# USC–SIPI REPORT #220

**Adaptive Noise Canceler for Narrowband and Wideband Interferences Using Higher-Order Statistics**

**by**

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**September 1992**

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# Adaptive Noise Canceler for Narrowband and Wideband Interferences Using Higher-Order Statistics*

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**Abstract**

A new higher–order statistics–based adaptive interference canceler is introduced to eliminate narrowband and wideband interferences in environments where the interference is non–Gaussian and a reference signal, which is highly correlated with the interference, is available. The new scheme uses higher–order statistics between the primary and reference inputs and employs a gradient–type algorithm for updating the adaptive filter coefficients. The higher–order statistics–based filter is independent of Gaussian uncorrelated noises and insensitive to both the reference signal statistics and the step size parameter. It is demonstrated, by means of extensive simulations, that the higher–order statistics–based filter can eliminate both narrowband and wideband interferences effectively. Compared with the second–order statistics–based filter, the higher–order filter converges faster and has smaller excess errors. In addition, as expected, the higher–order statistics–based filter outperforms the second-order filter when Gaussian uncorrelated noises are present.

*This work was supported by the Office of Naval Research under contract N00014–92–J–1034.

# I. Introduction

When a signal of interest (SOI) is corrupted by an additive interference and an auxiliary reference signal, which is highly correlated with only the interference is available, the elimination of the interference is accomplished by an adaptive noise canceling procedure. The reference signal is processed by an adaptive filter to match the undesired interference as closely as possible, and the filter output is subtracted from the primary input, which consists of the SOI and interference, to produce a system output. The objective of an adaptive noise canceler (ANC) is to produce a system output that best fits the SOI. Applications of ANC include the canceling of several kinds of interference in electrocardiography, speech, antenna sidelobe interference, and telephone circuits [15, 16].

In this paper, we assume that the interference is a narrowband or wideband signal. A conventional transversal ANC, which is denoted in this paper as ANC–SOS algorithm, utilizes the LMS algorithm and second–order statistics (SOS) [8, 16, 17]. The utilization of the ANC–SOS algorithm for eliminating narrowband interferences is well analyzed in [6] and [13]. However, applying the ANC–SOS algorithm in practice, we usually encounter two major difficulties. The first is that the ANC–SOS filter is affected directly by uncorrelated noises at the primary and reference inputs. The second is that the ANC–SOS algorithm is problem–dependent; i.e., it is very sensitive to both of the reference signal statistics and the choice of step size. For an ANC algorithm which is not sensitive to the reference signal statistics, several adaptive lattice structures have been introduced [5, 7, 11, 12]. However, Honig and Messerschmitt [9] have concluded that the lattice convergence speed is not independent of the input signal statistics and that the adaptive lattice filter does not always converge faster than its transversal counterpart. In addition, the lattice ANC algorithm in the presence of uncorrelated noises has not been studied.

Therefore, it is important to obtain an ANC algorithm which is independent of both uncorrelated noise sources and the statistics of the reference signal. The utilization of higher–order statistics (HOS) can provide such an adaptive filter in those applications where the interferences can be regarded as non–Gaussian processes and the uncorrelated noise sources as stationary, zero–mean, Gaussian processes. We will call this filter an adaptive noise canceler based on higher–order statistics (ANC–HOS). Higher–order statistics or spectra have been given a lot of attention lately due to their ability to preserve information of non-Gaussian stationary random processes [10]. Chiang and Nikias [1, 2, 3] have introduced adaptive schemes for time–delay estimation (or system identification) using third– and fourth–order cumulants. Dandawate and Giannakis [4] have developed a similar scheme as an ANC–HOS using the third–order cumulants and have shown that their filter is a better estimate of the relationship between the interference and reference signals in the case where an uncorrelated noise source is present only at the reference input. However, when the probability density function of the reference signal is symmetric, the third–order cumulants of the reference signal are identically zero [14]. Moreover, an ANC can not eliminate uncorrelated noises and the ANC system output is highly related to the noises. Thus, a reasonable objective should be the ability of the algorithm to restore the SOI. We compare the performance of the ANC–SOS algorithm and that of the ANC–HOS algorithm by means of errors between each system output and the SOI.

In this paper, we introduce a new ANC–HOS algorithm using $n$–th ($n > 2$) order cumulants to cancel narrowband and wideband interferences. We show that the ANC–HOS filter is independent of white or colored Gaussian uncorrelated noises and insensitive to both the reference signal statistics and the step size. It is important to note that the ANC–HOS algorithm may not work better than the ANC–SOS algorithm when the total power of the uncorrelated noises is quite large, because an ANC can not remove uncorrelated noises. However, we demonstrate that when the noise powers are small, but not negligible, the ANC–HOS filter can eliminate the interference more effectively than the ANC–SOS filter. The outline of the paper is as follows. The problem definition and preliminaries for an ANC algorithm are given in Section II. In Section III we introduce the ANC–HOS algorithm using a gradient–type algorithm and briefly describe the difficulties in utilizing the ANC–SOS algorithm. We develop the ANC–FOS filter using fourth–order cumulants to eliminate narrowband and wideband interferences in Section IV. Section V discusses the required computational complexity by the ANC-FOS algorithm. We present and compare performances of the ANC–FOS and the ANC–SOS algorithms for canceling several narrowband and wideband interferences, in Section VI. Conclusions are drawn in Section VII.

