Rate Control Using Spline-Interpolated R-D Characteristics

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ABSTRACT

Digital video's increased popularity has been driven large to a extent by a flurry of recently proposed international standards. In most standards, the rate control scheme, which plays an important role for improving and stabilizing the decoding and play-back quality, is usually not defined. Several techniques have been proposed to aim at the best possible quality for a given channel rate and buffer size. These approaches are complex in that they require the R-D characteristics of the input data to be measured. In this paper, we propose a method to approximate the rate and distortion functions to reduce the complexity of the optimization procedures while making a minimal number of a priori assumptions on the source data. In the proposed method, the R-D of image frames is approximated by spline interpolation functions, and inter-frame dependency (for P or B frames in MPEG) are modeled by a linear-constant function. The application to gradient-based rate-control scheme for MPEG shows that, for a typical MPEG encoder, by using the proposed model, the same performance can be achieved with only about 10 to 15 percent of computation cost.

Keywords : MPEG, video coding, buffer control, gradient search, Lagrange multipliers, spline approximations

1 INTRODUCTION

Digital video's increased popularity has been driven large to an extent by a flurry of recently proposed international standards. For example, MPEG [1] is already being used for digital video satellite broadcast and will be soon used in high-density Video-CDs. Another standard, H.263, is being targeted for low bit rate transmission over ISDN or phone lines and may soon replace H.261 as the preferred standard for videoconferencing. These standards, along with other video compressing schemes, share many common components, such as block transforms, macroblock structure, and motion compensated prediction.

However these standards only define the bitstream syntax and do not specify such essential components of the encoders as rate control and bit allocation. Bit allocation is the problem of selecting among a discrete set of possible choices which quantizers and modes of operation to use for each of the blocks in a frame or set of frames. The goal is to select a setting that provides good quality while not exceeding the global rate assignment for the frame. Rate control is concerned with a related problem, where in addition to optimizing quality the goal is to prevent the output buffer of the encoder from overflowing. Rate control scheme plays an important role for improving and stabilizing the decoding and play-back quality.

Many rate control schemes for constant-bit-rate encoding, such as the one in MPEG Test Models [2], use the buffer occupancy to determine the quantization setting. These approaches only take into account the rate, not the distortion, in the algorithm. Recently, there has been a growing interest in rate-distortion (R-D) optimal techniques for both bit allocation and rate control. There are several possible frameworks in which to optimize the performance of the rate control algorithm. A popular approach is to use models of the future frame's rate (and sometimes also distortion) and use control techniques to avoid overflow [3,4]. A second alternative is to *measure* the rate and distortion on the frames themselves, thus increasing the required optimization complexity but eliminating the dependency of the results on the choice of a good model. Examples of this approach can be found in [5–8] where techniques like Lagrangian optimization and dynamic programming have been used. These approaches are complex in that they require the rate and distortion characteristics of the input data to be measured. However they are well suited for environments, as those encountered in video coding, where a discrete set of operating points is available and where it may not be easy to find adequate "continuous" models for the data. In this paper our motivation is to approximate the rate and distortion functions to reduce the complexity of the optimization procedures, while making a minimal number of *a priori* assumptions on the source data.

Previous work on rate and distortion modeling has been based to a large extent on the exponential statistics model. For example, in [9,10], exponential expressions were used to model the relationship between the rate, distortion, and quantization step size in a macroblock. These types of models require low computation overhead since they are obtained based on parameters such as block variance which can be easily obtained from the input frames. However, these are continuous models which tend to be better when a large number of quantizers is used. Thus they may suffer from large errors because of the difficulty in modeling the highly nonlinear quantization and entropy coding process. In addition, these models do not take into account the dependencies that arise in the choice of quantizers for the reference frames and the predicted frames [6]. Even when these models take the dependencies into account, as in [11], they ignore some non-linear effects that are typical in video coding. For example, under the general intra/inter selection rule, there is no dependency if the quality of the reference frame is too low. Although data estimated through these models might be useful in the bit allocation stage of a buffer-state feedback rate control algorithm such as that in [2], the accuracy is not good enough for the optimal buffer control algorithm in [5,8].

In this paper we study models that are better suited to rate-distortion optimization in realistic video coding scenarios. We will focus on the two improvements motivated above, namely, we provide models that (i) make relatively few assumptions on the shape of the R-D characteristics and are thus suited when operating with a small number of quantizers and (ii) take into account the dependencies typical of video coding. These models are based on computing a few R-D points and interpolating the remaining points using spline functions. The price to pay for the increased accuracy is a somewhat higher complexity. The paper is organized as follows. In Section 2, we describe the formulation of the spline interpolation function and apply this function to optimal adaptive quantization for image compression. In Section 3, we present a scheme to model the frame dependencies for P and B frames. In Section 4, we apply the model to the MPEG video compression and present several experimental results. Finally, conclusions and future perspectives on this technique are given in Section 5.

