Abstract
This paper extends the work reported in SPE114222 for managing waterfloods using estimation of flow characteristics from only injection and production rates. The method first estimates the finite impulse response (FIR) curve corresponding to the fluid-flow between all injector-producer pairs. This FIR curve is analogous to the pressure curves obtained from pulse testing. Reservoir parameters, such as connectivity between wells, can be estimated from this curve, which can also be used to characterize variations of relative flow as a function of storage capacity (F-C plots), thus making it possible to quantify the heterogeneity of flow paths between wells.

Our proposed method is capable of identifying the flow channel relationships between all injector-producer well pairs. This helps in reservoir characterization, where information provided by the proposed procedure can describe the characteristics of flow-path between injector-producer pairs. It can also help in waterflood optimization by tracking sweep efficiency and balancing voidage-replacement. The method has been successfully tested and calibrated for simulated line drive patterns with various fracture geometry conditions. It successfully quantifies the interwell connectivity and the heterogeneous properties in the numerical simulations.

Our procedure has several advantages over pulse testing and tracer tests. In comparison to pulse testing, pressure data is not required and the only data needed are injection/production rates, which are often routinely available with high temporal resolution for many reservoirs. The quality of measured production rates affects the estimated properties. Because the proposed procedure can be routinely performed and implemented, the proposed procedure leads to a dynamic approach for reservoir and fluid flow mapping, where the results can be refined over time. Additionally, our procedure could also be used for better designs of pulse or tracer tests. Finally, once the flow channels are mapped, one can balance the flood and make diagnostic predictions about future response under any given injector scenarios.

Introduction
In waterflooding, injection rates can often be correlated with gross rates of the surrounding producers by monitoring pressure data. To estimate this correlation we can view the whole region where injectors-producers are located as a system, with the injectors as the system inputs, and the producers as the system outputs. Many methods have been used to estimate the directional transmissivity of flow based on injection and production rate data. What makes these approaches more attractive is that injection and production rate data are now routinely available with high temporal resolution for many reservoirs. Among these works, Heffer et al. use Spearman rank correlations, Panda and Chopra use artificial neural networks to estimate the relationships between injection and production rates. Albertoni and Lake estimate the effective flow units (called interwell connectivity in their work) based on a linear model using multiple linear regression (MLR) method. Yousef et al. improve this work by building a more complex model, named capacitance model in their work. In our previous work, we proposed an active method to further improving the estimation by avoiding collinearity between injection rates through careful selection of injection rate patterns.

Tracer testing has often been used for mapping high permeability channels in waterfloods and it provides useful reservoir parameter estimations, such as reservoir swept volume, fluid velocities, and flow geometry in a reservoir. For details, see Abbaszadeh-Dehghani et al., 1982, Abbaszadeh-Dehghani et al., 1984, and Oliver. However, conducting frequent tracer tests is often uneconomical and time consuming. Moreover tracer testing cannot provide a dynamic view of the system: an estimate of the waterflood characteristics is obtained after the testing, but the model parameters cannot be updated unless a new tested (e.g., using a different tracer) is conducted.
In this paper, we note that the problem of estimating reservoir properties from injection/production rates can be seen as a system identification problem, where injection rates are inputs and production rates are outputs. This could be seen as a standard multiple-input multi-output (MIMO) system, for which many powerful estimation tools have been developed. Based on previous work\cite{Johnson1,Johnson2,Johnson3}, it is quite reasonable to assume that the impact on production of each injector can be modeled as a linear FIR filter applied to the injection rates. This FIR curve can be analogous to the pressure changes caused by fluid injections. In order to capture the injector-producer relationships, different models developed by previous researchers, such as the streamline model, which describes the relationship between injectors and producers by many imaginary streamlines, and capacitance models, developed by Yousef et al.\cite{Yousef1,Yousef2}, which use only two parameters to describe the relationship between each injector-producer well pair.