## II. Problem Definition and Preliminaries

Let $\{x(k)\}$ and $\{z(k)\}$ denote measurements of the primary and reference sensors, respectively, satisfying

$$x(k) = s(k) + I(k) + n_p(k) \tag{1}$$

$$z(k) = w(k) + n_r(k) \tag{2}$$

where $\{s(k)\}$ is the signal of interest (SOI), $\{I(k)\}$ is the interference (narrowband or wideband), $\{w(k)\}$ is a reference signal highly correlated with the interference, and $\{n_p(k)\}$ and $\{n_r(k)\}$ are uncorrelated sensor noises. We assume that the SOI is zero–mean and any kind of a signal, i.e., deterministic or random, or a combination of both and uncorrelated with the interference and the reference signal. The reference signal is a stationary, zero–mean, non–Gaussian random process. The noises $\{n_p(k)\}$ and $\{n_r(k)\}$ are zero–mean, white or colored Gaussian, uncorrelated with each other and independent of the SOI, interference, and reference signal. Moreover, we assume that the relationship between the interference and reference signal can be represented by a linear–time–invariant (LTI) transformation so that

$$I(k) = \sum_j g(j)w(k-j). \tag{3}$$

Let $\{y(k)\}$ be an adaptive filter output

$$y(k) = \sum_{j=0}^{N-1} h(j)\, z(k-j) = \sum_{j=0}^{N-1} h(j)\,[w(k-j) + n_r(k-j)] \tag{4}$$

where $N$ denotes the number of taps and $\{h(n),\ n = 0, 1, \ldots, N-1\}$ is the adaptive filter coefficients. Then the ANC system output $\{e(k)\}$ is given by

$$e(k) = x(k) - y(k) = s(k) + n_e(k) \tag{5}$$

where $\{n_e(k)\}$ is the ANC system output noise. We can represent the system output noise as

$$n_e(k) = n_I(k) + n_p(k) - \sum_{j=0}^{N-1} h(j)n_r(k-j) \tag{6}$$

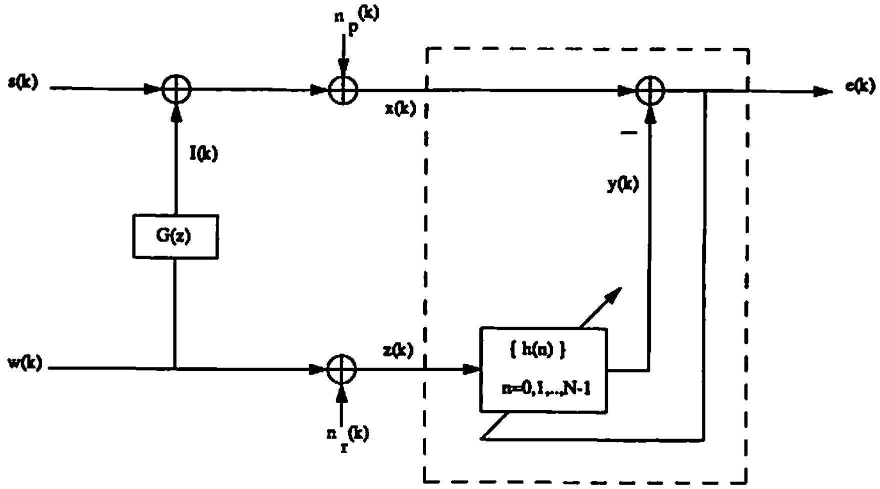


Figure 1: Adaptive noise canceler (ANC) with uncorrelated noises at the primary and reference inputs.

where

$$n_I(k) = I(k) - \sum_{j=0}^{N-1} h(j)\, w(k-j).$$

Although an ANC can eliminate the interference completely, the ANC system output contains the uncorrelated noises. Figure 1 shows the block diagram structure of an ANC system.

## III. ANC Based on Higher-Order Statistics (ANC–HOS)

In this section, we introduce an ANC–HOS algorithm using a gradient–type algorithm and HOS and show that its filter is independent of Gaussian uncorrelated noises. In addition, the difficulties in using the ANC–SOS algorithm in certain signal/interference environments will be described briefly.

### A. Basic Scheme of ANC–HOS

To develop an ANC–HOS algorithm, we need to assume that there exists at least one order $n$ such that the $n$–th ($n > 2$) order cumulants of the reference signal are not identically zero. Under this assumption, let $C_{xz\cdots z}(\tau_1, \ldots, \tau_{n-1})$ denote the $n$–th order cross–cumulants between the primary and reference inputs and $C_{yz\cdots z}(\tau_1, \ldots, \tau_{n-1})$ denote the $n$–th order cross–cumulants between the ANC–HOS filter output and the reference input. Since the $n$–th order cumulants of a stationary, zero–mean, Gaussian process are identically zero [10], we have

$$C_{xz\cdots z}(\tau_1,\ldots,\tau_{n-1}) = C_{Iw\cdots w}(\tau_1,\ldots,\tau_{n-1}) = \sum_j g(j)\, C_{w\cdots w}(j+\tau_1,\ldots,,j+\tau_{n-1}) \qquad (7)$$

and

$$C_{yz\cdots z}(\tau_1,\ldots,\tau_{n-1}) = \sum_{j=0}^{N-1} h(j)\, C_{w\cdots w}(j+\tau_1,\ldots,j+\tau_{n-1}). \qquad (8)$$

