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Computing With Words, When Words Can Mean Different Things to Different People

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Abstract

The main thesis of this paper is that *words mean different things to different people* and so there is *uncertainty* associated with words, which means that fuzzy logic (FL) must somehow use this uncertainty when it computes with words. Type 1 FL can not do this, but type 2 FL, as recently advocated by Karnik and Mendel ([4] – [8]) can. The paper links uncertainty and type-2 FL, and explains how this uncertainty can be translated into domain information for type 2 membership functions. Doing this lets us use these membership functions and type 2 FL to compute with words. The paper also quantifies the uncertainty for a collection of 16 words.

I. Introduction

Zadeh [16] lately has been advocating "computing with words" (CW) and using fuzzy logic (FL) to do this. For, example, he states: "... computing with words is a necessity when the available information is too imprecise to justify the use of numbers and ... when there is a tolerance for imprecision which can be exploited to achieve tractability, robustness, low solution cost, and better rapport with reality;" and, "... fuzzy logic is a methodology for computing with words." The main thesis of this paper is that *words mean different things to different people* and so there is *uncertainty* associated with words, which means that FL must somehow use this uncertainty when it computes with words. Type 1 FL can not do this, but type 2 FL, as recently advocated by Karnik and Mendel ([4] – [8]), can.

First, we will link uncertainty and type-2 FL (Section II). Then, we will demonstrate (Section III) that there is uncertainty associated with words, and we will quantify the uncertainty for a collection of 16 words. Finally, we will explain how this uncertainty can be translated into domain information for type 2 membership functions (Section IV), i.e., into a *footprint of uncertainty*. Doing this lets us then use these membership functions and type 2 FL to compute with words. Conclusions are provided in Section V.

II. Uncertainty and Fuzzy Logic

The concept of a *type-2 fuzzy set* was introduced by Zadeh [15] as an extension of the concept of an ordinary fuzzy set (henceforth called a *type-1 fuzzy set*). Some other contributors to the (small) literature about type-2 fuzzy sets are: Dubois and Prade [1] – [3], Karnik and Mendel [4] – [8], Kaufman and Gupta [9], Mizumoto and Tanaka [10], [11], Nieminen [12], Turksen [13], and Yager [14]. A type-2 fuzzy set is characterized by a *fuzzy* membership function, i.e., the membership value (or membership grade) for each element is a fuzzy set in [0, 1], unlike a type-1 set where the membership grade is a crisp number in [0, 1]. Such sets can be used in situations where there is uncertainty about the membership grades themselves, e.g., uncertainty in the shape of the membership function or in some of its parameters.

An example of a type-2 fuzzy set is depicted in Fig. 1. This is the situation of a triangular membership function (Fig. 1a) whose apex location, m, is uncertain. The apex may be anywhere from $m-\delta_1$ to $m+\delta_2$; so the triangular membership function may lie anywhere in the shaded region. The vertical projection of an arbitrary point along the horizontal axis, x^*, intersects the shaded region over a range of values $[\alpha,\beta]$. This range of values is called [8] the *primary memberships* of x. The membership grades of these primary memberships are called [8] *secondary memberships* of x. Three candidate secondary memberships are indicated in Fig. 1b. The term [8] *membership grade* for each x, represents all the primary memberships and their corresponding secondary memberships taken together. In this paper, we shall refer to the collection of primary memberships of x as the *footprint* *of uncertainty*. It is the shaded area in Fig. 1a, and, it is the footprint of uncertainty that is needed for words. Computing with words can be accomplished within the framework of type-2 FL, once we have established membership functions for the words. The key though is to establish the footprints of uncertainty for the words.