Our main contribution is providing a dynamic approach to estimate the flow characteristics between wells using only injection and production data. Our method can estimate the relationships between injection and production wells and characterize the degree of heterogeneity on the interwell regions. Note that this could also be done with data obtained from conventional tracer tests, but our approach (i) is a dynamic characterization system, monitoring and analyzing the flow characteristics and adjusting the estimations over time; (ii) has negligible cost as compared to tracer tests. This is because injection and production rate data is now routinely available with high temporal resolution for many reservoirs, so that estimates can be generated from data that is readily available, without requiring injection of chemical tracers.

In addition, our approach can also be used to obtain a rough idea about the flow characteristic before tracer tests are performed. Doing so will allow better tracer tests to be designed. Finally, once the FIR curves are estimated, we can make diagnostic predictions about production rates under any given injector scenarios.

Our proposed technique has been calibrated with a commercial reservoir simulator (CMG) under different scenarios.

**Injector-Producer Relationships**

In a waterflood, production rates are influenced by reservoir transmissibility bottom-hole flowing pressures and the pressure changes caused by fluid injections. In order to capture the injector-producer relationships, different models have been developed to describe the production rates caused by injection rates and other factors (for examples, see the references\cite{Johnson1,Johnson2,Johnson3}). As a starting point for our procedure, we discuss a general injector-producer model first.

**Injector-Producer Model**

We consider a general linear FIR model: production rates are partly determined by a linear combination of surrounding FIR filtered version of injection rates. That is, for the production rate $P_j(t)$ of a particular producer $j$ is:

$$P_j(t) = \sum_{i=1}^{N_i} I'_i(t) + \text{(Terms Indep. of Injection Rates)}$$

(1)

Where $I'_i(t)$ is a filtered version of the injection rate for injector $i$:

$$I'_i(t) = I_i(t) * h_{ij}(t) = \int_{\tau=0}^{t} I_i(t-\tau) h_{ij}(\tau)d\tau$$

(2)

$h_{ij}(t)$ is the FIR of the total flow-paths between injector $i$ and producer $j$, which accounts for any attenuation and delay between the injection and production rates. For example, $h_{ij}(0)$ at $t = 0$ corresponds to the instant influence to the production rates from the fluid injection. $h_{ij}(t)$ for nonzero $t$ correspond to the influence with at different time delay $t$, so the the integral of $h_{ij}(t)$ from $t = 0$ to $t = \infty$ represents the total production fluid caused by injecting a theoretical water injection “pulse” at injector $i$.

It is easy to show that this FIR-MIMO model is very general and can be used to approximate many different models developed by previous researchers, such as the streamline model, which describes the relationship between injectors and producers by many imaginary streamlines, and capacitance models, developed by Yousef et al.\cite{Yousef1,Yousef2}, which use only two parameters to describe the relationship between each injector-producer well pair.

The model in discrete form is:

$$P_j(n) = \sum_{i=1}^{N_i} I'_i(n) + \text{other terms}$$

(3)

$$I'_i(n) = I_i(n) * h_{ij}(n) = \sum_{m=0}^{L-1} I_i(n-m)h_{ij}(m)$$

(4)

where $L$ is the length of $h_{ij}(n)$.

With many injectors and producers, the system can be modeled as a standard linear FIR-MIMO system. Given this assumption our problems are:\textbf{1}) how to estimate the FIR coefficients in a robust manner; \textbf{2}) how to relate these estimated FIR coefficients to some important characteristics in the reservoir. We will propose our solutions to these two problems and discuss them in what follows.

Note that, because the daily operation data are discrete, from now on, we will use a discrete mathematical formulation.

