-2 (T2) person FSs, and the *inter-uncertainty* about a word is modeled using an equally weighted union of each person's interval T2 FS. Because an interval T2 FS plays such an important role in this model as well as in engineering applications of T2 FSs (e.g., [5]), we need to understand as much as possible about such sets and how they model uncertainties.

Definition 1. A *T2 FS*, denoted $\tilde{A}$, is characterized by a (3D) *T2 MF* $\mu_{\tilde{A}}(y,u)$, where $y \in Y$ and $u \in J_y \subseteq [0,1]$, i.e.,

$$\tilde{A} = \{((y,u), \mu_{\tilde{A}}(y,u)) | \forall y \in Y, \forall u \in J_y \subseteq [0,1]\} \tag{1}$$

in which $0 \leqslant \mu_{\tilde{A}}(y,u) \leqslant 1$. $\tilde{A}$ is also expressed as

$$\tilde{A} = \int_{y \in Y} \left[\int_{u \in J_y} f_y(u)/u \right] \Big/ y, \quad J_y \subseteq [0,1] \tag{2}$$

where $\int\int$ denotes union over all admissible y and u. For discrete universes of discourse $\int$ is replaced by $\sum$. In (2) $f_y(u)$ is called a *secondary grade*, and for an *interval T2 FS* all $f_y(u) = 1$. In (1) and (2) J_y is called the *primary membership* of y, and in (2) $\int_{u \in J_y} f_y(u)/u$ is called the *secondary MF*. Uncertainty about $\tilde{A}$ is conveyed by the union of all of its primary memberships, which is called the *footprint of uncertainty* (FOU) of $\tilde{A}$, i.e., $\mathrm{FOU}(\tilde{A}) = \cup_{y \in Y} J_y$.

When T2 FSs are used in a rule-based fuzzy logic system (FLS) (e.g., [3,5]), combined T2 fired rule consequent FSs are first mapped into a T1 FS by computing the centroid of the combined sets, and then this centroid is defuzzified into a number by computing its center of gravity. Note that the centroid of a T2 FS [2] is a 2D function—a T1 FS.

The most widely used T2 FS set to-date is the interval T2 FS. Here we shall examine the calculations of the centroid [1] and its defuzzified value for the special but very important case of an interval T2 FS whose FOU is *symmetrical*—a so-called *symmetrical interval T2 FS*. This case occurs very frequently because the FOU for a rule antecedent or consequent T2 FS (in a T2 FLS) is often prespecified to be symmetrical (e.g., the FOU for a Gaussian primary MF with variable mean $m \in [m_1, m_2]$ and standard deviation $\sigma \in [\sigma_1, \sigma_2]$ is symmetrical

[1] The *centroid* of an interval T2 FS is mathematically defined in Section 3.

Table 1
Centroid of a Gaussian primary MF with uncertain mean and interval set secondary MFs

$[m_1, m_2]$	$m_2 - m_1$	$[c_l, c_r]$	$c_r - c_l$
[5, 5]	0	[5, 5]	0
[4.87, 5.12]	0.25	[4.87, 5.12]	0.25
[4.75, 5.25]	0.5	[4.74, 5.25]	0.51
[4.62, 5.37]	0.75	[4.62, 5.37]	0.75
[4.5, 5.5]	1	[4.49, 5.50]	1.01
[4.25, 5.75]	1.5	[4.21, 5.78]	1.57
[4, 6]	2	[3.90, 6.09]	2.19
[3.75, 6.25]	2.5	[3.55, 6.44]	2.89
[3.5, 6.5]	3	[3.15, 6.84]	3.69

about $(m_1 + m_2)/2$). Symmetry also occurs when addition is performed on symmetrical interval T2 fuzzy numbers [1] (e.g., *About* 3 + *About* 10).

In [5] we presented two examples in which we calculated the centroid for two symmetrical interval T2 FSs. For those people who have not seen these examples we repeat them here, since they are very illuminating and will motivate the rest of this paper.