Note that (7) and (8) can be obtained by using the following facts: (i) the SOI and reference signal are uncorrelated, (ii) the SOI, interference, and reference signal are independent from the uncorrelated noises, and (iii) the SOI, reference signal, and noises are zero–mean processes. In addition, we can easily show that $C_{w\cdots w}(\tau_1,\ldots,\tau_{n-1}) = C_{z\cdots z}(\tau_1,\ldots,\tau_{n-1})$ and that (8) can be rewritten as

$$C_{yz\cdots z}(\tau_1,\ldots,\tau_{n-1}) = \sum_{j=0}^{N-1} h(j)\, C_{z\cdots z}(j+\tau_1,\ldots,j+\tau_{n-1}). \qquad (9)$$

Then, the "criterion of goodness" is defined as the sum of the squared errors between these two $n$–th order cumulants as in [3], viz.

$$\xi_g = \sum_{\tau_1} \cdots \sum_{\tau_{n-1}} [\, C_{xz\cdots z}(\tau_1,\ldots,\tau_{n-1}) - C_{yz\ldots z}(\tau_1,\ldots,\tau_{n-1})\,]^2 \qquad (10)$$

where $(\tau_1,\ldots,\tau_{n-1})$ may be defined to include the whole $(n-1)$–D plane $\mathcal{R}^{n-1}$. Using a proper domain $\Gamma \subset \mathcal{R}^{n-1}$, we can simplify the criterion of goodness. The criterion, $\xi$, becomes a special case of (10)

$$\xi = \sum \cdots \sum_{(\tau_1,\ldots,\tau_{n-1})\, \in\, \Gamma} [\, C_{xz\cdots z}(\tau_1,\ldots,\tau_{n-1}) - \sum_{j=0}^{N-1} h(j) C_{z\cdots z}(j+\tau_1,\ldots,j+\tau_{n-1})\,]^2 \qquad (11)$$

or

$$\xi = (\mathbf{C}_{xz\cdots z} - \mathbf{C}_{z\cdots z}\mathbf{H}_h)^T(\mathbf{C}_{xz\cdots z} - \mathbf{C}_{z\cdots z}\mathbf{H}_h) \qquad (12)$$

where $\mathbf{C}_{xz\cdots z}$ is an $M \times 1$ column vector and $\mathbf{C}_{z\cdots z}$ is an $M \times N$ matrix. $M$ denotes the number of overdetermined equations. Note that $M \geq N$. When the $m$–th row component of $\mathbf{C}_{xz\cdots z}$ is $C_{xz\cdots z}(\tau_1,\ldots,\tau_{n-1})$, the $m$–th row of $\mathbf{C}_{z\cdots z}$ becomes

$$[\, C_{z\cdots z}(\tau_1,\ldots,\tau_{n-1}), C_{z\cdots z}(\tau_1+1,\ldots,\tau_{n-1}+1), \cdots, C_{z\cdots z}(\tau_1+N-1,\ldots,\tau_{n-1}+N-1)\,]^T.$$

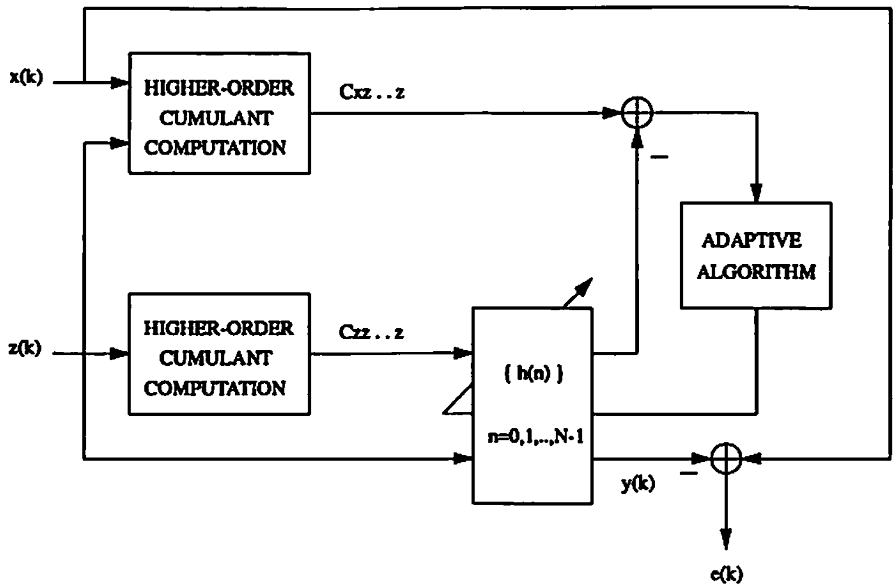


Figure 2: The configuration of the adaptive noise canceler using higher–order statistics (ANC–HOS).