**Estimation of FIR coefficients**

The FIR coefficients $h_{ij}(n)$ represent the information of how each producer is influenced by its surrounding injectors, so our first problem is how to estimate the FIR coefficients between all injector-producers well pairs. This method must be robust to some degree of noise because there are potential sources of error in production data. For example, in some fields only pump-off controller (POC)
data may be available on a daily basis, while more reliable estimates of production based on well-testing may only be available only once every few weeks. As a result, the pumping efficiency can only be precisely calculated every few weeks, and roughly be estimated in between. Thus the daily POC data, calibrated by well test data, will cause some uncertainty and we should treat them as noisy data.

Typical problems in system identification take as a starting point measurements of the behavior of a system under changing external influences (inputs to the system) and try to determine a mathematical relation between inputs and outputs at the system level, without trying to determine what is actually happening inside the system (see for example Ljung\(^{10}\)). Estimating the FIR coefficients of a linear FIR-MIMO system is a classical problem in system identification, and many robust methods have been developed, dealing with different kinds of noise. Because production data is a major source of error in our system, we adopt the output error (OE) model, as described by Ljung\(^{10}\).

Based on this assumption, from the system identification literature we choose to use a conventional prediction-error approach, namely, the least-square method with truncated FIR order, to estimate the FIR coefficients.

**Least-Square Method with Truncated FIR Order**

In the least-square method with truncated order we first truncate the FIR length to \(\alpha\) and treat it as an \(\alpha\)-th order FIR model, so that for each FIR \(h_{ij}\), we assume its non-zero coefficients are:

\[
H_j = [h_{ij}(0) \ h_{ij}(1) \ h_{ij}(2) \ ... \ h_{ij}(\alpha-1)]^T
\]  

(5)

That is, for each \(h_{ij}\), there are \(\alpha\) unknown coefficients that need to be computed. Then for each producer \(j\), we use the Least-Square (LS) criterion:

\[
V_j = \min_{\hat{h}} \sum_{i=1}^{N_j} \sum_{n} [P_{ij}(n) - I_i(n) * \hat{h}_{ij}(n)]^2
\]  

(6)

where \(n\) denotes the number of samples of injection and production data available.

The estimated FIR \(\hat{h}_{ij}\), which minimizes the LS criterion \(V_j\), is the best estimate for \(h_{ij}\) in the Least-Square Error sense.

In real world scenarios, the FIR length \(\alpha\) (representing the maximum possible delay of influence from injector to producer) is always unknown, so we calculate the estimated error \(V_j\) from \(\alpha=1\) to the maximum possible delay, and when the error \(V_j\) is smaller than some specific threshold value, we stop the calculation and the FIR length is chosen to be the most recently selected \(\alpha\). Repeating the same procedure for all producers, we can estimate the FIR coefficients between all injection and production well pairs.

Before moving on it is important to ask how much injection and production rate data is needed in order to get a reasonable estimation of FIR coefficients. The answer depends on the number of injectors in the system, the degree of collinearity of injection rates, and also the nature and magnitude of the noise. We will return to these issues in the Discussion section.

**Depicting Flow Characteristics Using FIR Curve**

Now we address the second question: how to relate these estimated FIR coefficients to some important flow characteristics in the reservoir. In our procedure, we use the FIR curves, formed by FIR coefficients, which are analogous to the curves obtained from pulse testing.

Also, our proposed FIR curve for each injector-producer pairs can be interpreted as an estimate of the flow influence from a specific injector to a specific producer, thus providing information similar to that provided by the tracer concentration curve. This motivated us to use this FIR curve as a rough estimate for the tracer concentration curve. In next section, we will show how to use the estimated FIR curve to estimate some important flow characteristics.

**Applications of FIR Curve**

We now discuss three applications for the estimated FIR curve; i) estimation of effective flow units, which are weights to indicate the effective contribution of injection wells to the total gross production in surrounding production wells, ii) computation of flow-storage capacity plots, which indicate the degree of heterogeneity between injection and production wells, and iii) prediction of production rates only from injection data.