Type-2 membership functions are used in type-2 fuzzy logic systems (FLSs). Figure 2 depicts the structure of a type-2FLS [6]-[8]; it is quite similar to a type-1 FLS, the only difference being that the antecedent and/or consequent sets in a type-2 FLS are type-2, so that each rule output set is type-2. *Extended* (by means of the Extension Principle) versions of type-1 defuzzification methods yield a type-1 set from the type-2 rule output sets. We call this process *type-reduction* rather than defuzzification, and the resulting type-1 set, the *type-reduced set*. The type-reduced set can then be defuzzified to obtain a crisp output. We call the combination of type-reduction and defuzzification *output processing*. An example of output processing is depicted in Fig. 3. The type-reduced set of a type-2 FLS shows the possible variation in the crisp output of the FLS due to uncertain natures of the antecedents and/ or consequents. It establishes a band of values around a crisp output value in much the same way that a confidence interval establishes a band about a point estimate when stochastic uncertainty is present; but, it does this for linguistic uncertainties. Such a band of uncertainty cannot be computed using a type-1 FLS. Due to space limitations, we cannot present any of the mathematical details that explain how to perform type-reduction and other type-2 operations. See [6]-[8] for these details. Type-2 FL software (freeware) is available on the web: *http://sipi.usc.edu/~mendel/software*.

II. Words and Uncertainty

We are interested in applying FL to situations where the meanings of words are associated with a scale 0-10, because we are interested in obtaining rule consequents from people using a survey, in which people will be given questions and their answers will be from a small vocabulary that covers this range. The discussions can easily be extended to arbitrary scales.

How can we cover such a scale with words? What is the smallest number of words that cover the interval 0-10? We will demonstrate that the answer to these questions depend on whether or not we include uncertainty associated with the words.

In order to answer these questions, we performed a survey. With the help of a social scientist (Prof. Sheila Murphy, USC Annenberg School of Communications), we established 16 words which we thought would cover the interval 0-10. No specific context was associated with these words; however, this in no way implies that the results described below are invariant to context. The words (which are referred to as *labels* in the survey) were randomized. Engineering undergraduate students were given the survey, whose wording is as follows: "Below are a number of labels that describe an interval or a 'range' that falls somewhere between 0 to 10. For each label, please tell us where this range would start and where it would stop. (In other words, please tell us how much of the distance from 0 to 10 this range would cover.) For example, the range 'quite a bit' might start at 6 and end at 8. It is important to note that *not all the ranges are the same size*." A table was provided to the students, so that they only had to fill in two numbers for each label. Survey results from 70 respondents are summarized in Fig. 4. For each label there are two circles with a solid line between them. The circles are located at the mean *start* and *end* points for the label. The dashed lines to the left of the left-hand circles and to the right of the right-hand circles, which each terminate in a vertical bar, equal one standard deviation, for the mean *start* and *end* points, respectively. Observe that standard deviations are not the same for the *start* and *end* values for each label.

Observe, also, from Fig. 4 that: (1) There is a gap between 'none' and 'very little,' implying that either another word should be inserted between them or they should be combined; for illustrative purposes, in the rest of this paper, we do the latter; (2) People seem to agree that 'none' starts at zero ... and there is very little uncertainty about this; (3) The same can not be said for the label 'a maximum amount'; the right-hand mean value for its range is 9.7571 and not 10; perhaps (as suggested by a reviewer of the journal version of this paper) people are often wary about assigning the highest possible numerical value to a question—for many people, there is always room for "more" of whatever is being asked about; (4) So, the 16 words do not quite cover the 0-10 interval, but this only occurs at the right-most extreme values; (5) There seems to be a linguistic gap between the labels 'some' and 'a moderate amount,' as evidenced by the small degree of overlap between these labels; perhaps this gap can be filled by adding the label 'somewhat moderate'; and, (6) The dashed portions of the intervals for each label represent the label's uncertainty.

What do we do with such uncertainty information, when we compute with words using (type-1) FL? Usually, in (type-1) FL we would tend to explicitly *ignore* the uncertainty, either by using just the [*start*, *end*] range for a label, or by perhaps being conservative, and using the range associated with the interval [*start* − standard deviation (*start*), *end* + standard deviation (*end*)]. As Fig. 4 indicates, it is not correct to do the former, and, as explained next, it is also not correct to do the latter.

The dashed lines in Fig. 4 represent linguistic uncertainty, in much the same way that standard deviation for a measured random quantity represents its uncertainty. When we work in the province of probability, we find it useful and important to distinguish between the mean and the standard deviation, so why should less be expected of us when we work with linguistic uncertainties? Unfortunately, type 1 FL can not let us distinguish the dashed part of an interval from the solid part. Type-2 FL can.

