A GRADIENT-BASED RATE CONTROL ALGORITHM WITH APPLICATIONS TO MPEG VIDEO

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ABSTRACT

In this paper, we present a rate control algorithm for MPEG video. The goal is to minimize the distortion while keeping the change in distortion between consecutive frames small. We formulate this goal as a constrained optimization problem and find a solution using an iterative gradient search method. To reduce the computation cost, we propose a model based on spline curves to approximate the rate distortion functions which also takes into account the frame dependencies. Simulations on short video sequences show that, at the same bit rate and buffer constraint, our technique generates output sequences with smaller and more stable mean square error than other approaches, while maintaining strictly constant bit rate for every group of pictures, at the expense of higher computation cost.

1. INTRODUCTION

Digital video compressed with the MPEG [1] standard has recently become increasingly popular for applications such as CD-ROM and Video-CD storage. These are asymmetric applications, where more computing power and processing time can be spent in encoding while the encoded data stream can be decoded with a relatively low cost. In decoding, data is read out from disk at a strictly constant speed thus making necessary a decoder buffer so that the video stream can be decoded and played back synchronously. Because the buffer control strategy affects only the complexity of the encoder, there is particular interest in strategies that might be complex but will reduce the required buffer size for a given video quality, or increase the quality for a given buffer size.

Many buffer control methods [2, 3] only take into account the rate, not the distortion, in the algorithm. Other methods control both the rate and quality by formulating the problem as a delayed decision constrained optimization problem and solving it using dynamic programming [4] or Lagrangian techniques [5, 6, 7]. The optimal approaches require additional encoding delay and complexity but provide optimal results in a rate distortion sense.

In this paper we formulate buffer control as a delayed decision optimization and we solve it using an iterative gradient search technique, which we introduced in [8]. In addition we propose a method for approximating rate and distortion functions by using spline curves which can be used to significantly speed up the search procedure.

2. PROBLEM FORMULATION

In MPEG, a video sequence is divided into Groups of Pictures (GOPs) with size equal to N frames. The GOP is a basic access unit that can be decoded independently. In this paper, we treat the GOP as a unit for buffer control.

The bit-rate and quality of MPEG video is controlled by a quantization scale, *mquant*, which can be changed over different macro blocks. In our formulation, the value of *mquant*, denoted as q, is kept constant over an entire frame. A complete system would consist of a rate control algorithm, such as the one described here, followed by an adaptive quantization strategy of some sort which would re-allocated bits among the macroblocks by changing their *mquant*. The rate derived from the rate control algorithm would then be used as the "rate budget". The buffer control problem is to assign the quantization scale q_i for the ith frame in a GOP such that the overall quality, measured by a pre-defined cost function, is optimized. Let $\mathbf{q} \equiv (q_0, q_1, \ldots, q_{N-1})^T$ be the quantization choices for the frames in a GOP. When the quantization scales are set to $\mathbf{q}$, we define the code length and mean square error of frame i as the rate and distortion functions, denoted by $r(i, \mathbf{q})$ and $d(i, \mathbf{q})$, respectively. $r(i, \mathbf{q})$ and $d(i, \mathbf{q})$ can be either actually measured during the encoding process, or calculated from an approximation model. By using a vector expression for $\mathbf{q}$, we are taking into account the "dependency" of the problem, i.e., the distortion/rate trade off for predicted/interpolated frames depends on the frames that were used to generate the prediction [6]. The buffer occupancy after frame i is coded is then:

$$b(i, \mathbf{q}) = b(i - 1, \mathbf{q}) + r(i, \mathbf{q}) - R \tag{1}$$

where R is the channel bit-rate in bits per frame. If $b(i, \mathbf{q})$ is smaller than zero, stuffing bits are padded to avoid underflow and $b(i, \mathbf{q})$ is assigned as zero.