**I. Estimation of Effective Flow Units**

Effective flow units, also called interwell connectivity, or well allocation factors in the literature, are defined as a set of weight factors that characterize the effective contribution of injection wells to the total gross production in surrounding production wells. The weights quantitatively indicate the communication between a producer and the injectors in a waterflood, and their estimation has been a frequent objective of recent work in the literature.\(^{3,8}\) For our general injector-producer model (FIR model), effective flow can be directly computed as the sum of the FIR coefficients corresponding to the injector-producer well pair. This can be easily seen by considering the definition of impulse response in a linear system, i.e., it measures the output of the system when excited by an impulse input. Thus, if an impulse represents one unit of water injection, the sum of coefficients in the FIR impulse response is the total response to that injection. Mathematically:

\[
\delta_{ij} = \sum_{n=0}^{L-1} h_{ij}(n)
\]  

(7)

Where \(\delta_{ij}\) denotes the effective flow units (interwell connectivity; well allocation factor) between injector \(i\) and producer \(j\). In practice, we often care about the efficiency of water injection, which accounts for cost, so these values...
should be normalized with respect to the total flow units for a given injector:

$$\bar{\delta}_{ij} = \frac{\delta_{ij}}{\sum_{j=1}^{N} \delta_{ij}}$$  \hspace{1cm} (8)

where $\bar{\delta}_{ij}$ denotes the normalized effective flow units between injector $i$ and producer $j$.

II. Flow-Storage Capacity Plot

Plotting flow capacity versus storage capacity has been proposed initially for estimating the injection sweep efficiency in the early petroleum literature. This method was usually shown in a flow-storage diagram (Lorenz plots or flow capacity plots) to highlight the relative flow to its associated volume. For example, if this plot shows that a large percentage of flow comes from only a small percentage of the pore volume, this will indicate there are likely to exist fast flow paths in the observed region. The existence of a fracture or high permeability channel can be strongly inferred from this information.

The flow-storage capacity plots can be used to characterize the degree of heterogeneity in a reservoir. Figure 1 shows an example for such a plot, with two example curves. The dashed line represents the homogeneous case with four uniform fractures, with each fracture having exactly a quarter of the flow and a quarter of the pore volume. So its corresponding F-C curve is a straight line with slope equal to 1. The solid line represents a heterogeneous case, with four different fractures. In this case, the curve includes the point $(x, y) = (0.3, 0.7)$, which indicates that some 70% of the flow is from 20% of the fracture network pore volume, i.e., there exists a fast flow path. The degree of heterogeneity was measured by the degree of departure from the homogeneous case. In other words, the increasing levels of heterogeneity are indicated by the movement of the Lorenz curve away from the straight line. Twice the area between the curve and the straight line is termed the Lorenz coefficient, $L_{c}^{11}$.

Shook proposed a procedure to approximate the true F-C plots using tracer tests data, in which the storage capacity, $C$, and the flow capacity, $F$, are calculated as:

$$C(t) = \int_{0}^{t} \frac{c(t) \tau}{C} \, d\tau$$  \hspace{1cm} (9)

$$F(t) = \int_{0}^{t} \frac{c(t) \tau}{F} \, d\tau$$  \hspace{1cm} (10)

$C(t)$ is the time-weighted reservoir volume “seen” by the tracer at time $t$. $F(t)$ is the fractional cumulative amount of tracer “delivered” to the production well via the pore volume, $C(t)$.

III. Prediction of Production Rates

The estimated FIR curve describes the influence of production rates from surrounding injection rates for each producer. Thus we can use FIR curve, combined with only injection rates of surrounding injectors, to predict the future production rates of each producer.