2 SPLINE INTERPOLATED R-D AND IMAGE COMPRESSION

In typical DCT-based compression the rate-distortion trade-off is controlled by a quantization scale. This parameters is used to compute the step size of the uniform quantizers used for the different DCT coefficients (see [12] for details). When an image block (or an entire image frame if constant quantization is used) is quantized and encoded with a specific quantization scale, q, the rate (the number of bits generated by the coder), $r(q)$, and the distortion (here the MSE is used), $d(q)$, can be calculated. Most of the computation cost in optimum rate control algorithms such as those in [5,6,13] comes from the computation of $r(q)$ and $d(q)$ for all applicable values of q[1]. Therefore, the computational cost can be reduced significantly if these two function values can be correctly estimated before actually quantizing and encoding the source data. However, due to the complex nonlinear properties of the quantization and entropy coding processes, it is difficult to predict the function value accurately enough by using simple mathematical expressions. In this paper, we propose an approach which calls for encoding the data and measuring the R-D functions, but *only on a small set of quantization scales* which we call "control points". Piece-wise polynomials, or splines, are then used to interpolate the function for other q's where the actual data has not been measured.

2.1 Formulation of Spline Interpolation Function

Because the rate and distortion functions are to be used in an optimization algorithm (gradient search, Lagrangian, etc.), the first-order derivative of these functions should be well-defined. A good candidate for the interpolation function would be the "interpolating cubic-spline", which possesses the second-order continuous property [14]. One disadvantage of this method is that the interpolation polynomials for any given segment (a segment is defined as a set of points between the two consecutive control points) depends on all the control points, i.e., it will require the coder to encode the source on all the control points even though only a small portion of the function data is required in the rate control algorithm. In this paper, we use another type of spline, which requires smaller computation cost, still possesses first-order continuity, and for which each segment depends only on four nearest control points.

We assume the control points are defined as (x_i, y_i), $i = 0 \dots M-1$, where M is total number of control points. Fig. 1 shows an example set of control points, where x_i represents the quantization scale (for MPEG, the applicable values are $\{1, 2, \dots, 31\}$), and y_i represents the actual measured rate or distortion. The function between two consecutive control points, x_i and x_{i+1}, is defined as

$$f_i(x) = a_i \cdot x^3 + b_i \cdot x^2 + c_i \cdot x + d_i \tag{1}$$

where $i = 0 \dots M-2$. There are $M-1$ polynomials, each corresponding to one segment. For each polynomial, the four parameters, a_i, b_i, c_i, d_i, can be derived from the four control points, (x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2}), by imposing the following two constrains:

1. The interpolated function should take the same values as the original one at the control points, hence:

$$x_i^3 \cdot a_i + x_i^2 \cdot b_i + x_i \cdot c_i + d_i = y_i \tag{2}$$

$$x_{i+1}^3 \cdot a_i + x_{i+1}^2 \cdot b_i + x_{i+1} \cdot c_i + d_i = y_{i+1} \tag{3}$$

2. The first-order derivative should be continuous on the control points. This condition can be achieved by defining the slope at control point x_i as (the first derivative of $f(x)$ is denoted as $f'(x)$):

$$f_i'(x_i) = f_{i-1}'(x_i) = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \tag{4}$$

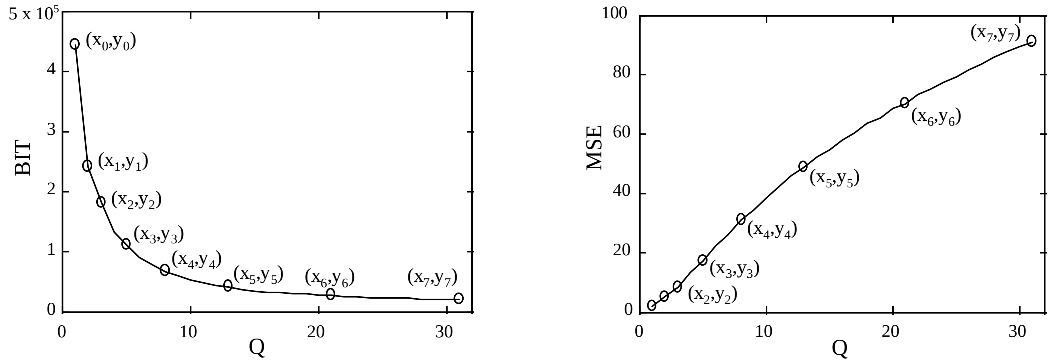
   By taking the derivative of (1) and substituting into (4) on the two end points of $f_i(x)$, we get

$$3x_i^2 \cdot a_i + 2x_i \cdot b_i + c_i = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \tag{5}$$

$$3x_{i+1}^2 \cdot a_i + 2x_{i+1} \cdot b_i + c_i = \frac{y_{i+2} - y_i}{x_{i+2} - x_i} \tag{6}$$

The four unknowns of $f_i(x)$, a_i, b_i, c_i, d_i, can be readily found from the set of equations (2), (3), (5), and (6).

[1] It has to be noted, though, that the complexity of computing the R-D data for n q for a given frame is not n times the complexity of encoding the frame. Indeed, the DCT itself only has to be computed once.



(a) Bits (b) MSE

Figure 1: Control points for typical (a) rate and (b) distortion curves. In this figure, a control point (x_i, y_i) represents that if the quantization scale is set to x_i, the measured rate or distortion value is y_i.

