Optimal rate control for video coding based on a hybrid
MMAX/MMSE criterion

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ABSTRACT

In this paper, we consider the problem of rate control for video transmission. We focus on finding off-line optimal rate control for constant bit-rate (CBR) transmission, where the size of the encoder buffer and the channel rate are the constraints. To ensure a maximum minimum quality is obtained over all data units (e.g., macro blocks, video frames or group-of-pictures), we use a minimum maximum distortion (MMAX) criterion for this buffer-constrained problem. We show that, due to the buffer constraints, a MMAX solution leads to a relatively low average distortion, because the total rate budget is not completely utilized. Therefore, after finding a MMAX solution, an additional minimization of average distortion criterion is proposed to increase overall quality of the data sequence by using remaining resources. The proposed algorithm (denoted MMAX+ as it incorporates both MMAX and the additional average quality optimization stage) leads to an increase in average quality with respect to the MMAX solution, while providing a much more constant quality than MMSE solutions. Moreover we show how the MMAX+ approach can be implemented with low complexity.

Keywords: Rate control, Video transmission, MMAX, MMAX+

1. INTRODUCTION

Future high bandwidth video applications, such as video-on-demand (VOD) will require transmission over the network of video compressed at a variable rate. Thus, a rate control has to be used, based on objectives such as coded video quality or data rate. Also, video transmission needs to be performed under delay constraints for real time playback, since video frames that arrive too late are useless.

After channel bandwidth or other constraints such as limited delay, total bit-budget or the size of codec buffers have been determined, a target quality measure should be chosen. Most previous work for image and video coding has been based on minimization of average distortion (MMSE). As a consequence, optimal bit allocation under various constraints for the MMSE criterion has been widely studied in the literature. Examples include bit allocation for arbitrary inputs and a discrete set of available quantizers,\(^1\) for dependent quantization,\(^2\) and optimal bit allocation under buffer constraint.\(^3\) A main drawback of the MMSE criterion is that the quality difference between frames can be large and some frames may be coded at relatively low quality even though the average quality is high. A minimum maximum distortion (MMAX) criterion has been proposed to prevent this heavy fluctuation of source quality.\(^4\) Solutions for the bit allocation problem under the MMAX criterion for both independent\(^5\) and dependent quantizers\(^4\) have been studied. Using this criterion, coding units having a significantly lower than average quality can be avoided. However, when multiple constraints are present, as when buffering is considered, the MMAX criterion by itself may be inefficient. This is because the MMAX optimization is terminated as soon as it cannot decrease the maximum overall distortion. For example, in the case of CBR transmission of a video sequence with buffer constraints, the maximum distortion frame could occur for a frame that is located in a period of several consecutive high complexity frames. Because these frames may require higher data rate than the given transmission rate, the buffer will tend to fill up. If this is the case the algorithm will stop because the distortion of the worst frame cannot be reduced without incurring in overflow.

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This means that additional rate could be used in other parts of the sequence, so that overall quality could be increased.

A criterion for minimizing distortion in lexicographical sense (MLEX) has been proposed as a modified MMAX approach to increase overall quality. In a MLEX criterion, two different solutions are arranged by a sorted list of their distortion in a non-increasing distortion order. Then a comparison of the distortions is based on considering the list starting from the 1st index. If the distortions in the first position are equal then the 2nd indices are compared. If the distortion of two solutions is different for a given index then the solution with smaller distortion in that index is the better one. Otherwise the comparison is continued through the following position until the distortion of two solutions are different. This criterion is used to find optimal bit allocation under CBR constraints where quantizer levels are used as a distortion measure. Since all frames have the same set of quantizer levels, it is shown that the optimal solution is determined by using constant quantizer segments. But, in general, if a distortion measure can take any arbitrary values, the proposed algorithm cannot be easily applied. In our work, we show how the algorithm used to find the MMAX solution with buffer constraints can be extended to find the MLEX solution as well.

As an alternative approach to increase overall quality after finding a MMAX solution, we propose to use a MMSE criterion for the remaining bit-budget. We denote this criterion MMAX+, because it adds additional targets to the MMAX criterion. Note that a MMSE criterion is used to break the tie among several MMAX solutions in a bit-budget constrained problem. However, in that work there is no additional bit-budget to be reallocated.

Both MMAX+ and MLEX will increase the average quality with respect to the MMAX solution (assuming an additional bit-budget is available). However, since MMAX+ explicitly targets average distortion it will lead to better average MSE than MLEX.