Example 1. In this example [5], we compute the centroid $[c_l, c_r]$ for a Gaussian primary MF [2] with uncertain mean $m \in [m_1, m_2]$ and certain standard deviation $(\sigma = 1)$, whose secondary MFs are interval sets. Table 1 summarizes the results for a range of $m_2 - m_1$ values, including the T1 case when $m_2 - m_1 = 0$. Observe that as the uncertainty about the mean increases $[c_l, c_r]$ increases as well, i.e., $c_r - c_l$ increases. Observe, also, that $[c_l, c_r]$ is always symmetrical about the T1 mean, $m = 5$, and that the average value of c_l and c_r is always equal to 5, regardless of the amount of uncertainty there is in m.

Example 2. In this example, we compute the centroid $[c_l, c_r]$ for the Gaussian primary MF with uncertain standard deviation $\sigma \in [\sigma_1, \sigma_2]$ and certain mean $(m = 5)$, whose secondary MFs are interval sets. Table 2 summarizes the results for a range of $\sigma_2 - \sigma_1$ values, including the T1 case when $\sigma_2 - \sigma_1 = 0$. Observe that as the uncertainty about the standard deviation increases $[c_l, c_r]$ increases as well, i.e., $c_r - c_l$ increases. Observe, also, that $[c_l, c_r]$ is again always symmetrical about the known mean, $m = 5$, and that the average value of c_l and c_r is always equal to 5, regardless of how much uncertainty there is about σ.

These observations have caused us to wonder whether or not there are general theoretical results about the centroid and its average (defuzzified) value for a symmetrical interval T2 FS. There are, and that's what the rest of this paper is about.

[2] A *primary MF* is a parameterized T1 MF whose parameters may vary over intervals.

Table 2
Centroid results for a Gaussian primary MF with uncertain standard deviation, and interval set secondary MFs

$[\sigma_1, \sigma_2]$	$\sigma_2 - \sigma_1$	$[c_l, c_r]$	$c_r - c_l$
[1, 1]	0	[5, 5]	0
[0.88, 1.13]	0.25	[4.80, 5.20]	0.40
[0.75, 1.25]	0.5	[4.60, 5.40]	0.80
[0.63, 1.38]	0.75	[4.40, 5.60]	1.20
[0.5, 1.5]	1	[4.18, 5.81]	1.62
[0.38, 1.63]	1.25	[3.93, 6.07]	2.14
[0.25, 1.75]	1.5	[3.59, 6.41]	2.82

2. Statements of main results

Our main result is summarized in the following:

Theorem. *Given a FOU for an interval T2 FS, one that is symmetrical about primary variable y at* $y = m$ (*e.g.*, Fig. 1), *then the centroid of such a T2 FS is symmetrical about* $y = m$, *and the average value* (*i.e.*, *the defuzzified value*) *of all the elements in the centroid equals m.*

Before proving this theorem, the results of which may seem intuitive to some readers, we explain its importance. If, for example, we begin with interval T2 fuzzy numbers, all characterized by symmetrical FOUs, and perform an operation (arithmetic, set-theoretic, non-linear function) on them that leads to another interval T2 fuzzy number with a symmetrical FOU, then the result of the combined centroid + defuzzification procedures, which are performed after these operations, could just as well have been obtained by treating the T2 fuzzy numbers as crisp and performing crisp operations on them. In short, *for such*

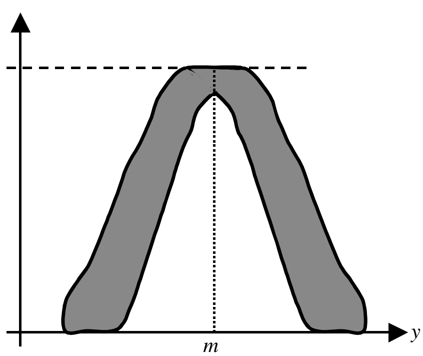

Fig. 1. A symmetrical FOU.