$\mathbf{H}_h$ denotes an $N \times 1$ ANC–HOS filter coefficient vector

$$\mathbf{H}_h = [\, h(0), h(1), \ldots, h(N-1)\,]^T .$$

The gradient of $\xi$ is given by

$$\nabla \equiv \frac{\partial \xi}{\partial \mathbf{H}_h} = 2\left( \mathbf{C}^T_{z\cdots z}\mathbf{C}_{z\cdots z}\mathbf{H}_h - \mathbf{C}^T_{z\cdots z}\mathbf{C}_{xz\cdots z} \right). \tag{13}$$

Then, the filter update equation takes the form

$$\mathbf{H}_h(k+1) = \mathbf{H}_h(k) - \mu(k)\nabla(k), \tag{14}$$

where $\mu(k)$ is the step size. Since the gradient $\nabla(k)$ consists of the $n$–th order cumulants of the primary and reference inputs which are not affected by Gaussian uncorrelated noises, the update equation (14) is independent of the uncorrelated noises. Figure 2 shows the block diagram structure of the ANC–HOS algorithm for eliminating the interference.

## B. ANC Based on Second–Order Statistics (ANC–SOS)

The ANC–SOS algorithm uses the LMS scheme based on SOS and the filter update equation is given by [8, 16]

$$\mathbf{H}_s(k+1) = \mathbf{H}_s(k) - 2\mu e(k)\mathbf{Z}(k) \tag{15}$$

where $\mathbf{H}_s(k)$ denotes the ANC–SOS filter coefficient vector, $\mu$ is a step size, and $\mathbf{Z}(k) = [z(k), z(k-1), \ldots, z(k-N+1)]^T$ is a reference input vector. If there are uncorrelated noises, then the ANC–SOS filter coefficients $\mathbf{H}_s(k)$ are affected directly by both noise sources because $\mathbf{Z}(k)$ contains $\{n_r(i),\ i = k, k-1, \ldots, k-N+1\}$ and $e(k)$ contains $\{n_e(k)\}$ as in (6). Thus, the ANC–SOS algorithm can not eliminate the interference effectively.

Even if there are no uncorrelated noises, the performance of the ANC–SOS algorithm is problem–dependent, i.e., it is very sensitive to a given problem. For a good narrow–bandwidth notch, the step size of the ANC–SOS algorithm should be satisfying the following relationship [6]

$$\frac{\mu N A^2}{4} \ll 1 \tag{16}$$

where $A$ is the amplitude of interference. Since these three parameters are closely related and we can control only $N$ and $\mu$, the performance of the ANC–SOS is highly dependent on $A$. For a relatively small amplitude of the interference, the ANC–SOS filter may converge very slowly. On the other hand, for a relatively large amplitude of the interference, the ANC–SOS filter may converge fast, but produce large excess errors after convergence. Through simulations, we have also found that when the amplitude of the interference is a fixed constant, the performance of the ANC–SOS algorithm is sensitive to the step size $\mu$. Thus, the performance of the ANC–SOS algorithm can deteriorate more severely when the interference is a sum of multiple narrowband signals or wideband signals. Table 1 shows a summary of the ANC–SOS algorithms.

## IV. ANC–HOS Applied to Sinusoidal Interference

Assuming that the interference is a sinusoidal signal whose phase is a random variable uniformly distributed over $[-\pi, \pi]$, we develop an ANC–HOS algorithm to eliminate the interference. Since the third–order cumulants of the interference are identically zero [14], we have to develop the ANC–FOS algorithm using fourth–order cumulants.

Table 1: Summary of the ANC–SOS Algorithm with $N$ taps

| | | |
|---|---|---|
| **Let:** | $x(i)$ | primary input at iteration $i$ |
| | $\mathbf{Z}(i) = [z(i), z(i-1), \ldots, z(i-N+1)]^T$ | reference input vector at iteration $i$ |
| | $e(i)$ | ANC-SOS system output at iteration $i$ |
| | $\mathbf{H}_s(i) = [h^i(0), h^i(1), \ldots, h^i(N-1)]^T$ | filter coefficient vector at iteration $i$ |
| **Initial Conditiions:** | $\mu = \mu_s/N, \quad \mu_s \ll 1$ | step size |
| | $e(0) = 0$ | |
| | $\hat{\mathbf{H}}_s(0) = \mathbf{H}_I$ | initial guess |
| **At iteration $i$:** | $i = 1, 2, 3, \ldots,$ | |
| | $\hat{\mathbf{H}}_s(i) = \hat{\mathbf{H}}_s(i-1) - 2\mu e(i-1)\mathbf{Z}(i-1)$ | |
| | $e(i) = x(i) - \hat{\mathbf{H}}_s^T(i)\mathbf{Z}(i)$ | |

For simplicity, we choose the domain $\Gamma = \{(\tau_1, \tau_2, \tau_3)\} \subset \mathcal{R}^3$ for the criterion of goodness, which satisfies the following conditions

$$0 \le \tau_1, \tau_2, \tau_3 \le M-1, \quad \tau_1 \ge \tau_2, \quad \tau_1 \ge \tau_3. \tag{17}$$

