Nanosignal Processing: Stochastic Resonance in Carbon Nanotubes That Detect Subthreshold Signals

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ABSTRACT

Experiments confirm that small amounts of noise help a nanotube transistor detect noisy subthreshold electrical signals. Gaussian, uniform, and impulsive (Cauchy) noise produced this feedforward stochastic-resonance effect by increasing both the nanotube system’s mutual information and its input−output correlation. The noise corrupted a synchronous Bernoulli or random digital sequence that fed into the thresholdlike nanotube transistor and produced a Bernoulli sequence. Both Shannon’s mutual information and correlation measured the performance gain by comparing the input and output sequences. This nanotube SR effect was robust: it persisted even when infinite-variance Cauchy noise corrupted the signal stream. Such noise-enhanced signal processing at the nanolevel promises applications to signal detection in wideband communication systems and biological and artificial neural networks.

Noise can help carbon nanotube transistors detect subthreshold electrical signals by increasing the transistor’s input−output mutual information or correlation. Several researchers have demonstrated the stochastic resonance (SR) effect for various types of threshold units or neurons.1−6 Experiments with p-type nanotube transistors confirmed the specific SR prediction based on the theoretical finding that simple memoryless threshold neurons exhibit SR for almost all finite-variance and infinite-variance noise types.7 The experiments used three types of additive noise (Gaussian, uniform, and infinite-variance1 Cauchy noise) and different combinations of subthreshold ON/OFF electrical signals. Figure 1 shows the nonmonotonic signature of SR for white Gaussian noise and the thresholdlike nonlinearity of the nanotube transistors.8−13 The modes of the mutual-information and correlation curves occurred for nonzero noise strength with a standard deviation of at least 0.01.

The nanotube experiments produced the SR effect for both the Shannon mutual information and the input−output correlation14 of noisy Bernoulli sequences. The mutual information \( I(S, Y) \) subtracts the noisy channel’s (the transistor’s) output conditional entropy \( H(Y|S) \) from its unconditional entropy \( H(Y) \): \( I(S, Y) = H(Y) - H(Y|S) \). The input signal \( S \) was a random binary voltage that produced a random output \( Y \) in the form of a transistor current. The correlation measure found the normalized zero-lag cross-correlation

\[
r_{xy}(l) = \frac{1}{N} \sum_{k=1}^{N} [x(k) - \bar{x}][y(k - l) - \bar{y}]
\]

of the two sequences with subtracted means. The measures did not assume that the nanotube detector had a special structure and did not impose a threshold scheme on the experiment.

Figure 1b shows the thresholdlike nonlinearity of the nanotube transistor in response to the noisy input signal. The transconductance \( G \) related the output drain-to-source current \( I \) to the input gate voltage \( V \) and the threshold voltage \( V_T \) in a memoryless signal function: \( I = G(V - V_T) \) if \( V \leq V_T \) and zero otherwise. We note that the threshold neuron model lacks the internal state dynamics of the FitzHugh–Nagumo (FHN) model.15 The transconductance \( G \) was negative because the pristine (undoped) nanotube transistors exhibited current−voltage characteristics that were consistent with p-type transistors. Linear regression extrapolated the nonlinearity and estimated the threshold voltage.

Each of the nanotube experiments (Supporting Information) applied 32 independent trials of 1000-symbol input sequences for 24 noise levels per type and over a range of gate voltages. The 24 sampled noise levels ranged from 0.001 to 1 standard deviation (dispersion for infinite-variance Cauchy) linearly on a logarithmic scale. The noisy input was
So the discrete-time Fourier transform was white because the noise samples were uncorrelated in time. 

produced a flat noise power spectrum over the interval \([0, 100 \text{ ksymbols/s}]\) near the end of each symbol interval to estimate the output sequence (Supporting Information). A histogram of the output sequence gave the discrete probability density function, which computed the unconditional Shannon entropy:

\[
H(Y) = - \sum_{i=1}^{N} p_i \ln p_i
\]  

for mutual information without converting the detector output into a binary sequence with a threshold scheme. Sorting the output sequence based on the input symbol and then applying the histogram gave the conditional output discrete probability density function \(P(Y|S = j) = p_j / p_i\) conditioned on the input symbols that computed the conditional entropy:

\[
H(Y|S) = - \sum_{i=1}^{N} \sum_{j=1}^{N} p_i p_j \ln \left( \frac{p_j}{p_i} \right)
\]  

The mutual information measure was the difference between the unconditional and conditional entropies:

\[
I(S, Y) = H(Y) - H(Y|S)
\]  