T2 fuzzy numbers and operations, if all that is desired is a crisp number after performing said operations on the T2 fuzzy numbers, then it is a waste of effort to perform the calculations using T2 mathematics. All knowledge about the uncertainties of the numbers, as captured by their T2 MFs, is lost at the end of the centroid + defuzzification procedures. Of course, the centroid of such T2 FSs still provides a useful measure of the uncertainties that have propagated through the operations. These observations lead us to the following:

Corollary 1. *If all that is desired is a crisp number after performing operations on interval T2 FSs, then for the use of such sets to make a difference to not using them (e.g., to using T1 FSs or just crisp numbers) the operations that are applied to them must lead to a T2 FS that has a non-symmetrical FOU.*

Interestingly enough, non-symmetrical FOUs occur in a Mamdani or TSK (Takagi–Sugeno–Kang) rule-based FLS. For example, in a Mamdani FLS, although the FOU for each fired rule is usually symmetrical (e.g., Fig. 2) this symmetry is (fortunately) lost when the fired rule T2 FSs are combined, e.g., by union (e.g., Fig. 3), height defuzzification, etc. Note that in Figs. 2 and 3 light shaded regions are the FOUs of two fired consequent sets, $\underline{f}^1$ and $\bar{f}_1$ are the lower and upper firing levels for Rule-1, and $\underline{f}^2$ and $\bar{f}_2$ are the comparable quantities for Rule-2. Formulas for these upper and lower firing levels can be found in [4] or [5].

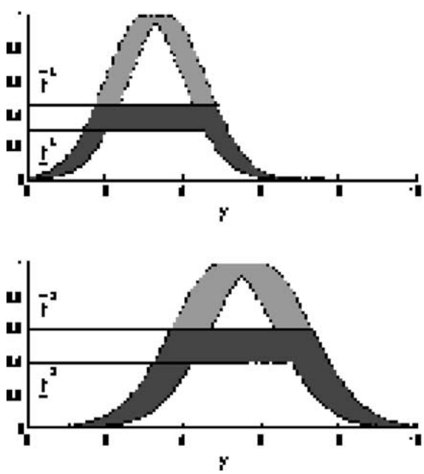

Fig. 2. Fired output sets (dark shaded regions) for two fired rules in an interval Mamdani T2 FLS when min t-norm is used.

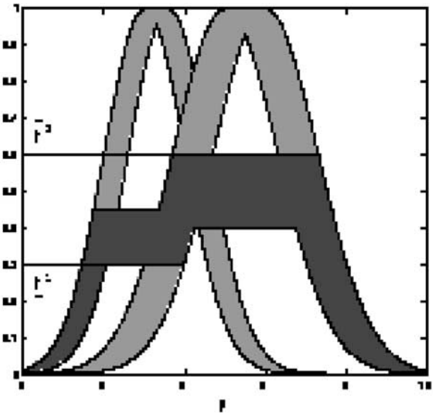

Fig. 3. Union combined output set for the two fired output sets in Fig. 2.

3. Preliminaries

The proof of theorem assumes continuous universes of discourse and only considers the FOU in Fig. 1 since that FOU is a quite general symmetrical FOU. In addition, we assume that: (1) the FOU is shifted to the origin, by means of the linear transformation of variables $y \to y - m$ (this is justified in Lemma 1) where m is the value of y about which the FOU is symmetrical; (2) the primary variable for the symmetric FOU is discretized into $N+1$ sample points where N is an even integer; and (3) each primary membership (see Definition 1) is discretized into the same number of levels, M. As a result of these assumptions, we direct our attention at the situation depicted in Fig. 4.

Before proceeding to a proof of the theorem, we provide some preliminaries.

An interval T2 FS whose primary membership at $y = y_i$ contains only one element $u = u_i$ (i.e., $J_{y_i} = u_i$) is denoted $(1/u_i)/y_i$. In this case, the secondary MF is the unit spike $1/u_i$.

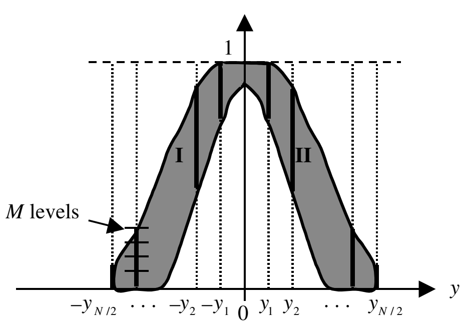

Fig. 4. Translated and discretized FOU. I and II denote the two symmetrical halves of the FOU.

