Quantizer Design and Distributed Encoding Algorithm for Source Localization in Sensor Networks

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***Abstract*— In this paper, we propose a quantizer design algorithm that is optimized for source localization in sensor networks. For this application, the goal is to minimize the amount of information that the sensor nodes have to exchange in order to achieve a certain source localization accuracy. We show that this goal can be achieved more efficiently when "application-specific" quantizers are used. Our proposed quantizer design algorithm uses a cost function that takes into account the distance between the actual source position and the position estimated based on quantized data. We also propose a distributed encoding algorithm that is applied after quantization and achieves rate savings by merging quantization bins without any degradation of localization performance. The merging technique in the encoding algorithm exploits the fact that certain combinations of quantization bins at each node cannot occur because the corresponding spatial regions have an empty intersection. We apply these algorithms to a system where an acoustic sensor model is employed for localization. For this case, we introduce the Equally Distance-divided Quantizer (EDQ), designed so that quantizer partitions correspond to a uniform partitioning in terms of distance. Our simulations show the improved performance of our quantizer over traditional quantizer designs. In addition, they show rate savings (32.8%, 5 nodes, 4 bits per node) when our novel bin-merging algorithms are used. Our results also show that an optimized bit allocation leads to significant improvements in localization performance with respect to a bit allocation that uses the same number of bits for each node.**

I. Introduction

In sensor networks, multiple correlated observations are available from many sensors that can sense, compute and communicate. Often these sensors are battery-powered and operate under strict limitations on wireless communication bandwidth. This motivates the use of data compression in the context of various tasks such as detection, classification, localization and tracking, which require data exchange between sensors. The basic strategy for reducing the overall energy usage in the sensor network would then be to decrease the communication cost at the expense of additional computation in the sensors [8].

One important sensor collaboration task with broad applications is source localization. Localization based on acoustic energy measured at individual sensors is considered in [3], where each sensor transmits unquantized acoustic energy readings to the central node, which then computes an estimate of the location of the source of these acoustic signals. Practical systems will require quantization of these energy readings before transmission and thus different quantizer designs should be compared in terms of localization error, defined as the average of the distance between the actual source location and its estimated value based on received quantized data. There are numerous examples of techniques for source localization. They have in common that the quantities being measured by the sensors can be linked to the position of the source.

Since standard design of scalar quantizers aims at minimizing the average distortion between the actual sensor reading and its quantized value, there is no guarantee that these quantizers will be optimal in the sense of minimizing localization error. Thus, we propose that quantizer design should be "application-specific". That is, to design optimal quantizers, a new metric should be defined that takes into account the accuracy of the application objective. For an example of application specific quantizer design for time-delay estimation see [6]. In this paper we consider as an application-specific metric the localization error, i.e., the difference between the actual source location and that estimated based on quantized data. A challenging aspect of this problem is that, while quantization has to be performed independently at each node, the localization error, which we wish to minimize, depends on the readings from *all* quantizers. Thus we have a problem where independent (scalar) quantizers for each node have to be optimized based on a global (vector) cost function.

To solve this problem, we have proposed an iterative quantizer design algorithm for the localization problem [2], as an extension of our earlier work [5]. We have applied our algorithm to a system where an acoustic sensor model [3] is considered. In [2], we have also studied the bit allocation problem (i.e., determining the number of bits to be used by each sensor) and provided a solution with the introduction of the Equally Distance-divided Quantizer (EDQ), which is simple and provides good performance in the acoustic sensor model case. Our approach is also applicable to general cases where sensors measure information that is a function of distance. Our experiments demonstrated the benefits of using application-specific designs. In particular, our bit allocation results showed that bits should be distributed so as to lead to partition of the sensor field that is as uniform as possible. Thus, for example, when several nodes are clustered together, the number of bits per node tends to be lower than when the same sensors are more spread out.

In this paper, we extend our previous work in [2] and propose a novel distributed encoding algorithm that can exploit redundancies in the quantization and is shown to achieve significant rate savings, while preserving source localization performance. In our problem a source signal is received and quantized by a series of distributed nodes. Clearly, in order to make localization possible, each possible location of the source has to produce a different set of readings at the nodes, so that the vector of readings uniquely determines the source location. Quantization of the readings at each sensor reduces the accuracy of the localization but again a vector of quantized readings at a set of sensors uniquely determines a region of the space where the source is located. A quantized value provided by an individual sensor can be linked to a region, with a shape that depends on the characteristics of the sensor. For example, in our paper we consider sensors that provide no directional information, leading to regions in the form of "rings" centered at the sensor location (see Figure 1); each of these "rings" corresponds to one quantization index. Thus, the central node operates by aggregating the information received from individual sensors: the source location, as illustrated in Figure 1, is estimated to be in the intersection of all the regions specified by the sensors.

Note that each source location leads to a set of quantized sensor measurements that correspond to a non-empty intersection. Thus, assuming there are no measurement errors, the central node will only receive those combinations of quantized measurements that correspond to real source locations. The key observation in this paper is that the number of real quantized vector observations is smaller than the *total* number of combinations of quantized values at the sensors. Thus, many arbitrary combinations of quantized readings at several sensors *cannot be produced* because the corresponding regions have an empty intersection.

In this paper we propose a novel algorithm that exploits this fact so as to reduce the overall rate required. With our method, we merge (non-adjacent) quantization bins in a given sensor whenever we determine that the ambiguity created by this merging can be resolved at the central node once information from other sensors is taken into account. Note that this is an example of binning as can be found in Slepian-Wolf and Wyner-Ziv techniques [1]. In our approach, however, we do not use any channel coding. Instead, we propose design techniques that allow us to achieve rate savings purely through binning, and provide several methods to select candidate bins for merging. Our experimental results show that significant gains be achieved with our proposed approach (e.g., over 30% reduction in rate with no change in localization performance). While our experimental results are provided for simple cases with a relatively small number of nodes and coarse quantization, the results do indicate that the potential savings achievable with our technique will increase with the number of sensors and at higher rates.