In order to capture the exponential-decay property, which is typically observed in R-D data[2], we choose the control points to be with the relation as $x_i = x_{i-1} + x_{i-2}$, so that, in the MPEG case, the set of eight control points becomes $\{1, 2, 3, 5, 8, 13, 21, 31\}$. However, on a typical video sequence at standard rate (e.g. CIF at 1.152 Mbps), the settings for $q = 1, 2$, or even $q = 3, 4$, are rarely used, hence, only 5 to 6 control points are required in most cases.

2.2 Application to Adaptive Quantization in DCT Image Compression

By applying the spline approximation model at DCT block level, we are able to reduce the computational load in searching for optimal adaptive quantization in DCT-based image compression. Suppose there are N blocks in an image, and for each block the rate and distortion for a given quantization setting are denoted as $r_i(q_i)$ and $d_i(q_i)$, where i is the index for block, and there are a finite number of available quantization settings. The optimum adaptive quantization problem is to determine quantization scales for all blocks $(q_0, q_1, \ldots, q_{N-1})$, such that the overall distortion is minimized:

$$\min_{(q_0, q_1, \ldots, q_{N-1})} \sum_{i=0}^{N-1} d_i(q_i) \tag{7}$$

subject to the following rate constrains (total bit-budget is R):

$$\sum_{i=0}^{N-1} r_i(q_i) \le R \tag{8}$$

The problem can be solved by the method of Lagrange multipliers, by repeatedly solving the following set of unconstrained problems for given λ's,

$$\min_{q_i} [d_i(q_i) + \lambda r_i(q_i)], \quad i = 0 \ldots N-1 \tag{9}$$

and search for λ, such that the constraint (8) is satisfied. This can be done efficiently by using the fast search method proposed in [15]. Note that the overhead for coding the quantization scale for each block can be ignored in the above formulation (but is included in counting the total bits). That overhead can be taken into account by using the techniques proposed in [13].

[2] Note that while approximately exponential characteristics are typical, the error incurred with our approach will normally be smaller, because we have more degrees of freedom, and the characteristics are not exactly exponential.

2.3 Simulations for the Adaptive Quantization

In this part, we encode the 512×512 grayscale Lena image using a modified JPEG encoder. The modification was made such that a quantization scale can be assigned for each DCT block as is done in MPEG. To test the effectiveness of the spline approximation model, we replace $r_i(q_i)$ and $d_i(q_i)$ by the approximated data, and run the adaptive quantization procedure. The results are shown in Table 1 and Fig. 2. We conclude from the results that, (i) the spline approximated model produces a much smoother R-D curve, which may have potential to be used to reduce the complexity of the search procedure in optimization algorithm, (ii) the result using approximated R-D is close to the one using actual R-D, and (iii) the constraint (8) may not able to be strictly satisfied due to the error in the approximated rate. In practice, this will not be a problem because the errors are typically small and the variations can be absorbed through buffering. By using the spline model, the computation complexity for the evaluating R-Ds is reduced to about 20% (6 control points instead of 31 settings). Note also that many blocks have R-D characteristics similar to that of Fig. 2(a) and the lack of smoothness in the shape makes our approach based on several control points more effective than exponential based models.

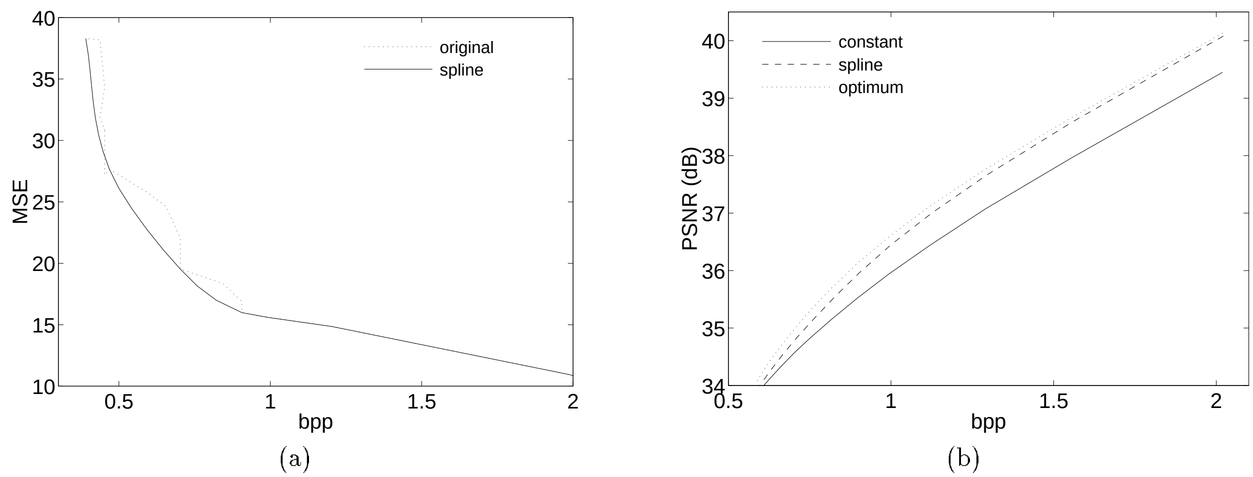


Figure 2: (a) Comparison between the original and spline interpolated rate-distortion function of a DCT block in the lena image, to illustrate the lack of smoothness in the original R-D characteristics. (b) PSNR curves of constant quantization, adaptive quantization using original data, and adaptive quantization using spline approximated data.

constant q.			a.q. with original R-D		a.q. with spline R-D	
q	bpp	PSNR	bits overflow	PSNR	bits overflow	PSNR
3	1.554	37.96	-34	38.66	1223	38.59
6	0.993	35.94	-17	36.58	2290	36.46
9	0.755	34.84	-2	35.32	2358	35.16
12	0.621	34.08	0	34.37	2095	34.26

Table 1: Adaptive quantization encoding of Lena image. *constant q.*: constant quantization; *a.q. with original R-D*: adaptive quantization using the original R-D; *a.q. with spline R-D*: adaptive quantization using spline approximated R-D. The *bits overflow* is the difference in bits between the actual number of bits generated and the bit-budget.