Cross correlation compared the input and the output symbol sequences and gave a scalar representation with its zero-lag value:

\[
r_{xy}(0) = \sum_{k=1}^{N} y(k)
\]  

Converting the input Bernoulli sequence to bipolar form (mapping ON to +1 and OFF to −1) made it approximately
Several SR researchers have found multiple modes in the plot of system performance against noise strength. The Cauchy-noise experiment produced a measurable SR effect for two of the four combinations of input voltages. Shown is an approximate SR effect for the subthreshold signal \( ON = 1.8 \) V and \( OFF = -1.6 \) V. The SR mode is at 0.04 standard deviation. Several SR researchers have found multiple modes in the plot of system performance against noise strength.\(^ {51-53}\) The limited dynamic range \([-5V, 5V]\) of the data acquisition equipment (Supporting Information) may have produced the second peak in the graph as a truncation artifact because it clipped large spikes when it realized the infinite-variance Cauchy noise. The clipping affected more than 0.1% of the noise only for dispersions greater than 0.01.

The experiment found the SR effect for mutual information and correlation for Gaussian and uniform noise and for four combinations of binary symbols: \((-2.0, -1.8), (-1.8, -1.6), (-1.6, -1.4),\) and \((-1.4, -1.2)\) V. Figure 1a shows the SR effect for additive white Gaussian noise and the subthreshold signal pair \( ON = -1.6 \) V and \( OFF = -1.4 \) V. The SR mode of the mutual-information curve is 6 times the value at minimal noise.

We also passed impulsive or infinite-variance white noise through the nanotube detector to test whether it was robust to occasional large noise spikes. We chose the highly impulsive Cauchy noise\(^ {7}\) for this task. This infinite-variance noise probability density function had the form:

\[
p(n) = \frac{\gamma}{\pi(n^2 + \gamma^2)}
\]

for zero location and finite dispersion \( \gamma \). Figure 2b shows that a diminished SR effect still persists for Cauchy noise with the subthreshold signal pair \( ON = -2.0 \) V and \( OFF = -1.8 \) V. Not all Cauchy experiments produced a measurable SR effect.

These SR results suggest that nanotubes can exploit noise in other signal-processing tasks if advances in nanotube device technology can overcome the problems of hysteresis and parasitic capacitance that affect logic circuits\(^ {25}\) and high-frequency signals.\(^ {26}\) The nanotube signal detectors might apply to broadband\(^ {27,28}\) or optical communication systems\(^ {29}\) that use submicroamp currents and attenuated signals in noise because our nanotube detectors used nanoamp current and could distinguish between subthreshold binary symbols. The

It divided the zero-lag cross correlation \( r_{SY}(0) \) by the square root of the energy of the input and the output sequences where the energy of a sequence is the same as the zero-lag value of its autocorrelation:

\[
[r] = \left[ \sum_{k=1}^{N} x(k) \right]^{2} = \left[ \sum_{k=1}^{N} x(k)(x(k) - \bar{x}) \right]_{k=0} = r_{XX}(0)
\]

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detectors might apply to parallel signal processing at the nanolevel because they could have a small minimum feature size in vast parallel arrays of nanotubes. The parallel detectors could apply to spread spectrum communications: each nanotube can act as an antenna that matches a separate frequency channel in frequency hopping and perhaps in other types of spread spectrum communications. A nanotube’s length can code for a given frequency while chemical adsorption can tune a nanotube’s threshold. The detectors could apply to spread spectrum communications: the nanotube detectors could interface with biological systems because an electrolyte can act as their gate. The nanotube detectors might also help implement pulse-train neural networks and exploit noise in biological or robotic systems because the detectors are threshold devices similar to spiking neurons.

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Supporting Information Available: Methods and Materials. This material is available free of charge via the Internet at http://pubs.acs.org.

References

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SUPPORTING INFORMATION

Materials and Methods

The experiment tested a p-type carbon nanotube field effect transistor (CNT-FET) as a threshold detector with subthreshold input signal plus noise. The input signal was a Bernoulli random variable corrupted by a zero-mean Gaussian, uniform, or zero-location Cauchy distributed noise. The experiment captured the detector output signal in response to the noisy input and compared against the Bernoulli input signal to determine the detector performance. The experiment sought evidence of the stochastic resonance (SR) effect – detection that improved with increasing noise strength before deteriorating with further increases in noise strength.
The CNT-FET signal detector consisted of a chemical vapor deposition (CVD) grown semiconductor carbon nanotube lying on a silicon dioxide insulation layer 500 nanometers (nm) thick and ohmically contacting titanium-gold electrodes (20 nm Ti, 60 nm Au) at both ends (figure S1). The metal contacts were the source and drain electrodes for electric current while the tube was the conduction channel. The p-doped silicon substrate beneath the silicon dioxide layer was the back gate that completed the field effect transistor that was the detector. The single-walled nanotube was three to five micrometers (µm) long and less than two nm in diameter according to atomic force microscopy.