One of the interesting observations from Fig. 4 is: the smallest number of labels that cover the interval 0-10 is 3 (e.g., 'none to very little,' 'a moderate amount,' and 'a maximum amount'), and this is only possible because of linguistic uncertainties, as reflected by the standard deviations associated with each label. So, linguistic uncertainty is good, in that it lets us cover the 0-10 range with a much smaller number of labels than without it (i.e., without it, there would be no overlap between the intervals for these 3 labels). Put another way, in the context of firing rules in a FLS, uncertainty can fire rules. This can not occur in the framework of type-1 FLSs; but, it can occur in the framework of type-2 FLSs. So, uncertainty has the potential to reduce complexity.

IV. Footprints of Uncertainty for Words

The footprint of uncertainty for a word depends on *how* interval information is obtained from people. To begin, we consider 3 possibilities, each associated with a different wording of a query. Each query begins in the same way: "Below are a number of labels that describe an interval or a 'range' that falls somewhere between 0 and 10.":

1. For each label, tell us where this range would start and where it would stop.
2. For each label, tell us where this range is centered and how far to the right of the center and how far to the left of the center the interval extends.
3. For each label, tell us where the range is centered and how long the interval is.

We used the first wording for the survey that is described in Section III. Its answers establish sample statistics for the two end-points of an interval. The answers to the third query also establish sample statistics for two points, the center and each end point, where each end point is assumed to be equidistant from the center. If an end-point bumps into 0 or 10, it is truncated at those values. The answers to the second query seemingly establish sample statistics for three points, the center and the left- and right-end points; however, this query has a serious linguistic shortcoming: the word 'center' implies that the interval will extend equally to the left and right of the center; but the rest of the query can cause it to extend by different amounts to the left and right of the center, which is a contradiction. So, we do not investigate the footprints of uncertainty for the second query. Because of space limitations, we present results only for the first wording, whose answers establish sample statistics for the two end-points of an interval.

Next, we present a construction procedure for determining the footprint of uncertainty for type-2 membership functions. In all cases we use triangular membership functions for the interior membership functions. Construction procedures for shoulder membership functions are described in the journal version of this paper. As a reminder, the secondary membership functions are located on the footprint of uncertainty.

Let a denote the average value for the left-hand point of the interval and b denote the average value for the right-hand point of the interval. The standard deviation for the location of the left-hand point is denoted $sd1$, and the standard deviation for the location of the right-hand point is denoted $sd2$. Points a and b are shown as solid circles in Fig. 5. We define the *uncertainty intervals* for the two points as $[a - sd1,\ a + sd1]$ and $[b - sd2,\ b + sd2]$, respectively.

There are two cases that we must consider: (1) $b - sd2 > a + sd1$ and (2) $b - sd2 < a + sd1$. In the first case (Fig. 5a) the uncertainty interval for point a does not overlap with the uncertainty interval for point b; but, in the second case (Fig. 5b), it does.

Case 1: $b - sd2 > a + sd1$ (see Fig. 5a). (1) Let $b - a = l$; (2) Locate the apex of the triangle at $l/2$, and assign it unity height; (3) The left-hand vertex of the triangle, on the horizontal axis, can range from $a - sd1$ to $a + sd1$; the region of uncertainty for the left-hand leg is the shaded left-hand triangle whose vertices are at: $(a - sd1, 0)$, $(a + sd1, 0)$, and $(l/2, 1)$; (4) The right-hand vertex of the triangle, on the horizontal axis, can range from $b - sd2$ to $b + sd2$; the region of uncertainty for the right-hand leg is the shaded right-hand triangle whose vertices are at: $(b - sd2, 0)$, $(b + sd2, 0)$, and $(l/2, 1)$; and, (5) The footprint of uncertainty is the union of all points in the two shaded triangles.