This paper is organized as follows. The problem formulation and target cost function are introduced in Section II-A. The quantizer design algorithm first proposed in [2], is presented in Section II-B. In Section III, the novel encoding algorithm is explained and we present an application to the case where an acoustic sensor model is employed in Section IV. Section V discusses the bit allocation problem. Simulation results are given in Section VI and the conclusions are found in Section VII.

II. Quantizer Design

A. Problem Formulation

Suppose that there are M nodes in a sensor field S and these nodes measure signals generated by a source assumed to be static during the localization process. We assume that the i-th sensor measures the source signal over a time interval k. This measurement is denoted by z_i and can be expressed as follows:

$$z_i(x,k) = f(x, x_i, \mathbf{P_i}) + w_i(k) \quad \forall i = 1, ..., M, \tag{1}$$

where the signal received by each sensor is assumed to be modeled by the function $f(x, x_i, \mathbf{P_i})$, where x is the source location, x_i is the position of node i, $\mathbf{P_i}$ is the parameter vector for the sensor model, and w_i is the combined noise term for the measurement noise and the modelling error that might exist. An example of $\mathbf{P_i}$ for an acoustic sensor case is given in Section IV. It is also assumed that the positions of all nodes $x_i, i = 1, ..., M$, are known and each node senses its observation $z_i(x,k)$ at time interval k, quantizes it and sends it to a central node, where all sensor readings are used to obtain an estimate of the source location $\hat{x}$.

Suppose that at node i we use a quantizer with L_i quantization levels, with a dynamic range of $[z_{min} \quad z_{max}]$. Denote $\alpha_i(.)$ the encoder at node i which generates a quantization index $j \in I_i = \{1, \ldots L_i\}$ for observations z_i that fall in the quantization bin Q_i^j. Denote $\beta_i(.)$ the decoder corresponding to node i, which maps the quantization index j to a reconstructed quantized observation $\hat{z}_i^j$.

As an example, $z_i(x,k)$ could be the energy of an acoustic signal during the k-th observation interval, where each interval has a predetermined duration. We assume that the central node will determine the location of a source based on $z_i(x,k)$'s obtained from all nodes. In some cases, one reading per node is used, while in other cases values of $z_i(x,k)$ for several k are needed for localization.

Clearly, for $z_i(x,k)$ to be useful for localization it must be a function of the relative positions of the source and the node. Thus there exists some function $g(.)$ that can provide an estimate of the source location $\hat{x}$ based on quantized observations

$$\hat{x} = g(\alpha_1(z_1), ..., \alpha_M(z_M)), \tag{2}$$

Note that the localization function $g(.)$ should be closely related to the sensor model $f(.)$. As an example, refer to (15) to (17) in Section IV.

To design the optimal quantizer at node i, i.e., the one that minimizes the localization error, we define a cost function $J_i(x)$ as follows

$$J_i(x) = \parallel z_i - \hat{z}_i \parallel^2 + \lambda \parallel x - \hat{x} \parallel^2 \quad \forall x \in S \tag{3}$$

where $\hat{z}_i$ is the reproduction value assigned to z_i. The cost function takes into account the error in reproducing the sensor reading, as well as the localization error, $\parallel x - \hat{x} \parallel^2$. The relative importance of the two terms in the cost function can be adjusted by selecting different values of the Lagrange multiplier, $\lambda \geq 0$. To see why this multiplier is needed note that the localization error term can only be computed given all the readings and cannot be separately optimized by each sensor. Thus, if we were to optimize localization, one particular reading z_i may have to be quantized in different ways, depending on the quantized values at other sensors. Clearly, in a real operating environment, quantization can only be based on z_i, and a given z_i will always be assigned to the same quantized value, since the i-th node cannot know the readings at other nodes without incurring a communication cost. Thus the actual encoding will be based on a cost function with $\lambda = 0$. Using a non-zero value for the multiplier during the design allows us to take into account the effect of quantization on localization while not deviating significantly from a regular quantizer design.

The overall optimal quantizer design is the one that can minimize the expected value of this cost function, averaged based on the probability density function of the source locations, $p(x)$:

$$J_{avg} = E(J_i(x)) = \int_S J_i(x) p(x) dx. \tag{4}$$

If no prior information is available about the relative likelihood of possible source locations, $p(x)$ could be made uniform over the sensor field. For the purpose of training our quantizer, we generate a training set of observations $\{z_1(x,k), ..., z_M(x,k)\}$ based on the sensor model, $f(x, x_i, \mathbf{P_i})$, with a given choice of $p(x)$. Our algorithm is aimed at finding M quantizers that minimize the averaged cost function J_{avg}. The cost function $J_i(x)$ can be rewritten in terms of the M quantizers

$$\begin{aligned} J_i(x, \alpha_i(z_i)) = & \parallel z_i - \beta_i(\alpha_i(z_i)) \parallel^2 + \\ & \lambda \parallel x - g(\alpha_1(z_1(x,k)), ..., \alpha_M(z_M(x,k))) \parallel^2 . \end{aligned} \tag{5}$$

B. Quantizer Design Algorithm

Our goal is to design a set of encoders, each operating independently on the observations of one node, so as to minimize the expected value of the cost function (5), when the entire vector $[\alpha_1(.), ...\alpha_M(.)]$ is used for localization. The algorithm should seek to design independent quantizers for each node, while taking into account their combined effect on localization. The generalized Lloyd algorithm (GLA) is used to design the encoder at each node. Since the cost function in (5) is dependent on the encoders at other nodes, we use an iterative procedure, where the quantizer at node i is optimized while the quantizers for the other nodes remain unchanged. This iterative method is based on that proposed in [5] and was first proposed for this localization problem in [2].