We have also proposed the MMAX+ criterion for video transmission over a VBR channel with token bucket policing. In this paper, we focus on video transmission over a CBR channel with a discrete set of quantizers available to code each frame. First we develop two algorithms to find the optimal MMAX solution. We introduce MMAX and MMAX+ criteria in this buffer-constrained problem, so that the best minimum quality of all frames is provided by the MMAX criterion and good overall quality is achieved by the MMAX+ criterion. We also propose an algorithm to reduce the complexity of finding the MMAX+ solution. Simulation results show that the solution of our proposed method gives almost same average quality as the MMSE solution and much better minimum quality, with lower complexity.

This paper is organized as follows: in section 2, algorithms to find the optimal solution under a MMAX criterion are presented. In section 3, algorithms to find the optimal MMAX+ solution with reduced complexity are proposed. Experimental results are provided in section 4. Conclusions are provided in section 5.

2. OPTIMAL RATE CONTROL IN A MMAX CRITERION

Video transmission is constrained by the maximum delay allowable, the encoder and decoder buffers and channel constraints such as channel rate and channel policing functions. In the CBR transmission case, it is possible to prevent the decoder buffer from underflowing (or overflowing) by keeping the encoder buffer from underflowing (or overflowing). Therefore, the constraints of this problem are transmission rate \( C \) and an encoder buffer size \( B \) since the delay is determined by \( C \) and \( B \) in a CBR case. We assume that one frame is coded every \( T \) seconds and immediately moved to the encoder buffer after encoding. Then the problem we are trying to solve using a MMAX criterion can be formulated as

\[
\min \{ \max(D_i) \} \quad \text{s.t.} \quad B_i \leq B \text{ for all } i, \tag{1}
\]

where \( D_i \) is the distortion of the \( i \)th frame \( (1 \leq i \leq S, S \text{ is the number of frames}) \), \( B_i \) is the buffer occupancy after the encoded \( i \)th frame is moved to the buffer. Since the data in the buffer cannot be negative, before moving the \( i \)th frame data, the buffer is zero even if it is in underflow. Therefore \( B_i \) can be determined as

\[
B_i = \begin{cases} 
B_{i-1} + R_i - C \cdot T, & \text{if } B_{i-1} \geq C \cdot T, \\
R_i, & \text{otherwise}, 
\end{cases} \tag{2}
\]
where $R_i$ is the bit-budget of the $i^{th}$ frame. In a given frame $i$, the $(R_i, D_i)$ pairs are determined by the selection of a quantization level $q_i$ ($1 \leq q_i \leq Q_i$). The algorithm to find the optimal MMAX solution can then be defined as follows:

**Algorithm 1:** Optimal bit allocation in a CBR channel with buffer constraints under a MMAX criterion

- **Step 0:** Initialize buffer occupancy by quantizing all frames with the coarsest quantization available to each frame.
- **Step 1:** Find the frame that has maximum distortion and decrease the quantization step size of that frame.
- **Step 2:** If the buffer is not in overflow then go to Step 1, otherwise STOP. The frame that has maximum distortion is the frame whose quantization changed just before buffer overflow. Obviously, the maximum distortion is the distortion of that frame without the final quantization change.

The bisection algorithm can also be applied to this problem. Note, however, that this algorithm may not be terminated since it is based on the bisection of distortions, and distortions can take arbitrary positive real values. This algorithm can be modified by pre-sorting all possible R-D data of all frames by a non-increasing distortion order. After sorting the data, a bisection method is applied to sorted indices in order to find the optimal solution. This modified algorithm can be described as follows:

**Algorithm 2:** Optimal bit-allocation in a CBR channel with buffer constraints under a MMAX criterion: Pre-sorting and Bisection

- **Step 0:** Sort all R-D data of all frames in a non-increasing distortion order, where the data in the sorted array are rates, distortions, quantization and frame numbers. Initialize buffer occupancy by quantizing all frames with the coarsest quantization available to each frame. Set this choice as a solution. Set the largest index of chosen quantization of all frames in the sorted array as $Init$. Set $Mid$ to $[(Init + Max)/2]$, where $Max$ is the maximum index of the array.
- **Step 1:** For each frame, choose the quantizer that has the lowest distortion among all the quantizers for that frame having an index lower than or equal to $Mid$.
- **Step 2:** If the buffer is not in overflow then update the solution and let $Mid = [(Mid + Max)/2]$ else let $Mid = [(Init + Mid)/2]$. If $Mid$ is not changed then STOP, otherwise go to Step 1.