Letting $\tau_1 - \tau_2 = m$, $\tau_1 - \tau_3 = n$, we can represent the criterion of goodness, $\xi$, as

$$\xi = \sum_{\tau_1=0}^{M-1} \sum_{m,n=0}^{\tau_1} [\, C_{xzzz}(\tau_1, \tau_1 - m, \tau_1 - n) - \sum_{j=0}^{N-1} h(j) C_{zzzz}(j+\tau_1, j+\tau_1-m, j+\tau_1-n)\,]^2$$

or

$$\xi = (\mathbf{C}_{xzzz} - \mathbf{C}_{zzzz}\mathbf{H}_f)^T(\mathbf{C}_{xzzz} - \mathbf{C}_{zzzz}\mathbf{H}_f) \tag{18}$$

where $\mathbf{C}_{xzzz}$ is an $\frac{M(M+1)(2M+1)}{6} \times 1$ column vector and $\mathbf{C}_{zzzz}$ is an $\frac{M(M+1)(2M+1)}{6} \times N$ matrix. $\mathbf{H}_f$ denotes an $N \times 1$ ANC–FOS filter coefficient vector.

Let us note that, in practice, the theoretical fourth–order cumulants need to be substituted by their estimates. Using the relationship between fourth–order moments and cumulants,

$$C_{xwyz}(k; \tau_1, \tau_2, \tau_3) = E\{x(k)w(k+\tau_1)y(k+\tau_2)z(k+\tau_3)\} - E\{x(k)w(k+\tau_1)\}\, E\{y(k+\tau_2)z(k+\tau_3)\}$$

$$-E\{x(k)y(k+\tau_2)\}\, E\{w(k+\tau_1)z(k+\tau_3)\} - E\{x(k)z(k+\tau_3)\}\, E\{w(k+\tau_1)y(k+\tau_2)\},$$

we obtain the estimate $\hat{C}_{xwyz}(k; \tau_1, \tau_2, \tau_3)$ in terms of estimates of fourth–order moments. Thus, when we have $\{x(1), x(2), \ldots, x(k+M+N-3), x(k+M+N-2)\}$ and $\{z(1), z(2), \ldots, z(k+M+N-3), z(k+M+N-2)\}$, for $0 \le \tau_1, \tau_2, \tau_3 \le M+N-2$, $\tau_1 \ge \tau_2$, and $\tau_1 \ge \tau_3$, we obtain

Table 2: Summary of the ANC–FOS Algorithm with $N$ taps

**Let:**

| | | |
|---|---|---|
| | $x(i)$ | primary input at iteration $i$ |
| | $\mathbf{Z}(i) = [z(i), z(i-1), \ldots, z(i-N+1)]^T$ | reference input vector at iteration $i$ |
| | $e(i)$ | ANC-FOS system output at iteration $i$ |
| | $\mathbf{H}_f(i) = [h^i(0), h^i(1), \ldots, h^i(N-1)]^T$ | filter coefficient vector at iteration $i$ |

**Initial Conditiions:**

| | | |
|---|---|---|
| | $M$ | parameter to decide the number of equations |
| | $f$ | forgetting factor |
| | $\mu_f < 1$ | step size parameter |
| | $C_{xzzz}(0;\tau,\tau-m,\tau-n) = 0, \quad \tau = 0,1,\ldots,M-1, \quad m,n = 0,1,\ldots,\tau$ | |
| | $C_{zzzz}(0;\tau,\tau-m,\tau-n) = 0, \quad \tau = 0,1,\ldots,M-1, \quad m,n = 0,1,\ldots,\tau$ | |
| | $\hat{\mathbf{H}}_f(0) = \mathbf{H}_I$ | initial guess |

**At iteration $i$:** $i = 1,2,3,\ldots,$

*For* $\tau = 0,1,\ldots,M-1, \quad m,n = 0,1,\ldots,\tau$

$$C_{xzzz}(i;\tau,\tau-m,\tau-n) = f \cdot C_{xzzz}(i-1;\tau,\tau-m,\tau-n) + x(i)z(i+\tau)z(i+\tau-m)z(i+\tau-n)$$

$$\hat{C}_{xzzz}(i;\tau,\tau-m,\tau-n) = \frac{1}{i}\, C_{xzzz}(i;\tau,\tau-m,\tau-n)$$

*For* $j = 0,1,\ldots,N-1$

$$C_{zzzz}(i;\tau,\tau-m+j,\tau-n+j) = f \cdot C_{zzzz}(i-1;\tau,\tau-m+j,\tau-n+j) + z(i)z(i+\tau+j)z(i+\tau-m+j)z(i+\tau-n+j)$$

$$\hat{C}_{zzzz}(i;\tau+j,\tau-m+j,\tau-n+j) = \frac{1}{i}\, C_{zzzz}(i;\tau+j,\tau-m+j,\tau-n+j)$$