3 INTER-FRAME DEPENDENCY MODEL FOR VIDEO

To apply the approximation model to video coding, we keep the quantization scale, q, constant over an entire frame, and use the spline function defined in previous section to approximate the frame-level rate and distortion as functions of q. We also make the motion estimation refer to the original reference frame, so that it only has to be computed once for each of the P and B frames, and does not have to be recomputed when the P-B frames are encoded to sample their respective R-D functions at the control points. For the I frames, the approach of the previous section can be directly applied. For the P and B frames, the R-D characteristics depend on the quality of their reference frames and we have thus to deal with multi-dimensional functions. We will now introduce methods that are less complex than full-blown multi-dimenional models while still capturing the inter-frame dependencies. Note that the ideas presented in this section are introduced in an MPEG framework, but are applicable to more general video coding enviroments.

3.1 Formulation of Inter-frame Dependency

We consider the first P frame in a GOP, and its reference I frame[3]. Because of the dependency, the rate and distortion functions become two-dimensional, i.e., they have the form $d(q_I, q_P)$ and $r(q_I, q_P)$, where q_I and q_P are the quantization scales for the I and P frame respectively. Now, the data has to be sampled in the two-dimensional space. One straightforward extension is to sample the data at the same 6 control points for each dimension (total 36 control points), but this requires many more computations. This is because, in order to compute the data for each additional control points along the q_I axis, the I frame has to be re-compressed and reconstructed again (involving DCT, quantization, de-quantization, and IDCT), and the P frame has to be re-encoded (involving prediction, DCT, quantization, and encoding). This complexity is much higher than the one for computing the data along the q_P axis (only involving quantization and encoding for the P frame). In this section, we introduce a model for inter-frame dependency which only requires two control points along the q_I axis.

Consider the fact that the rate-distortion characteristic of the predictive frame (P or B) depends on the quality of its reference frame(s). When the reference frame has smaller MSE, the prediction residue tends to be smaller, which results in a smaller rate and distortion in the predictive frame. Conversely, if the MSE in the reference is larger, not only the rate and distortion of the predictive frame will become larger, but also more macroblocks will be coded as "intra-block" (given the typical decision rules used in general MPEG encoders, e.g. in [16]), which will decrease the dependency on the reference frame. After some point, the predicted frame will be completely independent of the reference frame (see Fig. 3).

Suppose q_P fixed at a constant C, so that $d(q_I, q_P = C)$ becomes a one-dimensional function with variable q_I. The MSE of the reference frame (I-frame) is denoted as $d_I(q_I)$. Based on the above observation, the frame dependency for the distortion of a P frame is modeled as a linear increasing function with respect to $d_I(q_I)$ for $q_I \leq C$, and becomes a constant function for $q_I > C$, as shown in the following expression:

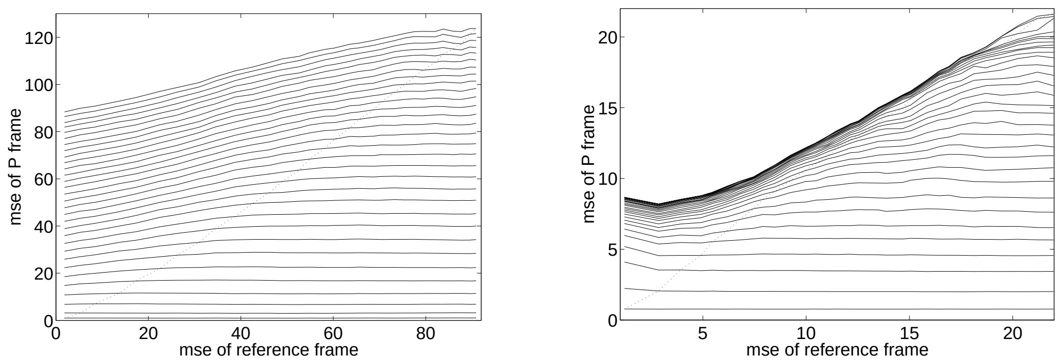
$$d(q_I, C) = \begin{cases} \alpha - \beta \cdot [d_I(C) - d_I(q_I)] & \text{if } q_I \leq C \\ \alpha & \text{if } q_I > C \end{cases} \tag{10}$$

where q_I is the only variable in the model. The two model parameters, α and β, can be determined by encoding and measuring the distortion at two values of q_I. For example, if the two values are chosen to be 5 and 13, and the same spline model with 6 control points (as in Section 2) is used along q_P axis, the set of 12 control points becomes:

$$\left\{ \begin{array}{cccccc} (5,3) & (5,5) & (5,8) & (5,13) & (5,21) & (5,31) \\ (13,3) & (13,5) & (13,8) & (13,13) & (13,21) & (13,31) \end{array} \right\} \tag{11}$$

To interpolate the function value for any given settings, say $(10, 10)$, the above interframe model is applied 4 times with C set to $\{5, 8, 13, 12\}$, so that the function values are derived at $(10, 5)$, $(10, 8)$, $(10, 13)$ $(10, 21)$. Then

[3] Note that the model still can be applied when the reference is another P frame.