First, we use historic injection and production rates to estimate the FIR curve $h_{ij}(n)$. Then the predicted production rates, which denoted as $\hat{P}_{j}$, for producer $j$ are:

$$\hat{P}_{j}(n) = \sum_{i=1}^{N} I_{ij}(n) + \text{bias}$$

$$= \sum_{i=1}^{N} I_{ij}(n) \cdot h_{ij}(n) + \text{bias}$$  \hspace{1cm} (13)

The bias value accounts for factors other than injectors that affect the production rates, such as production depletion curves. Here we assume the variations of production rates caused from other factors change slowly compared to injection rates, which is usually the case in a tight waterflood. Thus the bias is a constant value and can be estimated as

$$\text{bias} = \frac{1}{n} \sum_{k=0}^{n-1} \left( P_{j}(k) - \hat{P}_{j}(k) \right)$$  \hspace{1cm} (14)

Using this, we can predict the production rates at time $n$ using only previous production rates and injection rates up to time $n$.

Results

The technique was tested by applying it to a commercial reservoir simulator under different scenarios. The results of
these applications are presented and discussed in this section.

**Application: Estimating Interwell Connectivity**

We applied our procedure on a numerical simulator, CMG, with a line drive injection pattern with 6-injectors/3-producers scenario (Figure 2). In all cases we simulated two component water and oil fluid systems, and consider only vertical wells. Oil viscosity was set to 4 CP. The numerical simulation uses day as the time scale; that is, \( \Delta n = 1 \) day.

1-Homogeneous case

In this case, a single-layered homogeneous reservoir was created. This reservoir was set to a homogeneous permeability of 100 md everywhere, as shown in Figure 3. The reservoir pressure is set to 1000 psi.

The injection rates are chosen the same as in our previous paper (see Lee et al.6); that is, they are based on selected Haar wavelet sequences. Sequences in the set are highly non-collinear, and this design results in much smaller measurement errors compared to the situation when injection rates are highly collinear (see Discussion section). Note our method does not require any specific injection rates to be used, although carefully designed (uncorrelated) injection rates are highly recommended in order to decrease the error from noisy data.

Figure 4 and Table 1 show the results of estimated effective flow units by our procedure in this case. Because the reservoir is homogeneous, we expected the estimated effective flow units to be almost symmetric across the plane of symmetry and to decrease as the distance between injection and production wells increase. All estimates match well with the chosen reservoir conditions.

2-Multiple high permeability channels case

In this case, we consider a single-layered reservoir with an isotropic permeability of 0.1 md and where there are three high permeability channels with different lengths, and all roughly oriented in the 45 degree direction, as shown in Figure 5. The injection rates are set the same as in the homogeneous case.

Figure 6 and Table 2 are the results estimated by our procedure. Because the lengths of the three channels in this case are different, we expected that I1 and I4 will have larger effective flow units with respect to P1, as compared to the effective flow units between I2 , I5 and P2. The flow unites between I2 , I5 and P2 should be the second largest, and those between I3 , I6 and P3 will have the smallest effective flow units. Besides, because all the channels are in 45 degree direction, we expected I2-P1 should have larger flow units than I5-P1, and I3-P2 should be larger than I6-P2. The simulation results all agree with what we expected.

Comparing our results to those in our previous paper6, we see that better results are obtained. The estimated effective flow units capture the trend of high permeability channels oriented in the 45 degree direction, which was not captured from pervious work. So for this application, our new approach can be seen as an improvement of our previous work.

**Application: Flow-Storage Capacity Plot**

For this application, the same scenario (line drive injection pattern with 6-injectors/3-producers) and simulator settings are used. The flow-storage capacity plots are depicted for all injector-producer well pairs in both the homogeneous case and the multiple high permeability channels case. Here we also show the estimated FIR curve between all well pairs in multiple channels case, which is in Figure 7.

1-Homogeneous case

The results are shown in Figure 8, where we depict all F-C plots between each injector-producer well pairs. Because the reservoir is homogeneous, we expected all F-C Plots should be a straight line with the slope equal to 1. The results all agree with what we expected.