Case 2: $b - sd2 < a + sd1$ (see Fig. 5b). The construction procedure is exactly the same as in Case 1. Now, however, there is no open space between the two shaded triangles, because $b - sd2 < a + sd1$. So, the region of uncertainty is the shaded triangle whose vertices are at: $(a - sd1, 0)$, $(b + sd2, 0)$, and $(l/2, 1)$.

It is important to recall that the footprint of uncertainty is the domain for the type-2 membership function; secondary values sit on top of this domain, e.g., Fig. 1. Additionally, since intervals and end-point uncertainties are different for the words that's cover the 0-10 interval (Fig. 4), the footprints of uncertainty are word-specific.

V. Conclusions

We have demonstrated that computing with words involves linguistic uncertainties, and maintain that type-1 fuzzy logic can not incorporate such uncertainties, but, type-2 fuzzy logic can. Words can mean different things to different people, and it is this that causes the uncertainty. Intervals for words need to be established for each application—context is important; this can be done using a survey. We have demonstrated that for a list of 16 labels, uncertainty about these labels lets us cover the interval 0 –10 with as few as 3 labels; this can not be done using these labels if uncertainty is ignored. For the list of labels we used, it would take at least 6 of them to cover the interval with a sufficient degree of overlap between intervals (see Fig. 4). So, uncertainty has the potential to reduce complexity; but, it requires type-2 FL to do this.

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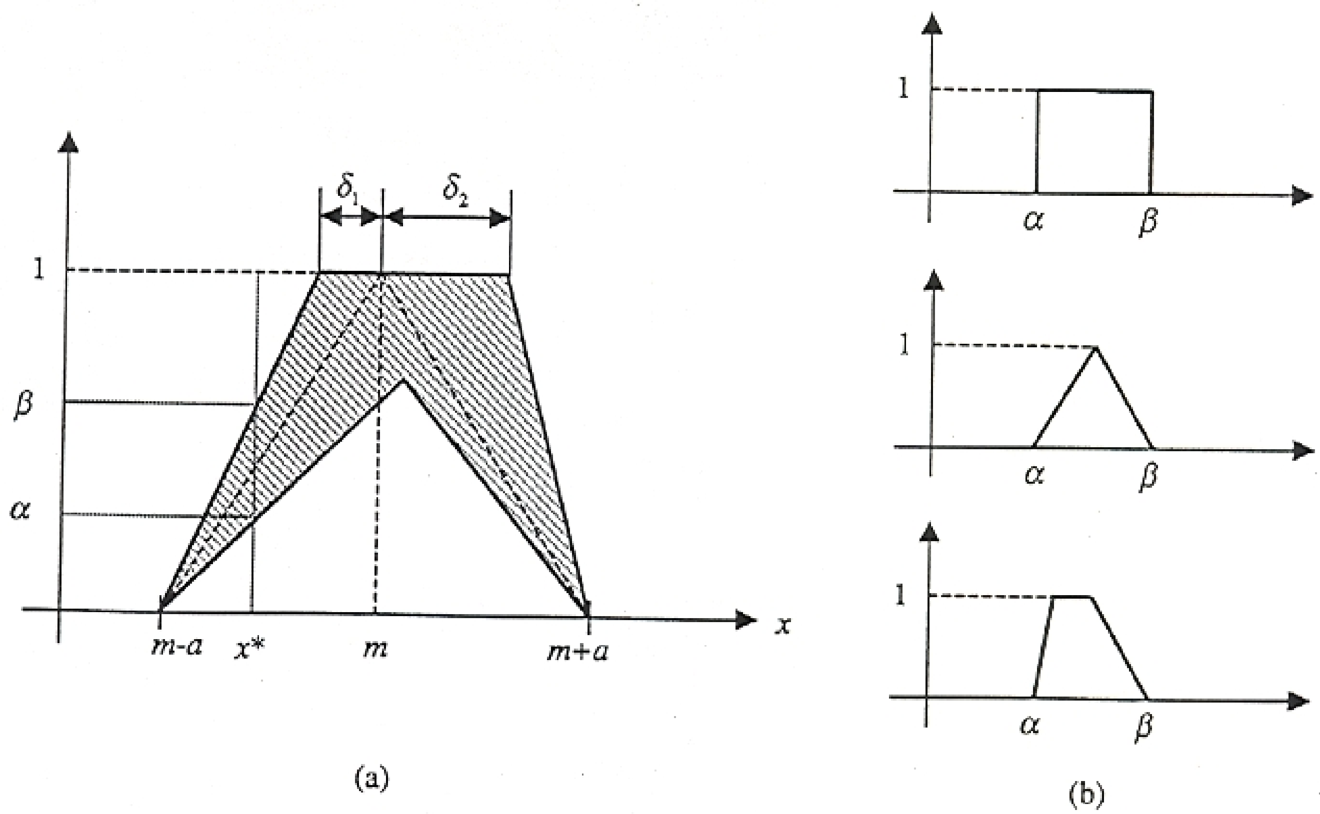