(a) Football sequence (b) Miss America sequence

Figure 3: MSE for the P frames from two video sequences, plot as function of MSE for their reference frames. Each solid line is a MSE curve for a given q in the predictive frame. The dotted line indicates the boundary where q for the predictive and reference frames are equal.

spline interpolation function is used to derive the value at $(10, 10)$ using the 4 derived data. The entire procedure is illustrated in Fig. 4 (a).

However, due to the difference in properties, a similar model does not work as well for the rate. From several video sequences, we have observed that, for the quantization scales between 3 and 24, the inter-frame dependency for rate is reasonably low. Hence, the following simple piece-wise linear model is used (suppose the two measured points for q_I are x_1 and x_2):

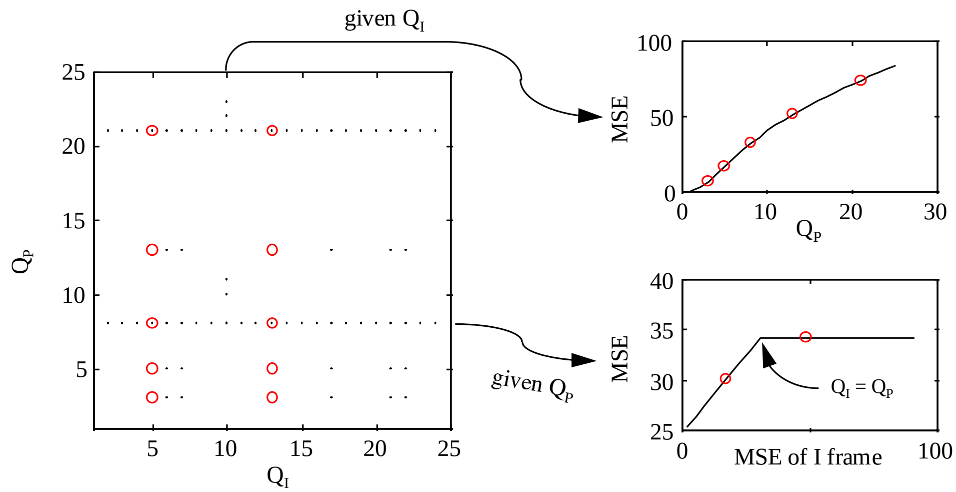
$$r(q_I, C) = \begin{cases} r(x_1, C) & \text{if } q_I \le x_1 \\ \{r(x_1, C)(d_I(x_2) - d_I(q_I)) + r(x_2, C)(d_I(q_I) - d_I(x_1))\}/\{d_I(x_1) - d_I(x_2)\} & \text{if } x_1 < q_I < x_2 \\ r(x_2, C) & \text{if } q_I \ge x_2 \end{cases} \tag{12}$$

For B frames, the MSE function becomes $d(q_I, q_P, q_B)$, where q_B is the quantization scale for the B frame itself, and q_I and q_P are the quantization scales for the two reference frames. To keep the computation simple, we first fix one reference frame by setting $q_I = c$, where c is one of the inter-frame control points, and we evaluate the dependency for the other reference frame by using the same model for P frames and get $d_1(c, q_P, q_B)$. We then fix the other reference frame and derive $d_2(q_I, c, q_B)$. Finally, $d(q_I, q_P, q_B)$ is defined as $\min(d_1(c, q_P, q_B), d_2(q_I, c, q_B))$. This procedure simulates part of the strategy for selecting "forward" or "backward" motion vectors in the MPEG encoder. The same model is also used for the rate function. There are a total of 18 control points to be measured if the same set of control points as in (11) is used. The entire process is illustrated in Fig 4 (b).

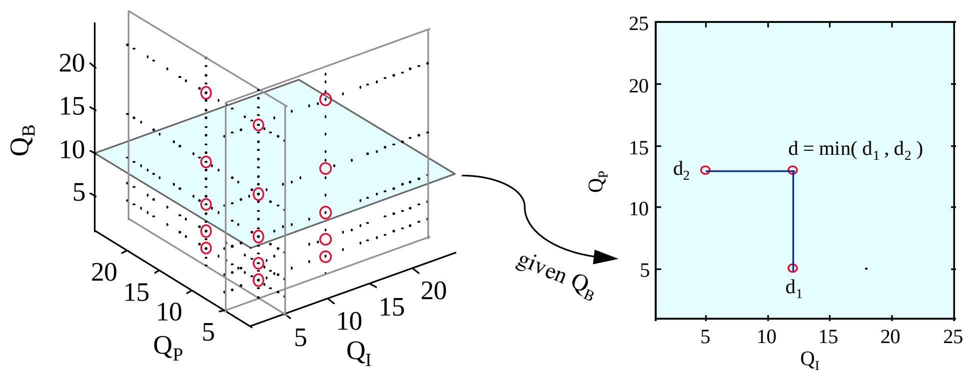
3.2 Model Compliance Test

We use the MPEG-2 encoder implementation of [16] to test the accuracy the approximation model, by the following steps: We first encode the frame, measure and record the MSE and code length, for every possible quantization settings. Based on the function values at the pre-defined control points ($\{1, 2, 3, 5, 8, 13, 21, 31\}$ for intra-coded frame, $\{5, 13\}$ for inter-frame dependency), we build the model using the procedure described in Section 2 and Section 3, and calculate the estimated rate and distortion values. The relative error is then calculated by