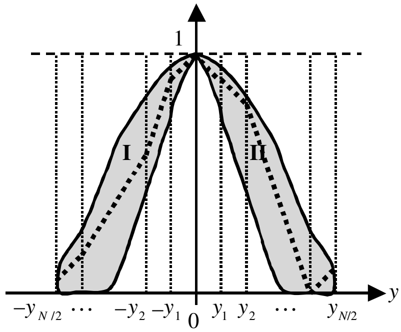

Fig. 5. Embedded (unsymmetrical) T2 FS. The dashed curve is the associated embedded T1 FS. At each of its values, the secondary grade equals 1. Although the embedded set is shown as a continuous curve, it only has values at $N+1$ discrete values of y.

Definition 2. For discrete universes of discourse Y and U, the jth embedded interval T2 FS $\tilde{A}_e^j$ has $N+1$ elements, [3] where $\tilde{A}_e^j$ contains exactly one element from $J_{y_{-N/2}}, \ldots, J_0, \ldots, J_{y_{N/2}}$, namely $u_{-N/2}^j, \ldots, u_0^j, \ldots, u_{N/2}^j$, each with its associated unity secondary grade, i.e.,

$$\tilde{A}_e^j = \sum_{i=-N/2}^{N/2} \left[1/u_i^j\right]/y_i, \quad u_i^j \in J_{y_i} \subseteq U = [0,1] \tag{3}$$

Note that $y_{-i} \equiv -y_i$ (see Fig. 4).

An example of an embedded interval T2 FS is depicted in Fig. 5. Because all secondary grades equal 1, we only show the domain [i.e., $\{(y_i, u_i),\ i=-N/2,\ldots,0,\ldots,N/2\}$] of the embedded T2 FS, which is also called an *embedded T1 FS*. When it is unnecessary to distinguish between an embedded T2 or T1 FS, then we just refer to such a set as an *embedded set*.

Fact 1. *There are at most M^{N+1} embedded sets.*

Proof. Consider the situation depicted in Fig. 4. Using the well-known Multiplication Rule in combinatorics, we obtain the result that there are exactly M^{N+1} embedded sets. Note that this number may be smaller if one or more points are common to *all* embedded sets (e.g., if the upper and lower MFs merge over a range of sample points). Hence, there are at most M^{N+1} embedded sets. □

[3] In earlier publications (e.g., [5,7]) we assumed that $\tilde{A}_e^j$ contained N elements. For a symmetrical FOU it is more convenient to assume that $\tilde{A}_e^j$ contains $N+1$ elements, where N is even, so that $N/2$ elements can be associated with each of the mirror images of the FOU (Regions I and II in Fig. 4), and one element is shared by the mirror images.

Fact 2. *Let $\tilde{A}_{\mathrm{e}}^{j}$ denote the jth T2 embedded set for an interval T2 FS $\tilde{A}$. Then $\tilde{A}$ can be represented as the union of all of its T2 embedded sets, i.e.,*

$$\tilde{A} = \sum_{j=1}^{M^{N+1}} \tilde{A}_{\mathrm{e}}^{j} \tag{4}$$

A proof of this useful representation for a T2 FS is given in [7] where it is called a *Representation Theorem*. It is valid for all kinds of T2 FSs, and not just for interval T2 FSs. While Mendel and John do not advocate using (4) for computing (because of the huge number of embedded T2 FSs), this representation is very useful for developing theoretical results about T2 FSs. We shall make use of it in our proof of the theorem.