Note that the quantizer design is performed off-line using the training set that is generated based on known values of $\mathbf{P_i}$ and $p(x)$ and thus the quantizer training phase makes use of information about all nodes, but when the resulting quantizers are actually used, each node quantizes the information available to it independently (i.e., using cost function $J_i(x)$ from (5) with $\lambda = 0$).

Given the number of quantization levels, L_i, at node i, the proposed algorithm is summarized as follows. For simplicity, in what follows, $z_i(x, k)$ is written as $z_i(x)$.

Step1 : Initialize the encoders $\alpha_i(.), i = 1, ..., M$. Set the thresholds ϵ_1 and ϵ_2, set $i = 1$, and set iteration indices $k = 0$ and $k_1 = 0$.

Step2 : Compute the cost function of (5).

Step3 : Define the region, V_i^j which corresponds to quantization bin Q_i^j:

$$V_i^j = \{x : J_i(x, \alpha_i = j) < J_i(x, \alpha_i = m), \forall m \neq j\} \quad (6)$$

where $j, m = 1, ..., L_i$. This is the spatial region where a source is located when it generates a signal that is quantized to index j by sensor i.

Step4 : Compute the average cost $J_{avg}^k = E_x(J_i(x))$

Step5 : If $\frac{(J_{avg}^{k-1} - J_{avg}^k)}{J_{avg}^k} < \epsilon_1$ go to Step 7; otherwise continue

Step6 : $k = k + 1$. Update the quantization bin Q_i^j as follows.

$$\begin{aligned} \hat{z}_i^j &= E(z_i(x)|x \in V_i^j) \\ Q_i^j &= [b_i^{j-1} \quad b_i^j] \quad \forall j = 1, ..., L_i \end{aligned} \quad (7)$$

where $b_i^j = \frac{1}{2}(\hat{z}_i^j + \hat{z}_i^{j+1})$, $b_i^0 = z_{i,min}$, $b_i^{L_i} = z_{i,max}$. The new value for $\hat{z}_i^j$ is chosen to be the expected value of the measured signal averaged over all possible locations in region V_i^j. The quantization bin is then redesigned so that an input z_i is assigned to the closest $\hat{z}_i$. Go to Step 2

Step7 : if $i < M \quad i = i + 1$ go to step 2;
else if $\frac{D^{k_1 - 1}(x, \hat{x}) - D^{k_1}(x, \hat{x})}{D^{k_1}(x, \hat{x})} < \epsilon_2$ Stop;
else $i = 1; k_1 = k_1 + 1$; Go to Step 2,
where $D^{k_1}(x, \hat{x})$ is given by $E(\| x - \hat{x} \|^2)$ at k_1th iteration.
A discussion of the robustness of our approach to model mismatches is left for Section VI.

III. Distributed Encoding Algorithm

The assumptions made in Section II-A still hold throughout this section.

A. Terminologies and Motivation

Let $S_M = I_1 \times I_2 \times ... \times I_M$ be the cartesian product of the sets of quantization indices. S_M contains $|S_M| = \prod_i^M L_i$ M-tuples representing all possible combinations of quantization indices. We denote S_Q the subset of S_M that contains all the quantization index combinations that can occur in a real system, i.e., all those generated as a source moves around the sensor field and produces readings at each sensor:

$$S_Q = \{(Q_1, ..., Q_M)|\ \exists x \in S, Q_i = \alpha_i(z_i(x)),\ i = 1, \dots, M\} \quad (8)$$

We denote S_i^j the subset of S_Q that contains all M-tuples in which the i-th node is assigned quantization bin Q_i^j:

$$S_i^j = \{(Q_1, ..., Q_M) \in S_Q | Q_i = j\}. \quad (9)$$

Thus, given Q_i^j, we can always construct the corresponding set S_i^j from the set S_Q. Note also that $\underline{S_i^j} \subset S_Q$,.

Along with this, we denote $\overline{S_i^j}$, the set of $(M-1)$-tuples obtained from M-tuples in S_i^j, where only the quantization bins at positions other than position i are stored. That is, if $(Q_1, ..., Q_M) = (a_1, ..., a_M) \in S_i^j$ then we have $(a_1, ..., a_{i-1}, a_{i+1}, ..., a_M) \in \overline{S_i^j}$. Clearly, there is a one to one correspondence between the elements in S_i^j and $\overline{S_i^j}$, so that $|S_i^j| = |\overline{S_i^j}|$.

As discussed in the introduction, there will be elements in S_M that are not in S_Q. Therefore, if we were to perform simple scalar quantization at each node it would be inefficient in terms of rate. This because a standard scalar quantizer would allow us to represent any of the M-tuples in S_M, but $|S_M| \geq |S_Q|$. What we would like to determine now is a method such that independent quantization can still be performed at each node, while at the same time we reduce the redundancy inherent in allowing all the combinations in S_M to be chosen. Note that in general, to determine that a specific quantizer assignment in S_M does not belong to S_Q requires having access to the whole vector, which obviously is not possible if quantization has to be performed independently at each node.

In our design we will look for quantization bins in a given node that can be "merged", without affecting localization. As will be discussed next, this is because the ambiguity created by the merger can be resolved once information obtained from the other nodes is taken into account. Note that this is the basic principle behind distributed source coding techniques: binning at the encoder, which can be disambiguated once side information is made available at the decoder (in this case quantized values from other nodes).