Under the assumption that rate and distortion of all quantization levels are pre-calculated, in Algorithm 1 Step 0 needs $S$ selections, Step 1 needs $\log S$ comparisons to find the frame that has maximum distortion and $S$ comparisons are needed to check buffer overflow in Step 2. Since the number of iterations is at most SQ where $Q$ is the maximum of $Q_i$, the complexity of Algorithm 1 is $O(S^2Q)$. But since the algorithm is terminated when it cannot improve the maximum distortion, in general, the complexity is much lower than this bound. In Algorithm 2, the complexity of merge-sorting is $SQ \log S$ since the data of each frame are already sorted (see
Fig. 2. Examples of computation of the effective buffer size (EBS) of different frames. The solid line represents the buffer occupancy of a MMAX solution. The height of the gray box is the EBS of the given frame and dashed lines show that the determined EBS does not induce buffer overflow. The EBS of frame “a” is determined by the residual buffer of the frame and that of frame “b” is determined by the residual buffer of a following frame. For frames “a” and “b”, the EBS is determined by the minimum residual buffer size of the current and following frames. The EBS of frame “c” is determined by the sum of the amount of underflow and the EBS of a frame after underflow. The EBS of frame “d” is determined by the residual buffer size of the frame because it is smaller than the EBS of the following frame.

Fig. 1). In Step 1, complexity of the entire iteration is at most $SQ$ since in each iteration already checked data do not need to be checked again. The number of iterations is at most $\log SQ$ and Step 2 needs $O(S)$ comparisons in each iteration, so that total complexity of Step 2 is $O(S \log SQ)$. Therefore the complexity of Algorithm 2 is $O(SQ \log S)$ whereas the complexity of the MMSE algorithm of this problem is $O(BSQ)$. Since in a video application, $B$ is relatively large, the complexity of the MMAX algorithm can be much lower than that of the MMSE algorithm.

The optimal MMAX solution may result in buffer underflow, especially in the case when several easily compressed frames are coded successively. As shown in (2), if the data in the buffer is smaller than the available channels of the frame interval then the buffer is in underflow and the channel is not fully utilized. Because underflow occurs at the encoder, we can use stuffing bits to prevent any problem. In this paper, we propose to use this “spare” bit-budget due to underflow in order to decrease the mean square error (MSE). We term this the MMAX+ approach as the MMAX solution is improved upon with an additional MSE criterion. MSE’s limitations are well known but, obviously, our MMAX+ technique could also be used with alternative additive distortion metrics.

3. OPTIMAL RATE CONTROL IN A MMAX+ CRITERION

After finding the MMAX solution, the problem we are trying to solve using a MMAX+ criterion can be formulated as

$$\min_{q_i} \left( \sum_{i=1}^{S} D_i \right) \text{ s.t. } B_i^M \leq B_i \leq B \text{ for all } i,$$  \hspace{1cm} (3)

where $B_i^M$ is the buffer state at the $i^{th}$ frame time when the MMAX solution is used. Note that we take the MMAX solution as the initial condition and we never reduce the bit allocation to a frame chosen by the MMAX approach, i.e., the additional step we propose can only increase the number of bits used.

It is important to note that the upper bound constraint in (3) is not tight. This is because the trace $B_i^M$ already incorporates the effect of transmitted bits. Thus any increase to $B_i$ over $B_i^M$ leads (if there was no buffer
underflow) to an increase in $B_i$ for $i' > i$, so that the overflow constraint could be violated for $i'$, even if it is not for $i$ (see Fig. 2 (b)). To reduce the upper bound of the buffer state of a frame, we introduce the concept of effective buffer size (EBS), where the EBS of a frame is the maximum bit-budget that can be used to increase the quality of the frame and such that no overflow occurs.

Obviously $EBS_i$ (the EBS of the $i^{th}$ frame) is smaller than or equal to the residual buffer size after selecting a quantizer according to the MMAX solution of the frame ($RBS_i$), the difference between the maximum buffer size and the buffer state of the MMAX solution (i.e., $RBS_i = B - B_i^M$), which varies from frame to frame. $RBS_i$ can also be explained as the maximum amount of data we can add for the $i^{th}$ frame without producing a violation of the delay constraint. Examples of computation of the EBS are shown in Fig. 2. In the figure, the EBS of frame “b” is determined by the minimum RBS of all frames from “b” onwards. This is because additional bits used at “b” will increase the buffer occupancy of all following frames (dashed lines in Fig. 2 (b)). However, if the buffer is in underflow at the $i^{th}$ frame interval (where the $i^{th}$ frame interval means the interval between the $i^{th}$ frame and the next frame) then the amount of underflow ($UF_i$, where $UF_i = \max(C \cdot T - B_i, 0)$) can also be added to the bit-budget of the $i^{th}$ frame without affecting the buffer state of future frames (See Fig. 2 (c)). In this example, given that $EBS_{i+1}$ is known, $EBS_i$ is computed by $EBS_i = \min(RBS_i, UF_i + EBS_{i+1})$. The EBS for a frame can be formed based on the following theorem.