2-Multiple high permeability channels case

In the multiple high permeability channels case, the flow-paths are much more complex than the homogeneous case, and we expected that the F-C Plots would no longer be a straight line. The results are shown in Figure 9. From the scenario, I1 should have more flow-paths to P2 and P3 than to P1, so we expected in the F-C Plots, I1-P2 and I1-P3 should deviate more from the straight line than I1-P1 pair. For I3, I3-P1 the deviation from the straight line should be even greater than for I3-P2 and I3-P3. Finally, for I6, I6-P1 should move away from the straight line than I6-P2 and I6-P3. The results all agree with what we expected.

**Application: Prediction of Production Rates**

For this application, we use a five-spot scenario with 5-injectors/4-producers and our simulated reservoir has two areas: one with permeability of 10 md and the other with permeability of 500 md with 5-spots scenario (Figure 10). The injection rates are the same as those used in Yousef6, which were obtained from a real oilfield. The injection rates are shown in Figure 11.

The total data available are about three thousand days. We used the first one thousand days as the training period, to estimate the FIR curve, and the rest time as the predicting period, to verify our predictions. The results are shown in Figure 14, which shows that the prediction results almost perfectly match the actual production rates generated by simulator. This provides further validation of our FIR-MIMO model and its applicability for production prediction. It is important to emphasize that the CMG simulator is not based on the FIR-MIMO model, so our results do support our assumption that this model provides a sufficiently accurate, yet simple, representation for the purpose of establishing injection/production relationships.

**Discussions**

In this section we first compare our procedure and the capacitance model. Then we address the minimum amount of data needed for model estimation and propose a
Comparison to the Capacitance Model

Yousef et al. proposed the capacitance model to describe the relationships between injection and production wells. In capacitance model, two parameters, $\lambda$ and $\tau$, are used to characterize the reservoir between each well pair, where $\lambda$ quantifies the connectivity and $\tau$ quantifies the fluid storage in the interwell regions. The shape of the influence between injectors and producers is basically determined only by $\tau$. Figure 12 shows different shapes with different time constant $\tau$. Obviously this is a simplified description because it used only one parameter to describe the influence shape on the interwell regions. To see the drawback of this, suppose the FIR curve was estimated for an interwell region containing three separate channels, with different attenuation and delay in each channel. Figure 13 is as an illustration for this example. This curve has three separate channels, which leads to an FIR curve that cannot be represented well by capacitance model, no matter the time delay $\tau$ is. On the other hand, our procedure can capture the heterogeneity of interwell regions caused by three separate channels. For example, the flow-storage capacity plot derived from our FIR curve is far away from the homogeneous case (straight line with slope equal to 1), which indicates there are many flow-paths between the injection and production wells. A similar F-C plot cannot be achieved using the capacitance model.

Amount of Data Needed for Model Estimation

In order to estimate the FIR coefficients using LS method, we need to decide the amount of data required to get a reasonable estimate, which will determine the time needed in order to obtain sufficient injection and production rate measurements.

For each producer, suppose there are $N_i$ surrounding injectors of interest and that the maximum possible flow delay between injector-producer well pairs is $T_{\text{max}}$. This means that for this producer, there are a total of $N_i(T_{\text{max}}+1)$ unknown coefficients. Besides, we also need another $T_{\text{max}}$ time for the transient behavior. In order to solve for these $N_i(T_{\text{max}}+1)$ unknowns, we need at least:

\[
\text{Data Number}_{\text{min}} = N_i \left( T_{\text{max}} + 1 \right) + T_{\text{max}}
\]

This means the minimum number of measurements needed is roughly proportional to the number of injectors in the system and the maximum possible delay. In real fields, the numbers of injection wells of interest is not often too large, because for each producer, we only need to consider neighboring injectors. Thus the number of measurements is often less than ten, depending on which well patterns are used in the field.