Figure S1 (a): Detector image. Atomic force microscope (AFM) image of the CNT-FET detector. The detector consisted of a semiconductor carbon nanotube lying across two Ti-Au electrodes (top and
The nanotube was three to five μm long and less than two nm in diameter according to the AFM. The nanotube was undoped.

**Figure S1 (b):** Nanotube layout. The metal electrodes cover the ends of the carbon nanotube. The nanotube lies on top of the thin (500 nm) silicon oxide layer. The underlying substrate has p-type (holes) doping and can conduct electricity to act as the backgate.
The CNT-FET signal detector was a threshold device and was the nonlinear system in the experiment with approximate threshold voltage \( V_T = -2 \) volts. The approximation linearly extrapolated the transistor current-to-gate voltage curve to find a voltage that would intercept the x-axis and would correspond to the OFF state (figure 1 (b) of the main text). The transconductance \( G \) related the output drain-to-source current \( I \) to the input gate voltage \( V \) and the threshold voltage \( V_T \): \( I = G \left( V - V_T \right) \) if \( V \leq V_T \) and zero otherwise. The transconductance \( G \) was negative for the p-type, pristine CNT transistors.

The experiment involved the following equipment (figure S2). A Hewlett Packard 4156 B semiconductor parameter analyzer (not shown) measured \( I-V_D \) and \( I-V_G \) curves that characterized the CNT-FET detector’s gate effect. A National Instrument PCI-MIO-16XE-10 multifunction data acquisition (DAQ) board generated the analog voltages that drove the transistor’s gate and biased the nanotube then measured the electric current flowing through the nanotube. A DL 1211 current-voltage preamplifier converted the detector’s output electric current (\( I_{DS} \)) to voltage for data acquisition (risetime set to 0.1 ms and sensitivity set to \( 10^{-8} \) A/V). Two resistors formed a voltage divider to divide the smallest voltage step by two and improved the resolution of the DAQ’s analog voltages. A personal computer running LabView driver controlled the input signal generation and the output measurement to test the CNT-FET detector. A cryostat isolated the detector electrically, kept it at room temperature, and maintained a rough vacuum to remove contaminants such as moisture. A subthreshold gate voltage without additive noise would keep the detector in the OFF state – the drain-to-source current would be in the pico-amp range.
Figure S2: Nanotube experiment setup. The threshold detector was a p-type CNT-FET. The input was the gate voltage and the output was the current of the CNT-FET detector. The DAQ board updated the input symbols about once every 10 ms to allow the data acquisition and amplifier hardware to reach steady state. An estimate of each output symbol was the average of 10 measurements that the DAQ made near the end of the symbol interval. Each experiment applied one type of additive white noise for 32 trials 1,000-symbol sequences and used 24 evenly spaced noise values that ranged from 0.001 to 1 standard deviations (dispersions for infinite-variance Cauchy noise).

The experiment generated digital signals in software and converted them to analog voltages to test the detector. An input $S$ consisted of a sequence of binary symbols $b$ plus white noise $n$: $s_i = b_i + n_i$. Each $b$ was independent, identically distributed (Bernoulli random variables), and took value $A$ with probability $p$ or $\overline{A}$ with probability $1-p$. The noise $n$ was independent of and synchronized with the binary symbols. Each $n$ was independent and identically distributed. Three types of distributions were available: Gaussian, uniform, and Cauchy. The binary symbols were subthreshold ($A = -1.8$ V and $\overline{A} = -1.6$ V, for example) with respect to the threshold voltage.

An output $Y$ was the detector’s current in response to each input $S$ at the gate. The p-type transistor
The experiment biased the nanotube at 200 mV and updated the input symbol about every 10 ms. The symbol interval was a compromise that produced data in quantity within limited lab time while allowing sufficient time for the preamplifier and DAQ to reach steady state. The experiment conducted 32 trials for each noise type and strength and for each pair of binary symbols. Each trial consisted of a 1000-symbol sequence. The data acquisition equipment measured and averaged ten samples of the detector output near the end of each symbol interval at a rate of 100 kilo-symbols per second to estimate the output symbol sequence. A comparison between the input sequence and the output sequence yielded the system performance.