Construct $\hat{\mathbf{C}}_{xzzz}(i)$ and $\hat{\mathbf{C}}_{zzzz}(i)$ such that

$$\hat{\mathbf{C}}_{xzzz}(i) = [\hat{C}_{xzzz}(i;0,0,0), \hat{C}_{xzzz}(i;1,0,0), \hat{C}_{xzzz}(i;1,0,1), \ldots, \hat{C}_{xzzz}(i;M-1,M-1,M-1)]^T$$

$$\hat{\mathbf{C}}_{zzzz}(i) = \begin{bmatrix} \hat{C}_{zzzz}(i;0,0,0) & \cdots & \hat{C}_{zzzz}(i;N-1,N-1,N-1) \\ \hat{C}_{zzzz}(i;1,0,0) & \cdots & \hat{C}_{zzzz}(i;N,N-1,N-1) \\ \hat{C}_{zzzz}(i;1,0,1) & \cdots & \hat{C}_{zzzz}(i;N,N-1,N) \\ \vdots & \vdots & \vdots \\ \hat{C}_{zzzz}(i;M-1,M-1,M-1) & \cdots & \hat{C}_{zzzz}(i;M+N-2,M+N-2,M+N-2) \end{bmatrix}$$

$$\hat{\nabla}(i) = 2\left[\hat{\mathbf{C}}^T_{zzzz}(i)\hat{\mathbf{C}}_{zzzz}(i)\hat{\mathbf{H}}_f(i) - \hat{\mathbf{C}}^T_{zzzz}(i)\hat{\mathbf{C}}_{xzzz}(i)\right]$$

$$\mu(i) = \mu_f \,/\, tr\,\{\hat{\mathbf{C}}^T_{zzzz}(i)\hat{\mathbf{C}}_{zzzz}(i)\}$$

$$\hat{\mathbf{H}}_f(i) = \hat{\mathbf{H}}_f(i-1) - \mu(i)\hat{\nabla}(i)$$

$$e(k) = x(k) - \hat{\mathbf{H}}^T_f(i)\mathbf{Z}(i)$$

Table 3: The Number of Multiplications per Iteration of the ANC–SOS and the ANC–FOS Algorithms

| type of algorithm | number of multiplications per iteration | | | | |
|---|---|---|---|---|---|
| ANC-SOS | $2(N+1)$ | $N=16$ | $N=24$ | $N=32$ | $N=64$ |
| | | 34 | 50 | 66 | 130 |
| ANC-FOS for Narrowband | $\frac{1}{6}M(M+1)(2M+1)(N+1)(N+4)+3N+1$ | $N=8$ | $N=8$ | $N=16$ | $N=16$ |
| | | $M=4$ | $M=5$ | $M=4$ | $M=5$ |
| | | 3265 | 5965 | 10249 | 18749 |
| ANC-FOS for Wideband | $\frac{1}{6}M(M+1)(2M+1)(N+1)(N+5)+3N+1$ | $N=8$ | $N=8$ | $N=16$ | $N=16$ |
| | | $M=4$ | $M=5$ | $M=4$ | $M=5$ |
| | | 3535 | 6460 | 10759 | 19684 |

$$\hat{C}_{xzzz}(k;\tau_1,\tau_2,\tau_3) = \frac{1}{k}\sum_{j=1}^{k} f^{k-j}x(j)z(j+\tau_1)z(j+\tau_2)z(j+\tau_3) \tag{19}$$

$$\hat{C}_{zzzz}(k;\tau_1,\tau_2,\tau_3) = \frac{1}{k}\sum_{j=1}^{k} f^{k-j}z(j)z(j+\tau_1)z(j+\tau_2)z(j+\tau_3) \tag{20}$$

where $0 < f \leq 1$ and $f$ is a forgetting factor which controls the shape of the window of data being used at each iteration. We choose $f = 1$ for the stationary case. Table 2 shows a summary of the ANC–FOS algorithm.

## V. Computational Complexity

The computational complexity of an algorithm is an important aspect in the adaptation process and therefore should be taken into account. We use the number of multiplications per iteration as a figure of merit. Table 3 shows the computational complexity of the ANC–FOS and the ANC–SOS algorithms. From Table 3, it is apparent that the number of multiplications required by the ANC–FOS algorithm is very large. We, therefore, conclude that the improved performance of the ANC-FOS algorithm over the ANC–SOS algorithm is achieved at the expense of more computations. Through simulations, however, we have also found that the ANC–FOS filter with a small $N$ can produce better results than the ANC–SOS filter having a large $N$.

## VI. Experimental Results

We consider some typical examples to compare the performance of the ANC–FOS algorithm with that of the ANC–SOS algorithm for eliminating narrowband and wideband interferences. Comparisons are presented in terms of the error between the SOI and its reconstructed version by each ANC algorithm. We assume that the SOI is deterministic BPSK having two states, $s_1$ and $s_0$, satisfying

$$s(k) = \begin{cases} \cos(2\pi f_s T\, k), & \text{for } s_1 \\ -\cos(2\pi f_s T\, k), & \text{for } s_0 \end{cases} \tag{21}$$

where $f_s T = 0.43$ and the duration of one state is 20 samples. The reference signal is assumed to be a sum of real-valued sine waves

$$w_i(k) = A_i \sin(2\pi f_i T\, k + \phi_i), \quad i = 1, 2, 3 \tag{22}$$

where $A_i$'s and $f_i$'s denote amplitudes and frequencies, respectively and $\phi_i$'s are independent random variables uniformly distributed over $[-\pi, \pi]$. Note that $f_1 T = 0.1$, $f_2 T = 0.25$, and $f_3 T = 0.3$. Each interference signal $\{I_i(k),\ i = 1, 2, 3\}$ is generated through three MA(2) systems excited by a reference signal $\{w_i(k),\ i = 1, 2, 3\}$. The corresponding MA coefficients equal [1, 0.1, −0.3], [1, 0.5, −0.1], and [1, −0.2, 0.2], respectively. In all simulations, we choose the step sizes $\mu = \mu_s / N$ for the ANC–SOS algorithm and $\mu(k) = \mu_f / tr\{\hat{C}^T_{zzzz}(k)\hat{C}_{zzzz}(k)\}$, where $\mu_f < 1$ for the ANC–FOS algorithm [2] to ensure the stability of the algorithms.