Merging of bins results in bit rate savings because fewer quantization indices have to be transmitted. To quantify the bit rate savings we need to take into consideration that quantization indices will be entropy coded (in this paper Huffman coding is used). Thus, when evaluating the possible merger of two bins, we will compute the probability of the merged bin as the sum of the probabilities of the merged bins. For example, suppose that Q_i^j and Q_i^k are merged into $Q_i^{min(j,k)}$. Then we can construct the set $S_i^{min(j,k)}$ and compute the probability for the merged bin respectively as follows:

$$S_i^{min(j,k)} = S_i^j \cup S_i^k \quad (10)$$

$$P_i^{min(j,k)} = P_i^j + P_i^k \quad (11)$$

where $P_i^j = \int_{A_i^j} p(x)dx$, $p(x)$ is the pdf of the source position and A_i^j is given by

$$A_i^j = \{x | (Q_1 = \alpha_1(z_1(x)), ..., Q_M = \alpha_M(z_M(x))) \in S_i^j\} \quad (12)$$

Suppose the encoder at node i merges Q_i^j and Q_i^k into $Q_i^l, l = min(j, k)$ and sends the corresponding index to the central node. The decoder will construct the set S_i^l for the merged bin using (10) and then will try to determine which of the two merged bins (Q_i^j or Q_i^k in this case) actually occurred at node i. To do so, the decoder will use the information provided by the other nodes, i.e., the quantization indices Q_m ($m \neq i$). Consider one particular source position $x \in S$

for which sensor i produces Q_i^j and where the remaining sensors produce a combination of $M-1$ quantization indices $\mathbf{Q} \in \overline{S_i^j}$. Then, for this x there would be no ambiguity at the decoder, even if bins Q_i^j and Q_i^k were to be merged, as long as $\mathbf{Q} \notin \overline{S_i^k}$. This follows because if $\mathbf{Q} \notin \overline{S_i^k}$ the decoder would be able to determine that only Q_i^j is consistent with receiving $\mathbf{Q}$.

The above argument holds for a specific $x \in A_i^j$, if it were to hold for *any* $x \in A_i^j$ then bins Q_i^j and Q_i^k can always be merged. With the notation adopted earlier this leads to the following definition:

Definition 1: Q_i^j and Q_i^k are identifiable, and therefore can be merged, iff $\overline{S_i^j} \cap \overline{S_i^k} = \emptyset$.

We consider here the case where there is no measurement noise (i.e., $w_i = 0$) and no parameter mismatches. In this case $Pr[(Q_1, ..., Q_M) \in S_Q] = 1$, i.e., only combinations of quantization indices belonging to S_Q can occur and those combinations belonging to $S_M - S_Q$, which lead to an empty intersection, never occur. Under these conditions any identifiable bins can be merged into one bin without any decoding error at the central node. The question that remains is how to merge identifiable bins in order to minimize the total rate used by the M nodes to transmit their quantized observations.

B. Proposed Encoding Algorithm

In general there will be multiple pairs of identifiable quantization bins that can be merged. Often, all candidate identifiable pairs cannot be merged simultaneously, i.e., after a pair has been merged, other candidate pairs may become non identifiable. In what follows we propose algorithms to determine in a sequential manner which pairs should be merged.

In order to minimize the total rate, an optimal merging technique should attempt to reduce the overall entropy as much as possible, which can be achieved by (1) merging high probability bins together and (2) merging as many bins as possible. It can be observed that these two strategies cannot be pursued simultaneously. This is because high probability bins (under our assumption of uniform distribution of the source position) are large and thus merging large bins tends to result in fewer remaining merging choices (i.e., a larger number of identifiable bin pairs may become non-identifiable after two large identifiable bins have been merged). Conversely, a strategy that tries to maximize the number of merged bins will tend to merge many small bins, leading to less significant reductions in overall entropy. In order to strike a balance between these two strategies we define a metric, W_i^j, attached to each quantization bin:

$$W_i^j = P_i^j - \gamma |S_i^j|, \tag{13}$$

where $\gamma \geq 0$. This is a weighted sum of the bin probability and the number of quantizer combinations that include Q_i^j. If P_i^j is large this would be a good candidate bin for merging under criterion (1), whereas a small value of $|S_i^j|$ will indicate a good choice under criterion (2). In our proposed procedure, for a suitable value of γ, we will seek to prioritize the merging of those identifiable bins having largest total weighted metric. This will be repeated iteratively until there are no identifiable bins left.

The proposed *global merging algorithm* is summarized as follows:
Step 1: Set $F(i,j) = 0$, where $i = 1, ..., M; j = 1, ..., L_i$, indicating that none of the bins, Q_i^j, have been merged yet.
Step 2: Find $(a,b) = \arg\max_{(i,j)|F(i,j)=0}(W_i^j)$, i.e., we search over all the non-merged bins the one with the largest metric W_a^b.
Step 3: Find $Q_a^c, c \neq b$ such that $W_a^c = \max_{j\neq b}(W_a^j)$ where the search for the maximum is done only over the bins identifiable with Q_a^b at node a. If there are no bins identifiable with Q_a^b, set $F(a,b) = 1$, indicating the bin Q_a^b is no longer involved in the merging process; if all the bins are merged, stop; otherwise go to Step 2.
Step 4: Merge Q_a^b and Q_a^c to $Q_a^{min(b,c)}$ with $S_a^{min(b,c)} = S_a^b \cup S_a^c$. Set $F(a, min(b,c)) = 1$. go to Step 2.

Given M quantizers, we can construct the sets, S_i^j and the metric $W_i^j, \forall i, j$, perform the merging using the proposed algorithm and find the parameter γ in (13) that minimizes the total rate. In the proposed algorithm, the search for the maximum of the metric is done for the bins of all nodes involved. However, we can take different approaches to the search, which are explained as follows.
Method 1: *Complete sequential merging.* In this method, we process one node at a time in a specified order. For each node, we merge the maximum number of bins possible before proceeding to the next node. Merging decisions are not modified once made. Since we exhaust all possible mergers in each node, after scanning all the nodes no more additional mergers are possible.
Method 2: *Partial sequential merging.* In this method, we again process one node at a time in a specified order. For each node, among all possible bin mergers, the best one according to a criterion is chosen (the criterion could be entropy-based and for example, (13) is used in this paper) and after the chosen bin is merged we proceed to the next node. This process is continued until no additional mergers are possible in any node. This may require multiple passes through the set of nodes.