Figure 1. (a) Triangle whose apex location, m, is uncertain, and varies in $[m-\delta_1, m+\delta_2]$; (b) possible secondary membership functions at point x^*.

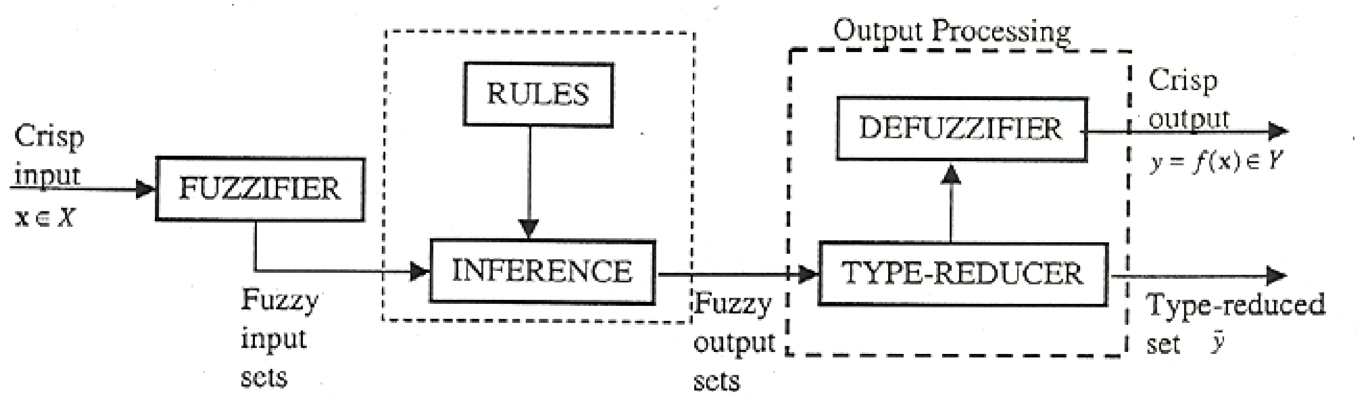

Figure 2. The structure of a type-2 FLS. Note that output processing consists of type-reduction followed by defuzzification.