We define the cost function as

$$J(\mathbf{q}) \;=\; D(\mathbf{q}) + wE(\mathbf{q}) \tag{2}$$

where

$$D(\mathbf{q}) \;=\; \frac{1}{N} \sum_{i=0}^{N-1} d(i, \mathbf{q}) \tag{3}$$

$$E(\mathbf{q}) \;=\; \frac{1}{N} \sum_{i=0}^{N-1} [d(i, \mathbf{q}) - d(i-1, \mathbf{q})]^2 \tag{4}$$

and w is the weighting coefficient between $D(\mathbf{q})$ and $E(\mathbf{q})$. The purpose of $E(\mathbf{q})$ in the cost function is to minimize the abrupt changes in quality and avoid "flicker" problems. Note that, although we choose MSE as a quality measure in this paper, it is also possible to use other measures that take visual perception into account.

The problem can now be formulated as that of finding $\mathbf{q}^*$ such that:

$$\mathbf{q}^* \;=\; \arg \min_{\mathbf{q}} J(\mathbf{q}) \tag{5}$$

subject to

$$q_i \in \{1, 2, \ldots, 31\}, \quad i = 0 \ldots N - 1 \tag{6}$$
$$b(i, \mathbf{q}) \leq b_{max}, \quad i = 0 \ldots N - 2 \tag{7}$$
$$b(N - 1, \mathbf{q}) \leq 0 \tag{8}$$

where b_{max} is the prescribed maximum buffer size. In (8), we force the final buffer occupancy to be less than or equal to zero, and pad stuff bits at the end of GOP to ensure all GOPs have the same number of bits.

This formulation leads to an integer programming problem with nonlinear cost function and nonlinear constraints. These characteristics make the optimization problem difficult. Such problems can be solved using dynamic programming techniques [4], where the true global optimal solution can be obtained at the expense of high computational cost. In this paper, we do not intend to obtain the global optimal solution. Instead, we are only looking for reasonable suboptimal solutions with reduced computation cost. To achieve our goal we introduce (i) an approximate gradient search technique and (ii) models for the rate and distortion functions, which combined achieve results close to optimal at a fraction of the complexity required to reach the optimal solution.

3. PENALTY FUNCTION AND GRADIENT SEARCH TECHNIQUE

Our first approximation is to change the integer-valued variable in (6) into a continuous one, so that many optimization techniques defined in continuous domain can be applied. The constraints of (7) and (8) can be taken into account by adding penalty functions to the cost, $J(\mathbf{q})$. The penalty functions are defined as

$$\begin{aligned} P_i(\mathbf{q}) &= \max(0, b(i, \mathbf{q}) - b_{max})^2 \\ Q(\mathbf{q}) &= \max(0, b(N - 1, \mathbf{q}))^2. \end{aligned} \tag{9}$$

The new cost function is

$$\phi(\mathbf{q}, c) = J(\mathbf{q}) + c \left(\sum_{i=0}^{N-2} P_i(\mathbf{q}) + Q(\mathbf{q}) \right), \tag{10}$$

where c determines the amount of the penalty. The original problem can be approximated by iteratively solving the unconstrained problem of minimizing $\phi(\mathbf{q}, c)$ as $c \to \infty$.

In order to solve the unconstrained problem efficiently, we make the assumption that the cost function is smooth. Our experiments [8] indicate that this assumption is usually realistic so that a nearly optimum point can be reached

using the gradient search technique. There are several iterative gradient search algorithms available for our problem [9, 8]. In this paper we consider the steepest descent method, by which the negative direction of the gradient vector $\nabla \phi(\mathbf{q})^T$ is used as the search direction, and the vector $\mathbf{q}$ is updated by the following

$$\mathbf{q}_{k+1} = \mathbf{q}_k - \alpha_k \nabla \phi(\mathbf{q}_k)^T \tag{11}$$

where α_k is a nonnegative scalar value obtained by minimizing the function

$$\varphi(\alpha) = \phi \left(\mathbf{q}_k - \alpha \nabla \phi(\mathbf{q}_k)^T \right) \tag{12}$$

using a line search procedure [9, 8]. (See [8] for details).

4. APPROXIMATING THE RATE DISTORTION FUNCTIONS

Our goal is to come up with reasonably good models of $r(i, \mathbf{q})$ and $d(i, \mathbf{q})$ that enable us to speed up the search for the optimal solution. Behavior at the macroblock level is difficult to model. For example, for the predictive frames (P and B), additional factors intervene such as the decision rules for selecting macroblock types as "intra" or "nonintra", and the strategy for the usage of motion vectors (forward, backward, or both). In this paper, we use piece-wise polynomials to approximate the frame level rate-distortion curves of $r(i, \mathbf{q})$ and $d(i, \mathbf{q})$.