These two methods can be easily implemented with minor modifications to our proposed algorithm.

C. Incremental Merging

The complexity of the above procedures is a function of the total number of quantization bins, and thus of the number of nodes involved. Thus, these approaches could potentially be complex for large sensor fields, i.e., M large. We now show that incremental merging is possible, that is, we can start by performing the merging based on a subset of N sensor nodes, $N < M$, and will be guaranteed that merging decisions that were valid when N nodes were considered will remain valid when all M nodes are taken into account. To see this, suppose that Q_i^j and Q_i^k are identifiable when only N nodes are considered. Then, since Q_i^j and Q_i^k are identifiable, $\overline{S_i^j(N)} \cap \overline{S_i^k(N)} = \emptyset$, where here N indicates the number of nodes involved in the merging process. By the property of the intersection operator $\cap$, we can claim that $\overline{S_i^j(M)} \cap \overline{S_i^k(M)} = \emptyset \quad \forall M \geq N$, implying that Q_i^j and Q_i^k are still identifiable even when we consider M nodes. Thus, we can start the merging process with just two nodes and continue to do further merging by adding one node (or a few) at a time without change in previously merged bins. When many nodes are involved, this would lead to significant savings in computational complexity. In addition, if some of the nodes are located far away from the nodes being added (That is, the dynamic ranges of their quantizers do not overlap with those of the nodes being added), they can be skipped for further merging without loss of merging performance.

IV. Application to Acoustic sensor model

As an example, we now consider source localization based on acoustic signal energy as proposed in [3], where an energy decay model of sensor signal readings is used for localization based on unquantized sensor readings. When an acoustic sensor is employed

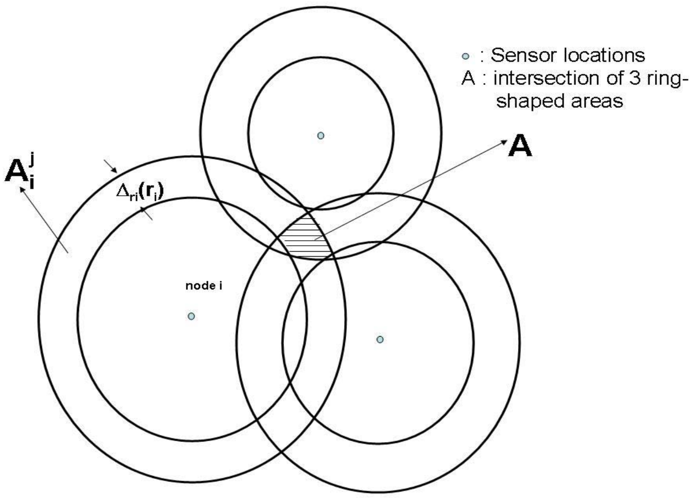

Fig. 1. Localization of the source based on quantized energy readings

at each node, the signal energy measured at node i over a given time interval k, and denoted by z_i, can be expressed as follows:

$$z_i(x,k) = g_i \frac{a}{(x-x_i)^\alpha} + w_i(k), \tag{14}$$

where the parameter vector $\mathbf{P_i}$ in (1) consists of the gain factor of the$i$-th sensor g_i, an energy delay factor α, which is approximately equal to 2, and the source signal energy a. The measurement noise term $w_i(k)$ can be approximated using a normal distribution. In (14), it is assumed that the signal energy, a takes values in the range $[a_{min} \quad a_{max}]$, while during the localization process a source generates a constant energy, which is assumed to be known to a central node where localization is performed based on quantized energy readings. In practice, the source energy can be estimated using the quantized energy readings for localization at the central node. Localization based on quantized observations is illustrated by Figure 1, where each ring-shaped area can be obtained from one quantized observation provided by a sensor. By computing the intersection of all the ring areas (one per sensor), it is possible to define the area where the source is expected to be located. Note that at least three observations are required to achieve a connected intersection. This can be written as follows

$$A = \bigcap_{i=i}^{M} A_i \tag{15}$$

where $A_i = A_i^j$ when $\alpha_i(z_i)$ produces the jth bin, Q_i^j. For this case, A_i^j is given by.

$$A_i^j = \{x : g_i \frac{a}{(x-x_i)^\alpha} \in Q_i^j\} \tag{16}$$

That is, A_i^j is the ring-shaped region obtained from the quantized bin Q_i^j that z_i falls into (Figure 1). Another expression for A_i^j can be also found in (12). If the source is uniformly distributed in the sensor field, the estimate, $\hat{x}$ would be the sample mean in the intersection A

$$\hat{x} = E(x|x \in A) \tag{17}$$

To avoid quantizer overload, the dynamic ranges of the M quantizers are initialized as $[z_{min} \quad z_{max}] = [\frac{a_{min}}{r_{max}^2} \quad \frac{a_{max}}{r_{min}^2}]$ where $[r_{min} \quad r_{max}]$ is the range within which each sensor is supposed to measure acoustic source energy. The value of r_{max} is set such that the probability that an arbitrary point inside the sensor field can be sensed simultaneously by at least 3 nodes should be close to 1 [7]. Assuming the distribution of the number of nodes in any given area $S = \pi r^2$ is Poisson with rate λS the probability, p is then given by

$$p = \sum_{i=3}^{\infty} \frac{e^{-\lambda \pi r^2} (\lambda \pi r^2)^i}{i!} \tag{18}$$

Given node density λ (nodes/m^2), we can compute $r_{max}(=2r)$ for a desirable value, p (say, 0.95). In this way, the likelihood of missing a source is minimized. To have finite dynamic rages, the value of r_{min} is chosen as a small nonzero value. Note that if more nodes are used, better quantization in each node is possible (the dynamic ranges will tend to be smaller). With this initialization step, the quantizer design as outlined in Section II-B can be used.