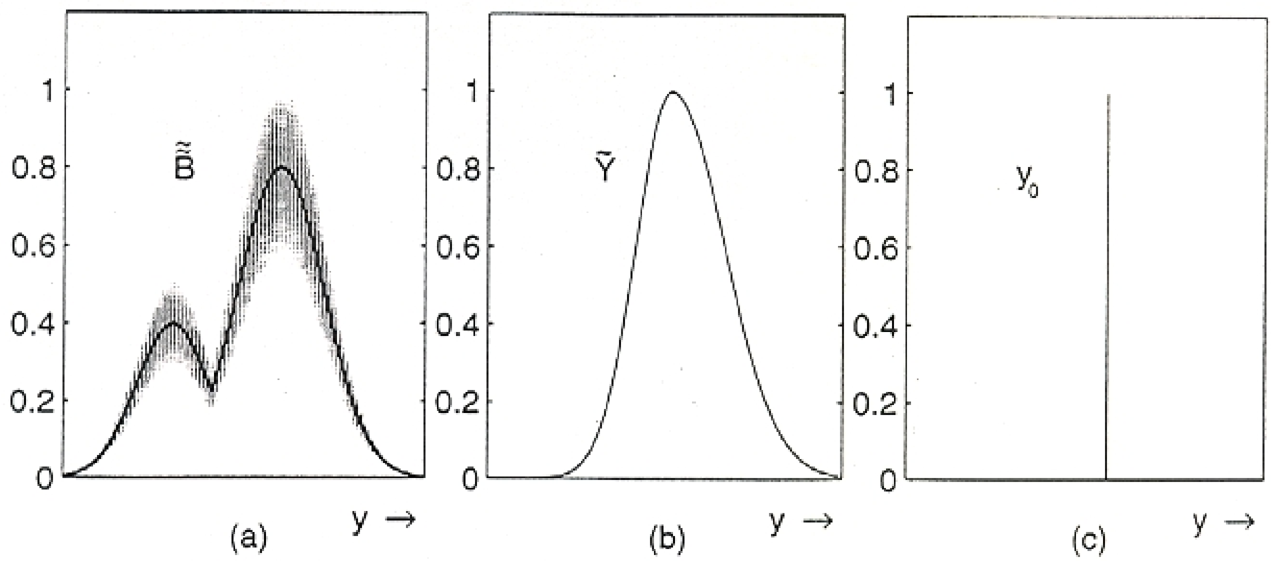

Figure 3. Pictorial representation of output processing. First, the individual rule output sets are combined in some manner to obtain type-2 fuzzy set (Fig. a); then, type-reduction is applied to to obtain the type-1 set (Fig. b); finally, the type-1 fuzzy set is defuzzified to produce the crisp output (Fig. c).

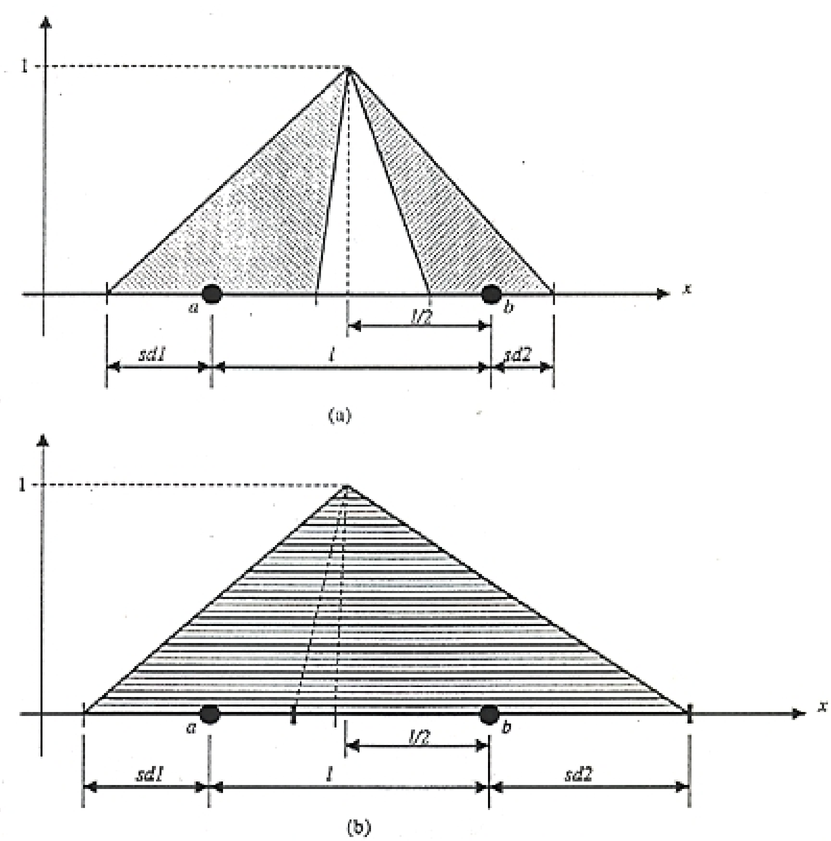

Figure 5. Footprint for secondary membership functions when end-point information is requested. (a) Uncertainty interval for *a* does not overlap with the uncertainty interval for *b*; and, (b) Uncertainty interval for *a* overlaps with the uncertainty interval for *b*.