4.1. Intra-Frame Approximation

We first consider the influence of quantization scales within a frame. This is useful not only for I frames, but also for P and B frames after their reference frames are fixed. The rate and distortion functions are, respectively, $r(q)$ and $d(q)$, where q is a scalar variable. The first step is to compute $r(q)$ and $d(q)$ on several fixed q's, called the "control points". In order to capture the exponential-decay property of $r(q)$, we choose 1, 2, 3, 5, 8, 13, 21, 31 as control points. The function values at these control points can be efficiently calculated by repeatedly quantizing and dummy-coding (encoding without generating output stream). The function value between two consecutive control points is interpolated by a cubic polynomial $f(q) = a \cdot q^3 + b \cdot q^2 + c \cdot q + d$ where the parameters a, b, c, d can be determined by imposing the zero and first-order continuity on the control points. For example, if the four consecutive control points are q_0, q_1, q_2, and q_3, the parameters for the segment between q_1 and q_2 can be derived by solving the following linear equations for the rate (For the distortion, r is replaced by d):

$$\begin{cases} f(q_1) &= r(q_1) \\ f(q_2) &= r(q_2) \\ f'(q_1) &= \frac{r(q_2) - r(q_0)}{q_2 - q_0} \\ f'(q_2) &= \frac{r(q_3) - r(q_1)}{q_3 - q_1} \end{cases} \tag{13}$$

where $f'(q)$ is the first-order derivative of $f(q)$. Note that, although we have defined 8 control points in our model, we only need 4 control points to determine the interpolated value for any given point.

4.2. Inter-Frame Dependency

The rate-distortion characteristic of the predictive frame (P or B) depends on the quality of its reference frame(s). When the reference frame has smaller MSE, the prediction residue tends to be smaller, which results in a smaller rate and distortion in the predictive frame. On the contrary, if the MSE in reference is larger, not only the rate and distortion of the predictive frame will become larger, but also more macroblocks will be coded as "intra-block" (by the decision rules used in [10]), which will decrease the dependency on the reference frame. After some point, it will be completely independent of the reference frame (see Fig. 1).
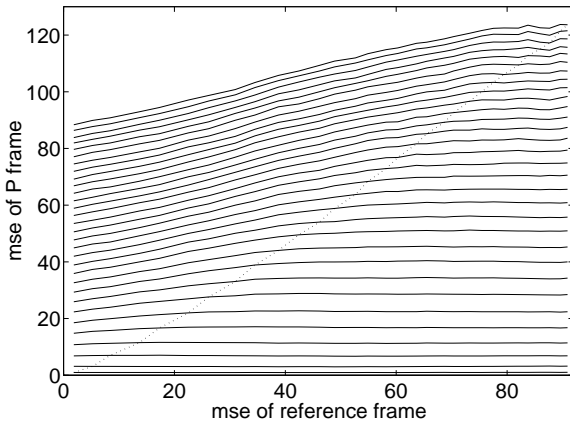

Figure 1: MSE for a P frame in football sequence, plot as function of MSE for its reference frame. Each solid line is an MSE curve for a given mquant in the predictive frame. The dashed line indicates the boundary where mquants for the predictive and reference frames are equal.

To model the frame dependency for the distortion, we denote the MSE of the predictive frame as $d(p, q)$, and the MSE of the reference frame as $d_r(q)$, where q and p are, respectively, the quantization scales for the reference and predictive frames. For each value of q, we define the following inter-frame dependency model:

$$
\begin{aligned}
d(p,q) &= d(q,q) - \alpha(q)\{d_r(q) - d_r(p)\} &&\text{if } p \le q \\
d(p,q) &= d(q,q) &&\text{if } p > q
\end{aligned}
\quad (14)
$$

where p is the only variable in the model. For each q, model parameters $d(q,q)$ and $\alpha(q)$ can be determined by encoding and measuring the distortion at two values of p. In our implementation, we only determine the model parameters for the q at the 8 control points defined in the previous subsection, and then, the entire $d(p, q)$ is reconstructed by using the intra-frame model from the control points. A similar model does not work as well for the rate. From several video sequences, we have observed that, for the quantization scales between 3 and 24, the inter-frame dependency for rate is reasonably low. The following model is used (suppose the two measured points for p are p_1 and p_2): use linear interpolation of $r(p_1, q)$ and $r(p_2, q)$ if $p_1 < p < p_2$, $r(p_1, q)$ if $p \le p_1$, and $r(p_2, q)$ if $p \ge p_2$.