Definition 3. The *centroid of the* jth embedded T2 FS is located at $y = c_{\mathrm{e}}^{j}$ ($j = 1,\ldots,M^{N+1}$) and its amplitude equals one; it is denoted $1/c_{\mathrm{e}}^{j}$, where

$$c_{\mathrm{e}}^{j} = \frac{\sum_{i=-N/2}^{N/2} y_i u_i^j}{\sum_{i=-N/2}^{N/2} u_i^j}, \quad u_i^j \in J_{y_i} \subseteq [0,1] \tag{5}$$

Note that c_{e}^{j} is the centroid of the embedded T1 FS that is associated with $\tilde{A}_{\mathrm{e}}^{j}$.

Fact 3. *The centroid, C, of an interval T2 FS can be thought of as being computed as follows:* (1) *compute the centroid of all embedded T1 FSs, c_e^j* ($j = 1,\ldots,M^{N+1}$), (2) *assign a unity MF to each of the resulting centroids, thereby obtaining the single-element T1 FS $1/c_e^j$* ($j = 1,\ldots,M^{N+1}$), *and* (3) *union all of these sets to obtain C, i.e.,*

$$C = \sum_{j=1}^{M^{N+1}} 1/c_{\mathrm{e}}^{j} \tag{6}$$

This follows directly from (4). By this fact, we can focus our attention at computing the centroids, c_{e}^{j}, of the embedded T1 FSs.

Next, we provide some preliminary lemmas that are used in our proof of the theorem.

Lemma 1. *Let $f(y)$ be a function (e.g., an embedded T1 FS) with center of gravity $\bar{f}$, and $g(y)$ be an m-translated version of $f(y)$, with center of gravity $\bar{g}$, i.e., $g(y) = f(y+m)$. Then,*

$$\bar{f} = \bar{g} + m \tag{7}$$

Proof. This follows directly from the fact that

$$\bar{g} = \frac{\int_{-\infty}^{\infty} y g(y)\,\mathrm{d}y}{\int_{-\infty}^{\infty} g(y)\,\mathrm{d}y} = \frac{\int_{-\infty}^{\infty} y f(y+m)\,\mathrm{d}y}{\int_{-\infty}^{\infty} f(y+m)\,\mathrm{d}y} = \frac{\int_{-\infty}^{\infty} (z-m) f(z)\,\mathrm{d}z}{\int_{-\infty}^{\infty} f(z)\,\mathrm{d}z} = \bar{f} - m \tag{8}$$

so that $\bar{f} = \bar{g} + m$. □

Lemma 2. *For an origin-shifted symmetrical FOU, all of its embedded T2 FSs are either symmetrical about $y = 0$ or are unsymmetrical about $y = 0$.*

Proof. This is trivial to prove, since all embedded T2 FSs (see Figs. 5 and 6) can only be symmetrical or unsymmetrical about $y = 0$. □

Lemma 3. *For an origin-shifted symmetrical FOU, there are exactly $M^{N/2+1}$ symmetrical embedded T2 FSs, each with a centroid equal to* 1/0.

Proof. Let $r(y_j)$ and $r(y_{-j})$ denote the amplitudes of the embedded T1 FS at the sample points y_j and y_{-j}, respectively, and $r(0)$ denotes the amplitude of the embedded T1 FS at $y = 0$. Recall, also, that $y_{-j} \equiv -y_j$. Then, we can express the centroid of any embedded T1 FS (see Figs. 5 and 6) as

$$c_{\mathrm{e}}^{j} = \frac{\sum_{j=1}^{N/2} y_j r(y_j) + \sum_{j=1}^{N/2} y_{-j} r(y_{-j}) + 0 \times r(0)}{\sum_{j=1}^{N/2} r(y_j) + \sum_{j=1}^{N/2} r(y_{-j}) + r(0)} \tag{9}$$

Invoking symmetry ($r(y_{-j}) = r(y_j)$), we can re-express (9) as:

$$c_{\mathrm{e}}^{j}(\text{symmetrical}) = \frac{a+b}{c+d} = 0 \tag{10}$$

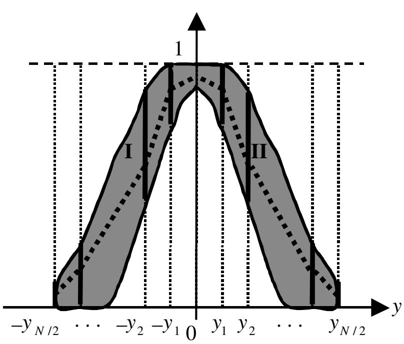

Fig. 6. Embedded (symmetrical) T2 FS.