**Theorem 1:** The EBS for a frame can be formed as

$$EBS_i = \begin{cases} RBS_i & : i \text{ is the last frame}, \\ \min(RBS_i, UF_i + EBS_{i+1}) & : \text{otherwise}. \end{cases}$$

Proof) At first, we consider the last frame. Since the buffer state is causal, increasing the rate for a frame only affects the buffer state of the current and future frames. Since no future frame exists, the EBS of the current frame is restricted by the buffer overflow of the current frame and it is determined by the RBS of the current frame (i.e., $EBS_S = RBS_S$.)

Next, we use the induction method to prove remaining parts. Since increasing the rate for a frame does not affect the buffer state of the previous frames, the solution is to choose the maximum bit-budget that does not result in buffer overflow at the current and future frames. For any $i$ ($1 \leq i \leq S - 1$), assume $EBS_{i+1}$ is known. Then $EBS_{i+1}$ guarantees no buffer overflow in all future frames. If the buffer is in underflow at the $i^{th}$ frame interval then the rate can be increased by the amount of underflow without changing the buffer state of the future frames. So in order to consider the overflow of future frames only, the solution is the sum of the amount of underflow at the $i^{th}$ frame interval ($UF_i$) and $EBS_{i+1}$. Since the rate can be increased at most the remaining buffer size of the $i^{th}$ frame ($RBS_i$), $EBS_i$ is determined by (4) ■

Therefore, the EBS is recursively computed from the last frame by using the equation (4). After computing the EBS for all frames, the problem in a MMAX+ criterion is redefined as

$$\min_{q_i} \left( \sum_{i=1}^{S} D_i \right) \text{ s.t. } B_i^M \leq B_i \leq EBS_i + B_i^M \text{ for all } i.$$

This new formulation now guarantees that increasing $B_i$ does not lead to overflow. The allowable quantization levels of the $i^{th}$ frame ($q_i$) are also reduced to ($q_i^M \leq q_i \leq Q_i^l$), where $q_i^M$ and $Q_i^l$ are determined by the MMAX solution and the upper bound of $B_i$ for each.

This rate control problem can be solved by using a dynamic programming method or a Lagrangian optimization method. Given the buffer constraints due to our goal to preserve the MMAX solution, the number of states in a dynamic programming method can be reduced significantly by computing the EBS. Fig. 4 shows an example of the range of the states. As mentioned in the previous section, the complexity of the MMSE algorithm is proportional to $B$ and $Q$. Since the EBS and the corresponding number of allowable quantization levels are
Figure 3. Comparison of experimental results of optimal solutions in different criteria. Used channel rate is 10 Mbps (i.e., 5 Mbits per a GOP interval) and the size of an encoder buffer is 20 Mbits. Therefore the maximum delay is 4 GOP intervals. Initial and final buffer states are at mid-buffer. (a) and (b) show the PSNR and bit-rate of each GOP respectively.

much smaller than $B$ and $Q$, the complexity of the MMAX+ algorithm is much lower than that of the MMSE algorithm.

4. EXPERIMENTAL RESULTS AND DISCUSSION

In order to verify the performance of the proposed algorithm, we implement this algorithm and test it with 1800 frames from the “Fire Birds” movie sequence. We use the Group of Pictures (GOPs) in MPEG as the basic data unit and the “closed GOP” option in MPEG2 is used to code each GOP independently. To gather R-D data of GOPs, each GOP is coded by using 159 different rates roughly between 1 Mbits and 13.64 Mbits (the difference between steps is roughly 80 Kbits.) Total sequence has 120 GOPs (each GOP has 15 frames,) and each GOP is coded by using an MPEG2 TM5 encoder.

Fig. 3 shows the optimal solution of MMAX, MMAX+ and MMSE criteria. As shown in the figure (a), the PSNR of MMAX+ is always higher than or equal to the PSNR of MMAX. Also the figure shows that the bit-rate fluctuation among GOPs of the MMSE solution is similar to that of the MMAX and MMAX+ solutions whereas the PSNR fluctuation among GOPs are much larger.