The encoding algorithm described in Section III-B is applied after quantization to achieve rate savings. The rate savings is computed after entropy-coding of the merged bins and compared with those for Method 1 and Method 2 in Section III-B.

V. Equally Distance-divided Quantizer and Bit allocation problem

Since each set of quantizers induces a partitioning of the sensor field, designing good quantizers for localization can be seen to be equivalent to making a good partition of the sensor field by adjusting the width, $\Delta_{r_i}(r_i)$ of the ring-shaped areas in Figure 1. If no prior information is available about the source location, $p(x)$ can be assumed to be uniform and thus choosing $\Delta_{r_i}(r_i)$ to achieve a uniform partitioning of the sensor field would seem to be a good choice. Intuitively, a uniform partitioning of the sensor field is more likely to be achieved when the ring-shaped areas have the same width, $\Delta_{r_i}(r_i) = const$ (this is certainly the case when the nodes are uniformly distributed). This consideration leads to the introduction of Equally Distance-divided Quantizers (EDQ), which can be viewed as uniform quantizers in distance, and such that $\Delta_{r_i}(r_i) = \frac{r_{max}-r_{min}}{2^{R_i}}, \forall i$. To justify the EDQ design, we performed a simulation (see Figure 2) which shows that EDQ provides good localization performance, which comes close to that achievable by the quantizer proposed in Section II-B. EDQ has the added advantage of facilitating the solution of the bit allocation problem.

Given a total number of bits, $R_T = \sum R_i$, the goal is to minimize the localization error by allocating different number of bits to each node. Even though the GBFOS algorithm [4] provides the optimal bit allocation, it would also require extremely large computational load, since it relies on the calculation of rate-distortion points at each iteration step, and the quantizers should be redesigned using the algorithm of Section II-B for each candidate bit allocation. Instead, in our experiments we use the GBFOS algorithm along with EDQ, which does not require quantizer redesign for each candidate bit allocation. With this approach one can use EDQ to compute easily the optimal bit allocation for the particular node configuration, and then use the technique proposed in Section II-B to design a quantizer for the given bit allocation.

VI. Simulation

The proposed quantizer was designed by the algorithm in Section II-B, using a training set with 1532 source locations generated with a uniform distribution in a sensor field of size $10 \times 10m^2$, where 5 nodes are randomly located (as shown in Figure 3). The model parameters are given by $a = 50, \alpha = 2, g_i = 1$ and $SNR = \infty$, and the localization error is computed by $E(\| x - \hat{x} \|^2)$. In Figure 2,

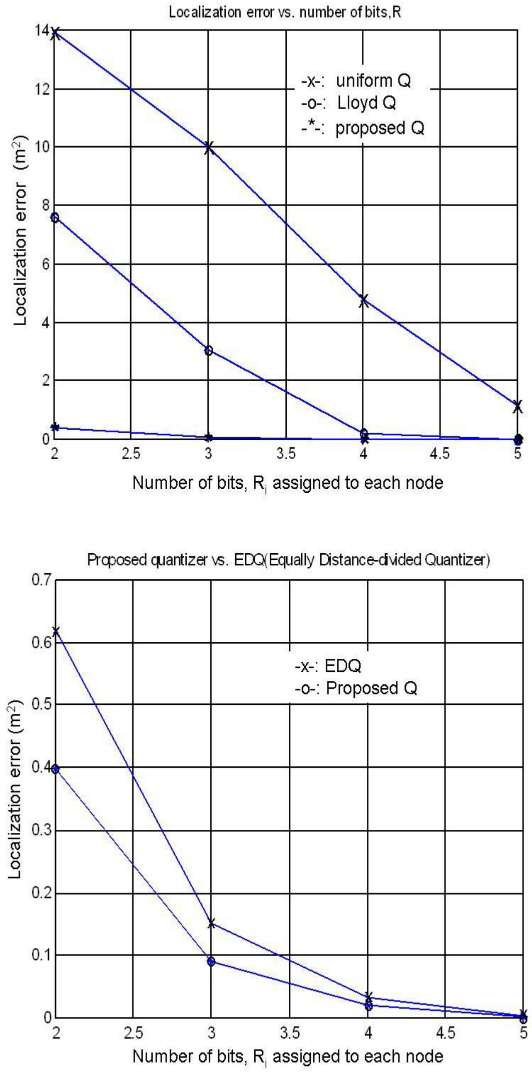

Fig. 2. Localization error vs. the number of bits, R_i assigned to each node. The localization error is given by $E(\| x - \hat{x} \|^2)$. 1481 source locations are generated with uniform distribution of a source location.

the localization error is compared with traditional quantizers such as uniform quantizers and Lloyd quantizers ($\lambda = 0$). Since the proposed quantizer makes full use of the distributed property of the observations, it can be seen to provide improved performance over the traditional quantizers. This can be also explained in terms of the partitioning of the sensor field, which is plotted in Figures 3 and 4. It is easily seen that our quantizer leads to a more uniform partitioning, which in turn reduces the localization error. In this simulation, we assume that when the source is very close to one of the nodes, the node position becomes an estimate of the source position. The localization error due to this assumption can be reduced by lowering the value of r_{min} at the expense of a larger dynamic range.