$$\text{relative\_error} = \left| \frac{\text{estimated\_value} - \text{original\_value}}{\text{original\_value}} \right| \tag{13}$$



(a)



(b)

Figure 4: Reconstruction of distortion model for (a) P frame and (b) B frame.

For I frame, the average and maximum relative errors are calculated over all the quantization scales. For P and B frames, the average and maximum relative errors are calculated over the typical operating range of quantization scales, which is from 3 to 24. The results are shown in Table 2. The results show relatively small errors for I frame, and also reasonably small for P frame, but it is somewhat large for B frame. Several sample graphical comparisons are shown in Fig. 5.

4 APPLICATION TO THE RATE CONTROL FOR MPEG

In this section, we apply the approximation model to MPEG video encoding. The gradient-based algorithm in [8] is used for the rate control, with the R-D for I frames substituted by the model defined in Section 2, and R-D for P and B frames substituted by the model defined in Section 3.

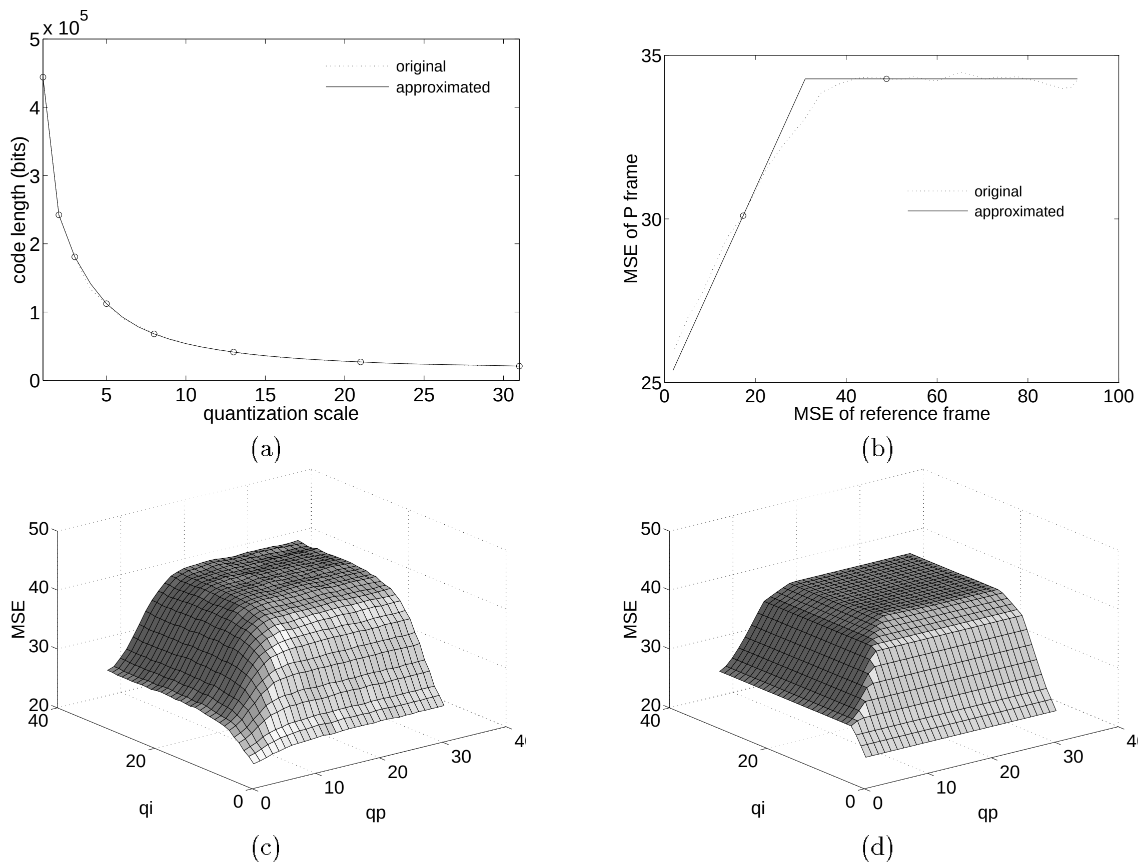


Figure 5: (a) Rate function of an I frame in the football sequence. The circles indicate the control points, which are chosen to capture the exponential-decay property of the rate function. (b) The dotted line is the MSE of a P frame in the football sequence, with respect to the MSE of its reference frame. The quantization scale of the P frame, q_P, is fixed at 8. The curve is approximated by a linear-constant function, indicated by solid line. The circles indicate the two control points, at $q_I = 5$ and $q_I = 13$. The corner point is at $q_I = q_P = 8$. (c) Original measured data and (d) reconstructed with B-frame model, of a B frame in the football sequence, as a function of q_I and q_P, with q_B fixed at 10.