For B frames, where two reference frames are involved, MSE function becomes $d(p_1, p_2, q)$, which is more difficult

to be characterized. In this paper, we simply evaluate the dependency for one reference frame at a time by using the above model, and we then pick the one with smaller MSE values. This procedure simulates part of the strategy for selecting "forward" or "backward" motion vectors in the MPEG encoder.

5. SIMULATIONS

Our software simulations are based on the MPEG-2 encoder implementation of [10].

5.1. Test on Gradient Search Algorithm

To show the effectiveness of our algorithm, we first test the gradient search algorithm defined in Section 3. The cost function for a given $\mathbf{q}$ is calculated by actually encoding the GOP, and measuring the rate and distortion values. The function only defined in the discrete integer grid of $\mathbf{q}$, which introduces two problems for the steepest descent algorithm. The first problem is on the calculation of gradient vector, where the derivative is required. In the simulation, the derivative is approximated by first-order difference. The second problem is that the exact line search can not be applied. Instead, we create a search path consist of the points that are closest to the line along the negative gradient direction, and then search through those points. See [8] for the details. One simulation result for the football sequence is shown in Figure 2.
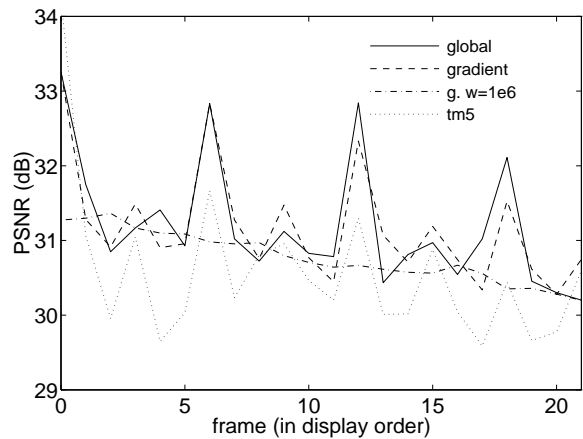

Figure 2: PSNR of the first 22 frames in Football sequence (352×240, at 1152 Mbps). The 1st curve, *global*, is obtained by exhaustive search. The 2nd, *gradient*, and 3rd curves correspond to our algorithm with $w = 0$ and $w = 10^6$, respectively. The last curve, *tm5*, was obtained using the Test Model 5 rate control algorithm [10].

We can see from the results that the solution of our algorithm is very close to the optimal one obtained by exhaustive search. Also, effectiveness of the squared difference of MSE, $E(\mathbf{q})$, is shown in the third curve, where w is set to 10^6 and our solution maintains nearly constant PSNR. Finally, compared to Test Model 5, note that we not only achieve better PSNR, but also make the following possible:

(1) constant rate for GOP, (2) smaller MSE variation between consecutive frames, and (3) solution in small-buffer case where TM5 fails. Our method offers the flexibility of choosing an appropriate cost function while ensuring that the buffer constraints are met. Additional results can be found in [8].

5.2. Model Compliance Test

We test the accuracy of approximation model defined in Section 4 by the following steps: We first encode I frame and P frame, measure and record the rate and distortion data, for every possible quantization settings. Then, pick the data at the following value for the control points: 1, 2, 3, 5, 8, 13, 21, 31 for intra frames, and 5, 13 for inter frames. Based on these control points, we build the model and calculate the estimated rate and distortion values. Then, the average and maximum relative errors are calculated for the typical operating range of quantization scales, which is from 3 and 24. The data from several different video sequences is shown in Table 1. The low error on the average shows that the model is accurate for most of the quantization settings, and some larger maximum errors indicates there are some cases where the model is incorrect.