where

$$a = \sum_{j=1}^{N/2} y_j r(y_j) + 0 \times r(0)/2 \tag{11}$$

$$b = -\sum_{j=1}^{N/2} y_j r(y_j) + 0 \times r(0)/2 \tag{12}$$

$$c = \sum_{j=1}^{N/2} r(y_j) + r(0)/2 \tag{13}$$

$$d = \sum_{j=1}^{N/2} r(y_{-j}) + r(0)/2 \tag{14}$$

Eq. (10) demonstrates that the centroid of each symmetrical embedded T2 FS can be expressed as 1/0.

That there are exactly $M^{N/2+1}$ such sets follows by observing that each of the symmetrical regions (I and II in Fig. 6) contains $N/2$ sampled values of the primary variable y, and shares the element at $y=0$. The latter element can be distributed equally in the two regions, as shown in each of the terms in (10). Doing this preserves the symmetry of the entire embedded set and allows us to talk about two symmetrical segments for that set, one in Region I and the other in Region II. Applying the previously used multiplication rule $N/2+1$ times, we obtain the value $M^{N/2+1}$. □

Lemma 4. *For a symmetrical FOU, there are exactly* $M^{N/2+1}(M^{N/2}-1)$ *unsymmetrical embedded T2 FSs, but they occur as pairs of mirror images (e.g., see Fig. 7).*

Proof. From Lemmas 2 and 3, the number of unsymmetrical embedded T2 FSs equals

$$M^{N+1} - M^{N/2+1} = M^{N/2+1}(M^{N/2} - 1).$$

Because Regions I and II are mirror images of each other about $y=0$, any embedded T2 FS in these regions must have a mirror image, e.g., see Fig. 7. □

Lemma 5. *Let* $g(y)$ *denote an unsymmetrical embedded T2 FS with centroid* $1/\bar{g}$, *and* $h(y)$ *denote the mirror image of* $g(y)$ *with centroid* $1/\bar{h}$. *Then,*

$$1/\bar{h} = 1/-\bar{g} \tag{15}$$

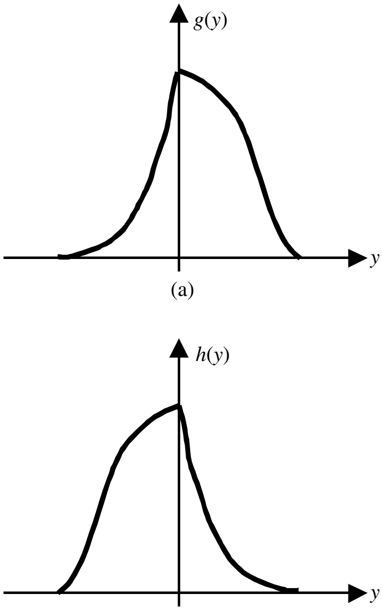

Fig. 7. Embedded type-2 fuzzy sets that are mirror images of each other, i.e., $g(y) = h(-y)$.

Proof. Using the facts that

$$1/\bar{g} = 1 \bigg/ \left[\int_{-\infty}^{\infty} y g(y)\, \mathrm{d}y \bigg/ \int_{-\infty}^{\infty} g(y)\, \mathrm{d}y \right] \tag{16}$$

$$1/\bar{h} = 1 \bigg/ \left[\int_{-\infty}^{\infty} y h(y)\, \mathrm{d}y \bigg/ \int_{-\infty}^{\infty} h(y)\, \mathrm{d}y \right] \tag{17}$$

setting $y = -t$ in (17), and then using the fact that $h(-t) = g(t)$, it follows that

$$\begin{aligned} 1/\bar{h} &= 1 \bigg/ \left[\int_{-\infty}^{\infty} t h(-t)\, \mathrm{d}t \bigg/ \int_{-\infty}^{\infty} -h(-t)\, \mathrm{d}t \right] \\ &= 1 \bigg/ - \left[\int_{-\infty}^{\infty} t h(-t)\, \mathrm{d}t \bigg/ \int_{-\infty}^{\infty} h(-t)\, \mathrm{d}t \right] \\ &= 1 \bigg/ - \left[\int_{-\infty}^{\infty} t g(t)\, \mathrm{d}t \bigg/ \int_{-\infty}^{\infty} g(t)\, \mathrm{d}t \right] = 1/-\bar{g} \qquad \square \end{aligned} \tag{18}$$