*Experiment 1 (Sensitivity to Step Size)*: To investigate how the step size affects the convergence rate of the ANC–SOS and the ANC–FOS algorithms, we compare them with several different step sizes when the narrowband interference is $\{I_2(k)\}$ with $A_2 = 1$. The numbers of taps of the ANC–SOS and the ANC–FOS filters are both 32. The results obtained by the ANC–SOS algorithm are shown in Fig. 3 when $\mu_s = 0.025, 0.05, 0.075$, and 0.1. We notice that a small value of $\mu_s$ causes slow convergence and a large value causes fast convergence. Figure 4 illustrates results obtained by the ANC–FOS algorithm when $\mu_f = 0.995, 0.9, 0.85$, and 0.8. There are no noticeable differences among these cases in terms of speed of convergence.

It has already been established that the convergence speed of the ANC–SOS algorithm is very sensitive to the step size when the amplitude of the reference signal is a fixed constant. For example, compare Fig. 3-(a) and Fig. 3-(d). The difference between two $\mu_s$'s is 0.075 and the actual difference between two step sizes is $\frac{0.075}{32} = 0.00234375$. On the other hand, the value of $\mu_f$ of the ANC–FOS algorithm does not seem to affect its convergence speed.

To cancel a single narrowband interference, we use $N = 32$ taps in the ANC–SOS filter and $N = 8$ taps, $M = 5$ in the ANC–FOS filter. When the interference is a sum of narrowband signals, we use $N = 32$ taps in the ANC–SOS filter and $N = 16$ taps, $M = 5$ in the ANC–FOS filter. Note that $\mu_s = 0.05$ for the ANC–SOS algorithm and $\mu_f = 0.995$ for the ANC–FOS algorithm in all narrowband interference cases.

*Experiment 2 (Narrowband Interferences Without Additive Uncorrelated Noises)*: We assume that there are no additive uncorrelated noises at both of the primary and reference inputs. In this experiment we consider two cases: (i) a single interference and (ii) sum of two interferences. The single interference is $\{I_2(k)\}$, where $A_2$ takes different values. Figure 5 shows the error curves generated by the ANC–SOS and the ANC–FOS algorithms. We see that when the number of taps and the step size are fixed, a small value of $A_2$ causes slow convergence of the ANC–SOS algorithm, whereas an large value of $A_2$ makes the ANC–SOS algorithm converge fast at the expense of large excess errors. On the other hand, the performance of the ANC–FOS algorithm is much less sensitive to the magnitude of $A_2$. The results from multiple interferences are shown in Fig. 6. The interference is $\{I_1(k) + I_2(k)\}$ with $A_1 = \sqrt{2}$ and $A_2 = 0.5$ or $A_1 = 0.5$ and $A_2 = 1$. Figure 5 and 6 demonstrate that the performance of the ANC–SOS algorithm is very sensitive to both of the magnitude of $A_2$ and the step size of the algorithm. Since both of the convergence speed and the excess error are important in the adaptation process, we conclude that the use of the ANC–SOS algorithm to eliminate a single narrowband interference is only appropriate when the amplitude of the reference signal is known a priori and is also fixed. In the case of multiple interferences, it is more difficult to choose values of the parameters of the ANC–SOS algorithm even though each amplitude of the interference sources is known and fixed, because there are no explicit relationships among them. The results obtained by the ANC–FOS algorithm show that we can use the ANC–FOS algorithm to cancel multiple interferences, as well as a single interference. Moreover, both of the convergence speed and the excess error of the ANC–FOS algorithm are less sensitive to both of the reference signal statistics and the step size. In addition, the error amplitude in the output of the ANC–FOS algorithm is much less than that of the ANC–SOS algorithm.

*Experiment 3 (Narrowband Interferences and Additive Gaussian Uncorrelated Noises)*: We assume that the uncorrelated noises are white Gaussian. We consider two cases in this experiment: (i) a single interference and (ii) a sum of two or three interferences. Figure 7 illustrates the error curves obtained by the ANC-SOS and the ANC–FOS algorithms, when the single interference is $\{I_2(k)\}$ with $A_2 = 2$ and the variances of the uncorrelated noises take values 0.001, 0.0025, 0.005, or 0.0075. In multiple interference cases, the error curves obtained by the ANC–SOS and the ANC–FOS algorithms are shown in Fig. 8 when both of the uncorrelated noise variances are 0.005 and the interference is $\{I_1(k)+I_2(k)\}$ with $A_1 = \sqrt{2}$ and $A_2 = 0.5$ or $\{I_1(k)+I_2(k)+I_3(k)\}$ with $A_1 = 0.5$, $A_2 = 1$, and $A_3 = 0.75$.
Although the performance of the ANC–SOS and the ANC–FOS algorithms is greatly influenced by the additive uncorrelated noises, the ANC–FOS algorithm converges faster and achieves less excess error than the ANC–SOS algorithm.