Error Analysis

In order to analyze the error, we suppose the production rates are noisy (the measurements on production rates are inaccurate). For each producer $j$, the production rate could be written as:

\[
P_j(n) = \sum_{i=1}^{N_i} h_{ij}(n) \ast I_i(n) + v_j(n)
\]

Where $v_j(n)$ represents the noise on the production rates of producer $j$. We can rewrite them in matrix form:

\[
P_j = S H_j + V_j
\]

where

\[
P_j = \begin{bmatrix} P_j(0) & P_j(1) & \ldots & P_j(K) \end{bmatrix}^T
\]

After some calculations, the covariance of estimated FIR $h_{ij}(t)$ for producer $j$ can be computed as:

\[
\text{Cov} \left( H_j \right) = \left( S^H S \right)^{-1} S^H R_{\text{SN}} S \left( S^H S \right)^{-1}
\]

where $S$ is the shifted version injection rates matrix:

\[
S^H = \begin{bmatrix} S_i^H \cr S_2^H \cr \vdots \cr S_M^H \end{bmatrix}
\]

\[
S^i = \begin{bmatrix} s_i(0) & s_i(1) & \ldots & s_i(L-1) & \ldots & \ldots & s_i(0) & s_i(1) & \ldots & s_i(L-1) \end{bmatrix}
\]

The vector $[s_i(0) \ s_i(1) \ \ldots \ s_i(L-1)]^T$ represents the injection rates of injector $i$. And $R_{\text{SN}}$ is the noise correlation matrix.

From this expression, we can see that the measurements in our method are mainly dependent on two factors: (1) The injection rates of all injectors; (2) Noise on the production rates (how good are the measurements of the production rates).

To see how injection rates can affect this method, we suppose the noise is white with mean zero, that is, the correlation matrix is $\sigma^2 I$, where $\sigma^2$ is the noise variance. Thus the covariance matrix of the measurements becomes:

\[
\text{Cov} \left( H_j \right) = \sigma^2 \left( S^H S \right)^{-1}
\]

The mean square error is:
Minimizing the MSE is equivalent to minimizing $\text{Tr}\{(S^H S)^{-1}\}$. That means that a set of injection rates will lead to lower error in estimation of FIR coefficients if they have (1) low out-of-phase autocorrelation and (2) low cross-correlation. Thus, designing a set of injection rates that are not so collinear with each other will tend to be important. In fact this is important for all injector-producer relationship estimation methods that use only injection and production rates (see Yousef et al.\textsuperscript{4} for more discussions). For techniques for designing a set of non-collinear injection rates, see Lee et al.\textsuperscript{6} for more details.

As for the second factor, it depends on the quality of production rate measurements. Some sensitivity analysis was performed by Yousef et al.\textsuperscript{4} with the conclusion that the daily rate data could be seen as being affected by substantial amounts of white noise. This also justifies the need for non-collinear injection rates, in order to get better measurements.

Also if we have a longer data, the diagonal terms in matrix $(S^H S)$ will become larger, so $\text{Tr}\{(S^H S)^{-1}\}$ will become smaller, meaning that we will achieve a smaller error in the estimation of FIR coefficients. With this analysis we can estimate the expected model accuracy given the noise levels, the level of collinearity of the injections rates and the amount of data available for model estimation.

Conclusions
We developed a method for estimating the direction and location of high permeability channels based on signal processing techniques applied to only injection and production rates. The FIR curve for interwell region is first calculated. Then estimation of effective flow units, and flow-storage capacity plot, can be applied to characterize the high permeability channels in the reservoir. The FIR curve can also be used for prediction of production rates.

The technique has been verified by numerical simulations. We evaluated several applications of this method on two different cases, and the results show that our estimated effective flow units and F-C plots match the characteristics of the reservoir in each case, and the prediction results of production rates match the data generated by numerical simulator very well.

The proposed procedure can overcome some limitations of existing methods. It can characterize relationships between well pairs and the degree of heterogeneity on the interwell regions.