4. Proof of theorem

The proof is in two parts. First, we prove that the centroid is symmetrical about $y=m$, and then we prove that the average (defuzzified) value of all the elements in the centroid equals m.

4.1. Symmetry of the centroid

We can think of computing the centroid of an interval T2 FS that has a symmetric FOU using (6), i.e., we must compute the centroids of its M^{N+1} embedded T2 FSs. From Lemma 3, we already know that the centroid of each of the $M^{N/2+1}$ symmetrical embedded T2 FSs equals 1/0. From Lemmas 4 and 5, we also know that the remaining $M^{N/2+1}(M^{N/2}-1)$ unsymmetrical embedded T2 FSs occur in pairs, one of which has centroid $1/d_i$ and the other of which has centroid $1/-d_i$, where d_i denotes the centroid of the respective embedded T1 FS, and

$$i = 1, 2, \ldots, \frac{1}{2}M^{N/2+1}(M^{N/2}-1)$$

Clearly, the collection of all of these centroids is symmetrical about $y=0$; hence, by (6), C must also be symmetrical about $y=0$. Using Lemma 1, we conclude that the centroid of the original unshifted interval T2 FS is symmetrical about $y=m$.

4.2. Defuzzified centroid

Next, we compute the average value, AVG, of all M^{N+1} embedded T2 FS centroids. AVG contains three m-shifted terms, as described in Section 4.1, i.e.,

$$\begin{aligned}
\text{AVG} &= \frac{mM^{N/2+1} + \frac{1}{2}M^{N/2+1}(M^{N/2}-1)(m+d_i)}{M^{N+1}} \\
&\quad + \frac{\frac{1}{2}M^{N/2+1}(M^{N/2}-1)(m-d_i)}{M^{N+1}} \\
\text{AVG} &= \frac{mM^{N/2+1} + mM^{N/2+1}(M^{N/2}-1)}{M^{N+1}} \\
\text{AVG} &= \frac{mM^{N+1}}{M^{N+1}} = m
\end{aligned} \tag{19}$$

This completes the proof of the theorem.

5. Consequence of the theorem

In the Karnik–Mendel method [2] for computing the centroid, $[c_l, c_r]$, of an interval T2 FS, two independent iterative computations must be performed, one for c_l, and one for c_r. As a result of our theorem, we can reduce these computations as follows:

Corollary 2. *Given a FOU for an interval T2 FS, one that is symmetrical about $y = m$, then $[c_l, c_r]$ can be computed by using the Karnik–Mendel method to compute c_l, after which c_r can be computed as*

$$c_r = 2m - c_l \tag{20}$$

Proof. This follows directly from the theorem in which we established that $m = (c_l + c_r)/2$. $\square$

So, for "symmetrical" interval T2 FSs, our theorem has led to a 50% reduction in centroid computations.

6. Conclusions

The results in this paper represent another successful use of the Mendel–John Representation Theorem for a T2 FS (Fact 2).

Our *centroid symmetry theorem* and Corollary 1 also help to explain and understand why performance improvements may be expected when using the defuzzified output of a type-2 FLS as compared to using the defuzzified output of a comparable type-1 FLS. The defuzzified type-2 outputs will be different from the defuzzified type-1 outputs due to the *non-symmetric* nature of the FOU of the T2 FS (e.g., see Fig. 3) prior to the calculation of its centroid and subsequent defuzzification. Our results also point up the importance for ultimately winding up with non-symmetric FOUs prior to defuzzification.

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