Cross correlation and mutual information provided comparison between the input and the output sequences and yielded two measures of detector performance. A cross correlation measured the similarity between the input and the output sequences. The correlation measure used the zero-lag value as a scalar representation of the cross correlation sequence between the input and the output:

\[
r_{xy}(l) = \sum_{k=1}^{N} s(k) y(k-l)
\]  \hspace{1cm} (S1)

\[
r_{xy}(0) = \sum_{k=1}^{N} s(k) y(k)
\]  \hspace{1cm} (S2)

A normalization scheme divided the zero-lag correlation by the square root of the energy of the input and the output sequences to give the normalized correlation measure where the energy of a sequence is the same as the zero-lag value of its autocorrelation:
\[ C(S,Y) = \frac{\sum_{k=1}^{N} s(k)y(k)}{\sqrt{\sum_{k=1}^{N} s(k)s(k)} \sqrt{\sum_{k=1}^{N} y(k)y(k)}} \]  
\[ = \frac{r_{sy}(0)}{\sqrt{r_{ss}(0)r_{yy}(0)}} \]  

This normalized correlation has the maximal value of one for any two identical sequences.

The mutual information was the difference between the output entropy \( H(Y) \) and the conditional output entropy \( H(Y|S) \) conditioned on the input:

\[ I(S,Y) = H(Y) - H(Y|S) \]  

where output entropy \( H(Y) = -\sum_{i=1}^{N} p_i \ln p_i \) used the symbol probabilities \( P(Y = Y_i) = p_i \) and conditional output entropy \( H(Y|S) = -\sum_{i=1}^{N} \sum_{j=1}^{N} p_{ji} \ln \left( \frac{p_{ji}}{p_j} \right) \) used the output probabilities conditioned on the input \( P_{y|s}(Y = Y_i|S = S_j) = \frac{p_{ji}}{p_j} \).

A histogram of each output sequence gave the discrete probability density function to compute the entropies. The use of a histogram avoids imposing an artificial software threshold scheme on the data. The histogram applied a fixed set of bin edges to each output sequence so that the bins represented a fixed set of discrete symbols. The normalization ensured that symbol probability density functions summed to unity.

The nanotube detector exhibited some hysteresis in its gate effect but not enough to prevent the SR effect. The hysteresis (figure S3) affected the transistor's gate effect: threshold voltage shifted based on the direction of the input voltage change. Charge trapping by water molecules on the silicon dioxide surface was one possible mechanism of hysteresis. We kept the detector in vacuum to reduce the
hysteretic effect but some effect persisted even after 72 hours in vacuum. Again the device hysteresis
did not prevent the observation of the SR effect.

The experimental data exhibited a gate effect consistent with a transistor in a plot of input sequence
versus output sequence (figure 1 (b) of main text). The figure suggested that the signal sequences in the
experiment encountered little changes in the threshold effect. The experiment used subthreshold
Bernoulli symbols and signals that had short hold times, rapid voltage transitions, and small voltage
changes. This voltage scheme differs from the large voltage range and slow voltage-sweep transitions
that characterized the hysteresis in figure S3. The experiment yielded evidence of the SR effect in spite
of the hysteretic mechanisms.
Figure S3: Nanotube transistor gate effect. A semiconductor parameter analyzer produced the $I$-$V_G$ curve that showed the transistor current as a function of gate voltage. The gate effect showed some hysteresis: the threshold voltage and the transconductance varied depending on the direction of the gate voltage sweep. The hysteretic effect did not prevent observation of the SR effect. This set of the $I$-$V_G$ curve shows a threshold shift in contrast to the effective $I$-$V_G$ curve (figure 1 (b) in main text) that the detector produced in response to the experiment’s noisy input signal.

A $\chi^2$ test rejected the similarity between the beta probability density function $\beta(0.5,5)$ and either the
mutual information or the correlation \((p<0.0001)\). The test statistics between the beta and the mutual information and between the beta and the correlation were much greater than the \(\chi^2\) critical value for 24 degrees of freedom.

A Kolmogorov-Smirnov test rejected the similarity between the beta cumulative density function \(\beta(0.5,5)\) and either the mutual information or the correlation \((p<0.0001)\). The test statistic between the beta and the mutual information was 0.3955 and was 0.4997 between the beta and the correlation.