The distributed encoding algorithm was applied to the system where 5 nodes are located as shown in Figure 3 and an acoustic sensor model is employed at each node. In obtaining the metric in (13), the source distribution is assumed to be uniform. Each time the number of bits assigned to each node varies from 2 to 4, the encoding algorithm was applied to the 5 quantizers which are designed by the proposed algorithm in Section II-B. Table I provides results of the new encoding algorithm under the different merging techniques outlined in Section III-B. Methods 1 and 2 are as described in Section III-B, Method 0 refers to the approach where entropy coding is applied but there is no merging, and Method 3 is the global merging algorithm discussed in that section . Note that the total rate consumed by the 5 nodes was computed based on independent entropy coding at each of the nodes. We can observe that even with relative low rates (4 bits per node) and a small number of nodes (only 5) significant rate gains (up to 30%) can be achieved with respected to Method 0, which does not exploit the special characteristics of S_Q and S_M.

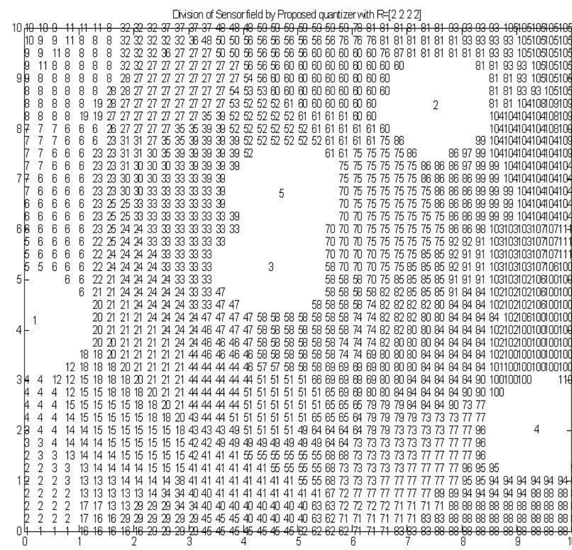

Fig. 3. Partitioning of Sensor field ($10 \times 10m^2$) (grid= 0.25×0.25) by proposed quantizers. All partitioned regions are numbered, so that a region is filled with the same number and $R_i = 2$

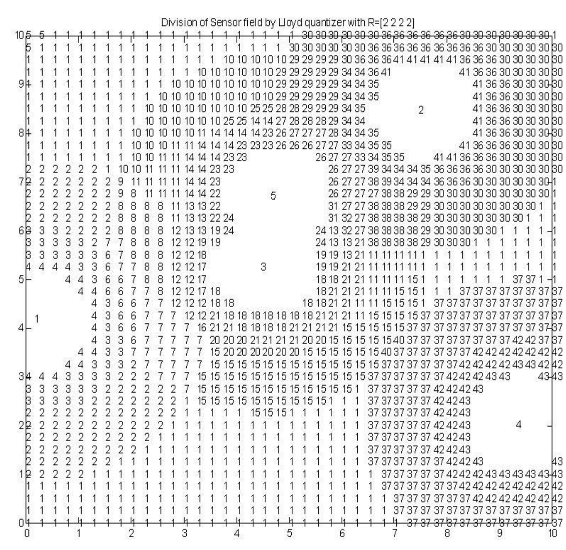

Fig. 4. Partitioning of Sensor field ($10 \times 10m^2$) (grid= 0.25×0.25) by Lloyd quantizers. All partitioned regions are numbered and $R_i = 2$

The proposed quantizer was evaluated under various types of mismatch conditions. In each test we modified one of the parameters with respect to what was assumed during quantizer training. The simulation results are tabulated in Table II. In this experiment, 1481 and 1176 source locations in a sensor field of size $10 \times 10m^2$ were generated under the assumption of a uniform distribution and a normal distribution, respectively. For each source location, localization is performed using the true parameters, even when there is mismatch. The proposed quantizers showed good performance for the various parameter perturbations. That is, there is no need to redesign quantizers when there are tolerable parameter mismatches. In a large sensor field, they also provided good results with respect to traditional quantizers in Table III.

In the same node configuration as in Figure 3, the bit allocation was conducted using EDQ to search for the optimal bit allocation R^*, that would give the minimum localization error. It can be seen that nodes 3 and 5 are so close to each other that they provide redundant information for localization and thus the optimal solution allocates few bits to both these nodes. In fact, in our example, at relatively low rates (an average of 2 bits per node) it is more efficient to send information from only three nodes (node 1,2 and 4), i.e., allocating zero bits for the other two nodes (node 3 and 5). In Table IV, the localization errors were computed using EDQ and our proposed quantizer designed for several different bit allocations respectively, showing that bit allocation is important to achieve good localization performance.

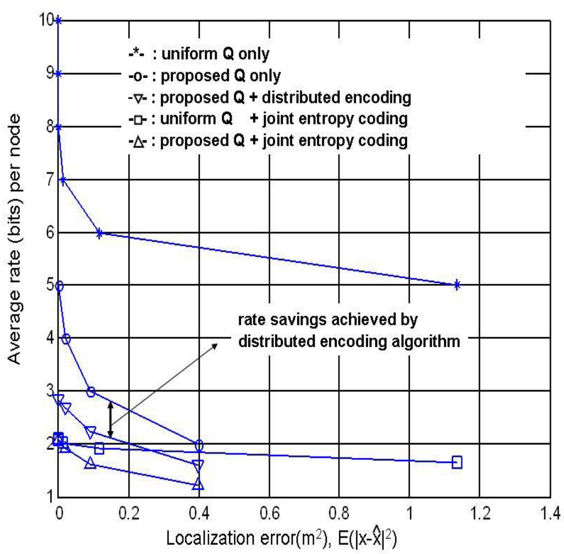

Fig. 5. Comparison of performance of the proposed techniques (proposed quantizers and distributed encoding algorithm) with techniques based on uniform quantizers and joint entropy coding.