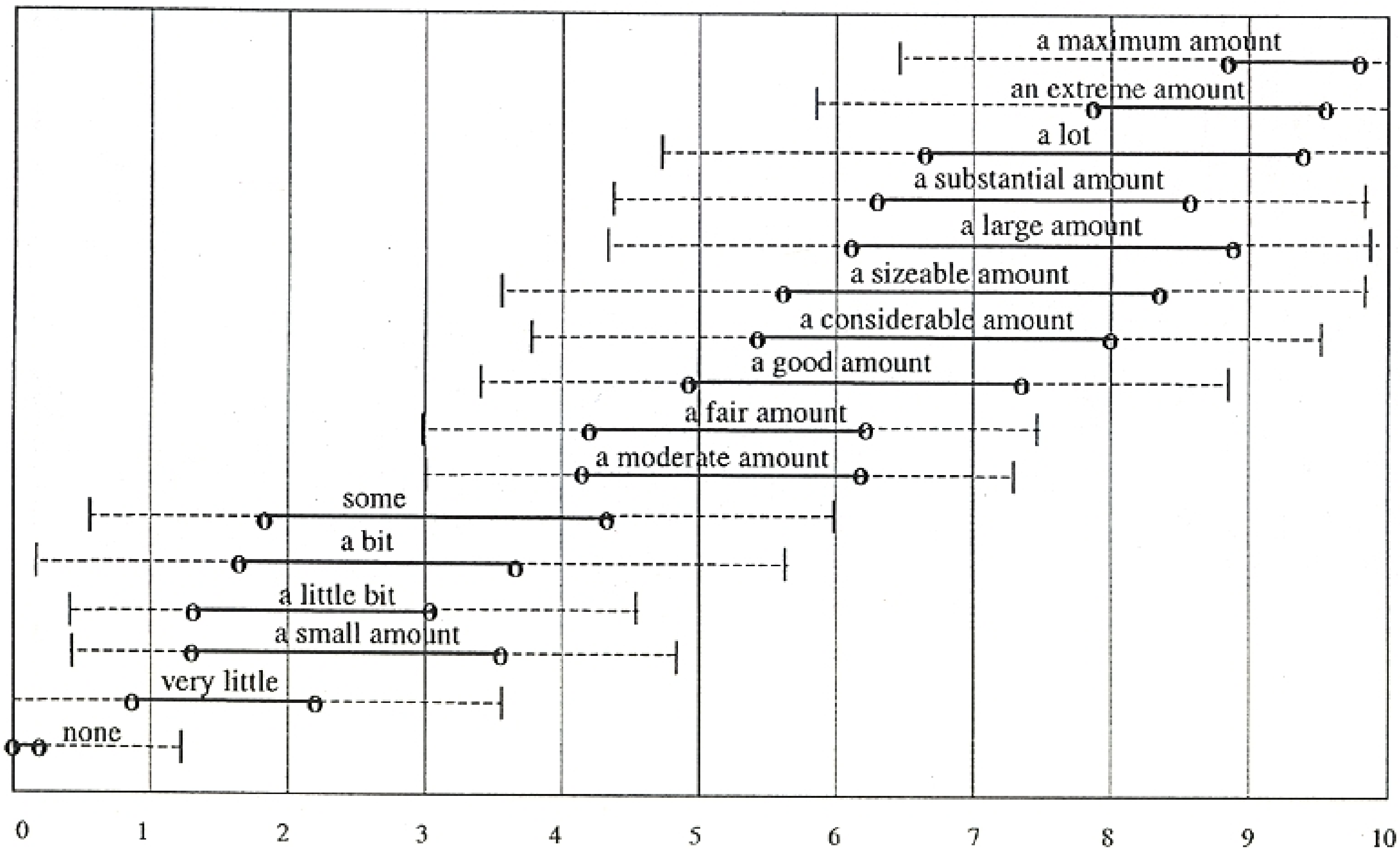

Figure 4. All 16 labels and their intervals and uncertainty bands.