4.1 Gradient-Based Rate-Control Algorithm

The rate-control problem is defined as assigning the quantization scales q_i to the ith frame in a GOP such that the overall quality, measured by a pre-defined cost function (here MSE is used), is optimized. Let $\mathbf{q} \equiv (q_0, q_1, \ldots, q_{N-1})^T$ be the quantization choices for the frames in a GOP. When the quantization scales are set to $\mathbf{q}$, we define the code length and mean square error of frame i as the rate and distortion functions, denoted by $r(i, \mathbf{q})$ and $d(i, \mathbf{q})$, respectively. The buffer occupancy after frame i is coded is then:

$$b(i, \mathbf{q}) = b(i-1, \mathbf{q}) + r(i, \mathbf{q}) - R \tag{14}$$

where R is the channel bit-rate in bits per frame. If $b(i, \mathbf{q})$ is smaller than zero, stuffing bits are padded to avoid underflow and $b(i, \mathbf{q})$ is assigned as zero. The cost function is defined as

$$J(\mathbf{q}) = \sum_{i=0}^{N-1} d(i, \mathbf{q}) \tag{15}$$

	I frames				P frames			
	MSE		BITS		MSE		BITS	
	avgerr	maxerr	avgerr	maxerr	avgerr	maxerr	avgerr	maxerr
Football	0.88%	7.01%	1.04%	6.32%	0.39%	6.60%	0.66%	8.41%
Claire	0.83%	4.37%	0.34%	3.28%	0.88%	12.30%	2.49%	33.02%
Susie	1.08%	6.10%	0.89%	6.11%	1.24%	15.88%	2.92%	15.88%
Miss America	0.95%	3.84%	0.65%	7.18%	0.89%	11.03%	3.27%	45.82%

	Football B frames			
	MSE		BITS	
	avgerr	maxerr	avgerr	maxerr
B1	2.28%	17.56%	2.89%	17.78%
B2	2.29%	14.73%	3.15%	19.65%

Table 2: Relative errors. For I frames, the statistic is over the entire quantization scale range. For P and B frame, it is calculated over the range from 3 to 24. (*avgerr*: average error, *maxerr*: maximum error.)

The problem can now be formulated as that of finding $\mathbf{q}^\star$ such that $J(\mathbf{q})$ is minimized, subjected to

$$b(i,\mathbf{q}) \leq b_{max}\ , \quad i = 0 \ldots N-2 \quad \text{and} \quad b(N-1,\mathbf{q}) \leq 0 \tag{16}$$

where b_{max} is the prescribed maximum buffer size. Note that we force the final buffer occupancy to be less than or equal to zero. Stuffing bits will be then padded at the end of GOP to ensure all GOPs have the same number of bits. To solve this constrained optimization problem, we convert the constraints of (16) into a set of penalty functions:

$$P_i(\mathbf{q}) \;=\; \max(0, b(i,\mathbf{q}) - b_{max})^2 \quad \text{and} \quad Q(\mathbf{q}) \;=\; \max(0, b(N-1,\mathbf{q}))^2. \tag{17}$$

which are added to the cost $J(\mathbf{q})$:

$$\phi(\mathbf{q}, c) = J(\mathbf{q}) + c\left(\sum_{i=0}^{N-2} P_i(\mathbf{q}) + Q(\mathbf{q})\right), \tag{18}$$

where c determines the amount of the penalty, which is simply set to a relatively large value. And finally, we use steepest descent method to solve the unconstrained problem, by which the negative direction of the gradient vector $\nabla\phi(\mathbf{q})^T$ is used as the search direction, and the vector $\mathbf{q}$ is updated by the following

$$\mathbf{q}_{k+1} = \mathbf{q}_k - \alpha_k \nabla\phi(\mathbf{q}_k)^T \tag{19}$$

where α_k is a nonnegative scalar value obtained by minimizing the function

$$\varphi(\alpha) = \phi\left(\mathbf{q}_k - \alpha\nabla\phi(\mathbf{q}_k)^T\right) \tag{20}$$

using a line search procedure [17]. To apply the above approximation model and speed-up the computations, the functions $r(i,\mathbf{q})$ and $d(i,\mathbf{q})$ are substituted by the approximated data. Because of the model error in rate, the original strictly-constant-rate for GOPs may be no longer satisfied, but we expect the buffer constraints will still be satisfied most of the time because most of the errors is from the B frames, which consume the fewest number of bits.

Consider the fact that the model errors in B frames are relatively large, we can further improve the solution by re-allocating bits for the B frames. First, the I and P frames are encoded using the solution from the model, and then, the total number of bits remains for B frame is calculated. Finally, the bit allocation for B frames is optimized by using the same Lagrange method as in Section 2.2. The Lagrange method can be applied because all the B frames are independent to each other after their reference frames (I and P) are fixed. By using this approach, the solution is improved and the strictly-constant-rate for GOPs are satisfied again.