Fig. 4 shows the EBS of each GOP. Note that in the figure, the buffer state of MMAX is always positive since it includes the new coming data ($R$) at each GOP time. In Fig. 4, the EBS of the GOPs between 40 and 110 is zero and this means we cannot increase bit-rate of these GOPs (otherwise encoder buffer is overflowed after moving the 110th GOP data into the buffer.) This also explains the reason that the PSNR of the MMAX and MMAX+ solutions of the GOPs between 40 and 110 is identical in Fig. 3 (a).

To compare the performance, we also developed the algorithm to find the optimal MLEX solution by changing Algorithm 1 slightly (i.e., after finding a MMAX solution, instead of terminating the algorithm, keeping the iteration to minimize the 2nd largest distortion and then to minimize the following largest distortion until any distortion cannot be lowered).

Table 1 shows the experimental results for each criterion. As expected, the minimum PSNR of the MMAX solution is higher than that of the MMSE solution. The MMAX+ criterion improves the average PSNR around 0.4 dB, achieving a value that is near the average PSNR of the MMSE solution. The standard deviation of the MMAX solution shows that the PSNR of each GOP is very similar but the maximum PSNR is relatively high.
Figure 4. Encoder buffer state and effective buffer size. The solid line indicates the buffer state of the MMAX solution and the vertical distance between the dashed and solid lines indicate the effective buffer size of each frame.

Table 1. Performance (PSNR) comparison of proposed MMAX and MMAX+, MMSE and MLEX optimal solutions. The constraints used are same as those in Fig. 3.

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<tbody>
<tr>
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<td>39.44</td>
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<td>0.494</td>
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Table 2. Performance (PSNR) comparison in different maximum delay. The number in the “Method” column indicates maximum delay in GOP interval units. Therefore the sizes of encoder buffers are 40 Mbits and 10 Mbits for each. Initial and final buffer states are at mid-buffer (i.e., 20 Mbits and 5 Mbits for each).

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<tr>
<td>MMAX (8)</td>
<td>38.33</td>
<td>0.120</td>
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The reason for this is that some GOPs have simple content and so the PSNR of these GOPs at minimum rate (1Mb/s) determines the maximum PSNR (see the PSNR of the 9th frame in Fig. 3(a)). Although the complexity to find the MMAX+ solution highly depends on the buffer state of the MMAX solution, in this experiment, the complexity of the MMAX+ algorithm is roughly 10 times lower than that of the MMSE algorithm.

In Table 2, the performance of each method is compared for different values of the maximum delay, or equivalently, different sizes of the encoder buffer. The results show that minimum PSNR of the MMAX solution and average PSNR of the MMSE solution are increased (decreased) and the difference of average PSNR between MMAX+ and MMSE solutions is increased (decreased) as maximum delay is increased (decreased). Because larger buffer size (with increased delay) means that the problem is not as constrained, we can find a better solution (better average PSNR) in a MMSE criterion. Also, if the encoder buffer size is increased then local fluctuation of the bit-rate of GOPs can be absorbed and buffer is not easily overflowed by consecutive complex GOPs. Therefore the minimum PSNR of the MMAX solution is increased and the additional bit-budget available due to the buffer underflow is decreased. Since the bit-budget for the MMAX+ (or MLEX) solution is decreased, the average PSNR cannot be improved much (and so the average distortion of the MMAX+ and MLEX solutions is almost same when maximum delay is 8). In the other case, if the encoder buffer size is reduced, it is more likely that buffer overflow can occur (i.e., minimum PSNR in MMAX is decreased) and the remaining bit-budget due to buffer underflow is increased (i.e., average PSNR in MMAX+ is increased) and the difference of the average PSNR between MMAX+ and MLEX is increased.). As a result, the performance of a MMAX criterion is improved (i.e., more bit-budget is used for a MMAX solution) as maximum delay is increased, and therefore the benefits of a MMAX+ criterion are not as significant in that case.

5. CONCLUSIONS

In this paper, we developed the optimal bit allocation algorithm of CBR transmission in MMAX and MMAX+ criteria. The MMAX+ criterion is introduced to improve total quality by using the remaining channel bandwidth under the MMAX criterion. Also an algorithm for finding the effective buffer size is proposed. The effective buffer size is used to reduce the number of possible states of each frame and as a result, the complexity of the algorithm to find the optimal MMAX+ solution is reduced.

REFERENCES