Relative Errors for I frames				
	MSE		BITS	
	avgerr	maxerr	avgerr	maxerr
Football	0.88%	7.01%	1.04%	6.32%
Claire	0.83%	4.37%	0.34%	3.28%
Susie	1.08%	6.10%	0.89%	6.11%
Miss America	0.95%	3.84%	0.65%	7.18%

Relative Errors for P frames				
	MSE		BITS	
	avgerr	maxerr	avgerr	maxerr
Football	0.39%	6.60%	0.66%	8.41%
Claire	0.88%	12.30%	2.49%	33.02%
Susie	1.24%	15.88%	2.92%	15.88%
Miss America	0.89%	11.03%	3.27%	45.82%

Table 1: Relative errors for I and P frames. *avgerr*: average error, *maxerr*: maximum error.

5.3. Gradient Search with Approximation Model

Our goal in modeling the rate and distortion functions is not so much the accuracy of the model but the speed up factors it allows while maintaining a near optimal solution. In this part, we integrate the two parts and run a simulation using gradient search algorithm with approximated rate distortion functions. By using the model, the computation complexity is greatly reduced, because now, we only have to evaluate the function value at fixed control points. Also, because the rate and distortion functions are approximated by a set of continuous piecewise polynomials, the cost function is well-defined in the real-valued space. So, the derivative of the cost function can be derived in close form, and the line search can be performed exactly. After the algorithm has converged, we simply round the value **q** to its

nearest integer and use it to encode the sequence. The simulation results for the football sequence show that *our solution using models matches the one obtained by measurings the actual rates and distortion functions, with only 1/10 of the computations.* This good performance for the football sequence can be contributed to relatively small model error, as shown in Table 1. More extensive testing and enhancement is left for future work.

6. CONCLUSIONS

In this paper, we have demonstrated the feasibility of using a gradient-based optimization algorithm for buffer control. By using this technique, we are able to achieve strictly constant bit rate per GOP, increase for overall quality (in MSE sense), while decreasing the variation of qualities between different frames. We also successfully reducing the computations by introduce a rate-distortion model based on piece-wise polynomials.

7. REFERENCES

[1] Inform. Technology - Generic Coding of Moving Pictures and Associated Audio, ITU Draft Rec. H.262, ISO/IEC 13818-2, Mar. 1994.

[2] MPEG video simulation model three, ISO, coded representation of picture and audio information, 1990.

[3] C.-T. Chen and A. Wong. A self-governing rate buffer control strategy for pseudoconstant bit rate video coding. *IEEE Trans. on Image Proc.*, 2(1):50–59, Jan. 1993.

[4] A. Ortega, K. Ramchandran, and M. Vetterli. Optimal trellis-based buffered compression and fast approximation. *IEEE Trans. on Image Proc.*, 3(1):26–40, Jan. 1994.

[5] D. W. Lin, M.-H. Wang, and J.-J. Chen. Optimal delayed-coding of video sequences subject to a buffer-size constraint. In *Proc. of SPIE Visual Communications and Image Processing '93*, Cambridge, MA, Nov. 1993.

[6] K. Ramchandran, A. Ortega, and M. Vetterli. Bit allocation for dependent quantization with applications to multiresolution and MPEG video coders. *IEEE Trans. on Image Proc.*, 3(5):533–545, Sept. 1994.

[7] J. Lee and B. W. Dickinson. Joint optimization of frame type selection and bit allocation for MPEG video encoders. In *Proc. of ICIP 94*, volume II, pages 962–966, Austin, Texas, 1994.

[8] L.-J. Lin, A. Ortega, and C.-C. J. Kuo. Gradient-based buffer control technique for MPEG. In *Proc. of SPIE Visual Communications and Image Processing '95*, Taipei, Taiwan, May. 1995.

[9] D. G. Lueberger. *Linear and Nonlinear programming.* Addison-Wesley, 1984.

[10] MPEG software simulation group, MPEG-2 encoder v. 1.1a. URL: ftp://ftp.netcom.com/pub/cfogg/mpeg2/mpeg2codec_v1.1.tar.gz.