4.2 MPEG Encoding Results

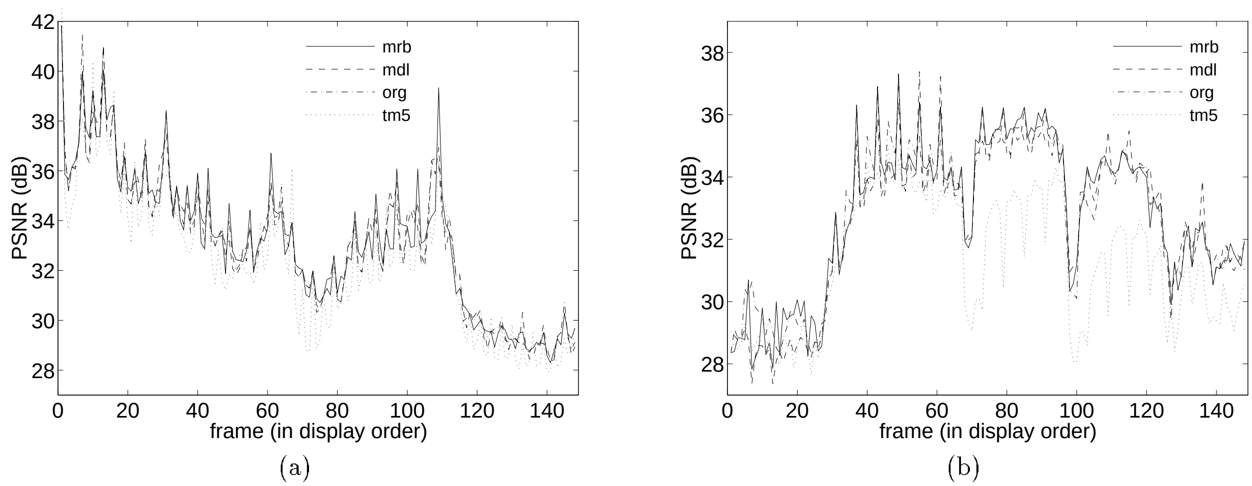


Figure 6: PSNR of image frames for (a) football, and (b) table tennis. In each figure, *mrb*: gradient-based method using the approximated R-D by the proposed model, with additional bit-re-allocation for B frames; *mdl*: gradient-based method using the approximated R-D only; *org*: gradient-based method using the original measured R-D; *tm5*: Test model 5 algorithm.

	Football		Table Tennis	
	PSNR	Complexity	PSNR	Complexity
Model R-D	33.13	1.68	32.64	1.71
B-Frame Re-Alloc.	33.17	1.70	32.80	1.73
Original R-D	33.17	8.87	32.74	11.35
Test Model 5	32.43	1.00	31.25	1.00

Table 3: Average PSNR and computation complexity with different encoding method. The second second row is based on the model R-D with additional bit-re-allocation for B frames. The computation complexity is relative to the Test Model 5 algorithm.

In the experiment the MPEG-2 encoder implementation of [16] to encode two video sequence: football and table tennis. Both of them are in CIF format, and coded at 1.152 Mbps using four different configurations: (i) gradient-based method with the approximated R-D from the proposed model; (ii) use (i) with additional bit-re-allocation for B frames using Lagrange method; (iii) gradient-based method with the original R-D; and (iv) Test Model 5 (TM5) algorithm[4]. The results are shown in Fig. 6 and Table 3. The computation complexities shown in the Table are relative to the Test Model 5 algorithm, and are estimated based on the procedures in [16], where (i) 13 mults and 29 adds are required for each 8×1 DCT; (ii) two-step search[5] method is used for the motion estimation (takes about 85 to 90 percent of overall computations in a single-pass encoding). We assume the memory is large enough to hold all the intermediate data including the motion vectors, reconstructed reference frames, DCT coefficients, etc., so that many of the operations only have to be done once during the evaluation of R-D data on the control points.

The results show that, by using the approximated model, the number of computations are reduced significantly with very little loss in PSNR. With bit-re-allocating on B frames, we are able to achieve the same PSNR with

[4] Note that in the rate control used in TM5 adaptive quantization is used within frames, while we are using constant quantization. Because the adaptive quantization in TM5 is not aimed at minimizing the MSE a further $0.3dB$ or so could be gained from operating at constant quantization. However since adaptive quantization is needed in TM5 to prevent buffer overflow we still use TM5 with adaptive quantization for our comparison.

[5] Spiraling outward full search for full-pixel displacement, followed by the search for 8 neighboring half-pixel displacement.

only a fraction of computation overhead. The results also show that, for the table tennis sequence in Fig. 6 (b), the optimum method is capable of adjusting to the scene changes much faster than the test model 5 algorithm.

5 CONCLUSION AND FUTURE WORK

From the above experiments, we have demonstrated that our proposed model provides a good estimation of rate-distortion characteristic for any given quantization settings. The first applications to the gradient-based rate control algorithm shows the same performance can be achieve with only 15 to 20 percent of computation costs. It can also be applied applied to other optimal rate control techniques such as the dynamic programming approaches. In addition, our model is also useful for bit-allocation for other rate-control techniques or constant-quality variable-bit-rate encoding scheme. In the next stage, we will incorporate our model into these techniques and measure its performance in terms of the accuracy and computation reduction.

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