Nomenclature

$C$ = Storage Capacity

$F$ = Flow Capacity

$h_{ij}$ = finite impulse response (FIR) for interwell region between injector i and producer j

$\hat{h}_{ij}$ = estimated FIR coefficients between injector i and producer j

$I_i =$ injection rate ($L^3/t$)

$I'_{ij} =$ filtered version of injection rate ($L^3/t$)

$L =$ Length of finite impulse response (FIR)

$N_i =$ number of injection wells

$N_p =$ number of production wells

$P_i =$ liquid production rate ($L^3/t$)

$P_j =$ predicted liquid production rate ($L^3/t$)

$R_N =$ covariance matrix of noise

$S =$ injection rate sequences matrix

$T_{\text{max}} =$ Maximum possible delay in FIR curve

$V_j =$ least-square criterion for estimating FIR coefficients

Greek letters

$\alpha =$ the truncated FIR length

$\lambda =$ weights in capacitance model

$\sigma^2 =$ noise variance on production rates

$\tau =$ time constant in capacitance model

$\delta_{ij} =$ effective flow units between injector i and producer j

$\delta_{ij} =$ normalized effective flow units between injector i and producer j

Subscripts and superscripts

$i =$ injector index

$j =$ producer index

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References


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Figure 1: An example for Flow-Storage Plot. The dash line is homogeneous case; the solid line is the heterogeneous case.

Figure 2: Line drive scenario: 6 injectors and 3 producers.
Figure 3: Model on numerical simulator CMG - single layered homogeneous reservoir with an isotropic permeability of 100 md.

Figure 4: Estimated effective flow units in homogeneous case. The flow units are represented by arrows that start from injector and point to producer in each injector-producer pair. The longer the arrow, the larger value of the effective flow units between the two wells.

Table 1: Estimated effective flow units in homogeneous case plotted in Figure 4.

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<td>0.399</td>
<td>0.300</td>
</tr>
<tr>
<td>Inj 6</td>
<td>0.183</td>
<td>0.298</td>
<td>0.519</td>
</tr>
</tbody>
</table>

Figure 5: Model on numerical simulator CMG - single layered reservoir with an isotropic permeability of 0.1 md. There are three high permeability channels with different length in the reservoir.

Figure 6: Estimated effective flow units in multiple high permeability channels case.

Table 2: Estimated effective flow units in multiple high permeability channels case plotted in Figure 6.

<table>
<thead>
<tr>
<th></th>
<th>Pro 1</th>
<th>Pro 2</th>
<th>Pro 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inj 1</td>
<td>0.719</td>
<td>0.142</td>
<td>0.140</td>
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<tr>
<td>Inj 2</td>
<td>0.410</td>
<td>0.578</td>
<td>0.012</td>
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<tr>
<td>Inj 3</td>
<td>0.065</td>
<td>0.485</td>
<td>0.450</td>
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<tr>
<td>Inj 4</td>
<td>0.575</td>
<td>0.254</td>
<td>0.171</td>
</tr>
<tr>
<td>Inj 5</td>
<td>0.141</td>
<td>0.793</td>
<td>0.066</td>
</tr>
<tr>
<td>Inj 6</td>
<td>0.008</td>
<td>0.310</td>
<td>0.682</td>
</tr>
</tbody>
</table>
Figure 7: FIR curve for multiple high permeability channels case.

Figure 8: Flow-Storage Capacity Plots for homogeneous case.
Figure 9: Flow-Storage Capacity Plots for multiple high permeability channels case.

Figure 10: Model on numerical simulator CMG - single layered reservoir with two areas: one with permeability of 10 md and the other with permeability of 500 md with five-spot scenario.

Figure 11: Injection rates used in this five-spot scenario simulation.

Figure 12: Injection unit-step and the filtered injection rates at different values of the time constant $\tau$.

Figure 13: An example of estimated FIR curve which consist of three separate channels 1, 2 and 3.
Figure 14: The prediction results compared with simulated production rates for all producers. The blue line is the production rates by predicting method, and the red line is the produced by numerical simulator CMG.