Finally, we address the question of how our technique compares with the best achievable performance for this source localization scenario. As a bound on achievable performance we consider a system where (i) each node quantizes its observation independently and (ii) the quantization indices generated by all nodes for a given source location are jointly coded (in our case we use the joint entropy of the vector of observations as the rate estimate). This approach can be applied to both the original quantizer designed and the quantizer obtained after merging.

Note that this is not a realistic bound because the joint coding cannot be achieved unless the nodes are able to communicate before encoding. Note that in order to approximate the behavior of the joint entropy coder via distributed source coding techniques one would have to transmit multiple observations of the source energy from each node, as the source is moving around the sensor field. Some of the nodes could send observations that are directly encoded, while others could transmit a syndrome produced by an error correcting code based on the quantized observations. Then, as the central node receives all the information from the various nodes it would be able to exploit the correlation from the observations and approximate the joint entropy. This method would not be desirable, however, because the information in each node depends on the location of the source and thus to obtain a reliable estimate of the measurement at all nodes one would have to have observations at a sufficient number of positions of the source. Thus, instantaneous localization of the source would not be possible. The key point here, then, is that the randomness between observations across nodes is based on the localization of the source, which is precisely what we wish to observe.

For the node configuration in Figure 3, the average rate per node was plotted with respect to the localization error in Figure 5. As can be seen from Figure 5, our proposed techniques (proposed quantizers in Section II-B and distributed encoding algorithm in Section III-B) outperform techniques based on uniform quantization. For this particular configuration we can observe a gap of less than 1 bit/node, at high rates, between the performance achieved by our proposed quantizer with distributed encoding and that achievable with the same quantizer if joint entropy coding was possible. We also observe that our proposed techniques lead to a very significant gain (at low localization error rates) as compared to standard uniform quantization (around 5 bits/node). In summary, our techniques provide substantial gain over straightforward application of known techniques and come close to the optimal achievable performance.

VII. Conclusion

In this paper, we have proposed a distributed encoding algorithm for source localization in sensor networks. In the experiments based on the acoustic sensor model, our approach provides significant rate savings without any degradation of localization performance. In the future, we will work on the case where the source signal energy is unknown. For this case, a new distributed localization algorithm should be developed.

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TABLE I

Total rate in bits (Rate savings) achieved by various merging techniques. The rate savings is computed by $\frac{R_T - \mathbf{totalrate}}{R_T} \times 100$, where $R_T = \sum R_i$, R_i is the number of bits assigned to node i. The rate in Method 0 is computed without merging and Method 3 represents the global merging algorithm in Section III-B

R_T	Method 0	Method 1	Method 2	Method 3
10	9.9 (0.3%)	8.7 (13.4%)	8.7 (13.4%)	8.1 (18.9%)
15	14.8 (1.4%)	11.5 (23.7%)	11.9 (20.9%)	11.2 (25.4%)
20	19.5 (2.3%)	13.5 (32.3%)	13.6 (32.0%)	13.4 (32.8%)

TABLE II

LOCALIZATION ERROR (LE) OF THE PROPOSED QUANTIZERS DUE TO VARIATIONS OF THE SIGNAL ENERGY a, AND THE MODELLING PARAMETERS. LOCALIZATION ERROR (LE) (m^2) IS GIVEN BY $E(\| x - \hat{x} \|^2)$. LE (NORMAL) IS FOR TEST SET FROM NORMAL DISTRIBUTION AND LE (UNIFORM) FROM UNIFORM DISTRIBUTION. THE PROPOSED QUANTIZERS ARE DESIGNED WITH $R_i = 3, a = 50, \alpha = 2, g_i = 1$ AND $SNR = \infty$ FOR UNIFORM DISTRIBUTION.

Source energy a	40	45	50	55	60
LE(normal)	0.065	0.070	0.078	0.098	0.119
LE(uniform)	0.071	0.061	0.072	0.077	0.094
Delay factor α	1.6	1.8	1	2.2	2.4
LE(normal)	0.167	0.124	0.078	0.046	0.055
LE(uniform)	0.307	0.120	0.072	0.078	0.694
Gain factor g_i	0.6	0.8	1	1.2	1.4
LE(normal)	0.046	0.065	0.078	0.119	0.173
LE(uniform)	0.071	0.071	0.072	0.094	0.132
SNR(dB)	20	40	60	80	100
LE(normal)	1.470	0.167	0.082	0.079	0.078
LE(uniform)	2.381	0.123	0.083	0.073	0.072

TABLE III

COMPARISON BETWEEN PROPOSED QUANTIZER AND TYPICAL QUANTIZERS FOR LARGE SENSOR FIELD $(20 \times 20m^2)$ WHERE 15 NODES ARE DEPLOYED AND EACH NODES USES 2 BITS FOR ITS QUANTIZER.

Quantizer type	Localization Error
Uniform Quantizer	6.9206
Lloyd Quantizer($\lambda = 0$)	1.8994
Proposed Quantizer($\lambda \gg 1$)	0.1643

TABLE IV

LOCALIZATION ERROR (m^2) FOR VARIOUS SETS OF BIT ALLOCATIONS WHERE R^* WAS OBTAINED BY GBFOS USING EDQ GIVEN $R_T = \sum R_i = 10$. LOCALIZATION ERROR IS GIVEN BY $E(\| x - \hat{x} \|^2)$

Sets of bit allocations	EDQ	Proposed Quantizer
$R^* = [4\ 3\ 0\ 3\ 0]$	0.1533	0.1105
$R = [3\ 3\ 0\ 4\ 0]$	0.1615	0.1200
$R = [3\ 4\ 0\ 3\ 0]$	0.1543	0.1227
$R = [3\ 2\ 2\ 3\ 0]$	0.3005	0.2014
$R = [2\ 2\ 2\ 2\ 2]$	0.6199	0.3975

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