*Experiment 4 (Wideband Interferences (AM) Without Additive Uncorrelated Noises)*: The wideband interference is $\{I_2(k)\}$ with time–varying amplitude $A_2$. We choose $N = 16$ taps and $\mu_s = 0.1$ for the ANC–SOS filter and $N = 8$ taps, $M = 5$, and $\mu_f = 0.995$ for the ANC–FOS filter. Figure 9 shows the AM reference signal and the error curves generated by the ANC–SOS and the ANC–FOS algorithms when $A_2$ has abrupt changes in value. Figures 10 illustrates the results when $A_2$ is slowly time–varying. In both cases, the ANC–FOS algorithm converges faster with smaller excess errors. For a realistic AM case, we assume that there are time–varying disturbances in the reference amplitude generated through a lowpass filter excited by a uniformly distrubuted random process. The cutoff frequency is $f_cT = 0.05$. The AM signal and the results obtained by the ANC-SOS and the ANC-FOS algorithms are shown in Fig. 11. In all slowly time-varying AM cases, we see that the ANC–FOS algorithm outperforms the ANC–SOS algorithm in terms of convergence and amplitude of output errors.

*Experiment 5 (Wideband Interference (FM) Without Additive Uncorrelated Noises)*: Assuming that the amplitude of reference signal is fixed, its frequency is time–varying. The relationship between the reference signal and the interference is a MA(2) system whose coefficients equal [1, 0.5, -0.1]. Note that the parameters of the ANC–SOS and the ANC–FOS algorithms are same as in Experiment 4. For a realistic FM signal, we have generated time–varying frequency disturbance through a lowpass filter with $f_c T = 0.05$ excited by a uniformly distributed random process. Figure 12 shows the time–varying frequency of the reference signal and the error curves obtained by the ANC–SOS and the ANC–FOS algorithms.
When there are time-varying disturbances in frequency, the error in the ANC-SOS algorithm is much larger than that in the ANC-FOS algorithm. The result demonstrates that the ANC-FOS algorithm outperforms the ANC-SOS algorithm in the FM cases. Note, however, that both of the ANC–SOS and ANC–FOS algorithms are much more sensitive to perturbations in frequency than in amplitude of the reference signal.

## VII. Concluding Remarks

We have introduced in this paper the ANC–HOS formulations using a gradient–type algorithm and higher–order statistics. To eliminate narrowband and wideband interferences, we have developed the ANC–FOS algorithm using fourth–order cumulants. We have shown that the ANC–FOS filter is independent of Gaussian uncorrelated noises and have considered its required computational complexity. In addition, through simulations, we have shown that the performance of the ANC–FOS algorithm is not sensitive to both the step size and the reference signal statistics. On the other hand, the performance of the traditional ANC–SOS algorithm can be greatly influenced by the reference signal statistics, the step size, and additive uncorrelated noises. In the ANC–SOS algorithm, when the step size is fixed, a small value of the amplitude of the interference source causes slow convergence, whereas a large value causes large excess errors. Although the ANC–FOS algorithm needs more computational complexity and more delays to compute gradients than the ANC–SOS algorithm, the ANC–FOS algorithm converges much faster with less excess errors than the ANC–SOS algorithm. We have demonstrated that the ANC–FOS algorithm can eliminate wideband interferences as well as narrowband interferences very effectively.

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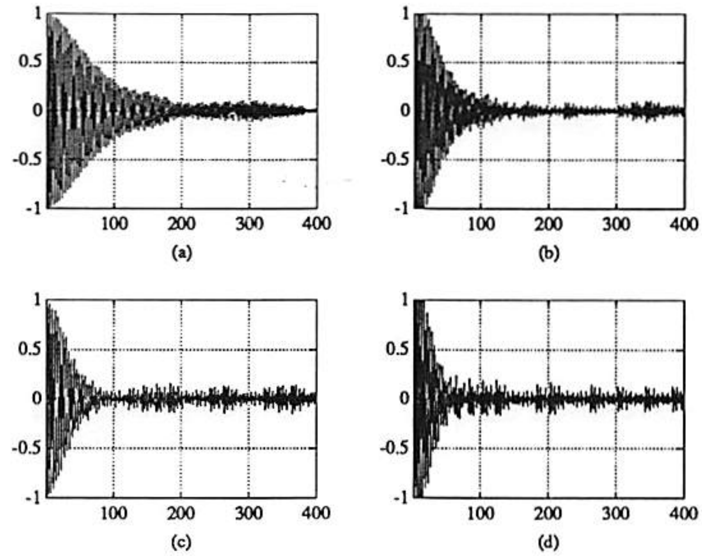


Figure 3: *Experiment 1*: For interference $\{I_2(k)\}$ with $A_2 = 1$, error curves obtained by the ANC–SOS algorithm ($N = 32$) when $\mu_s$ is (a) 0.025 (b) 0.05 (c) 0.075 (d) 0.1.

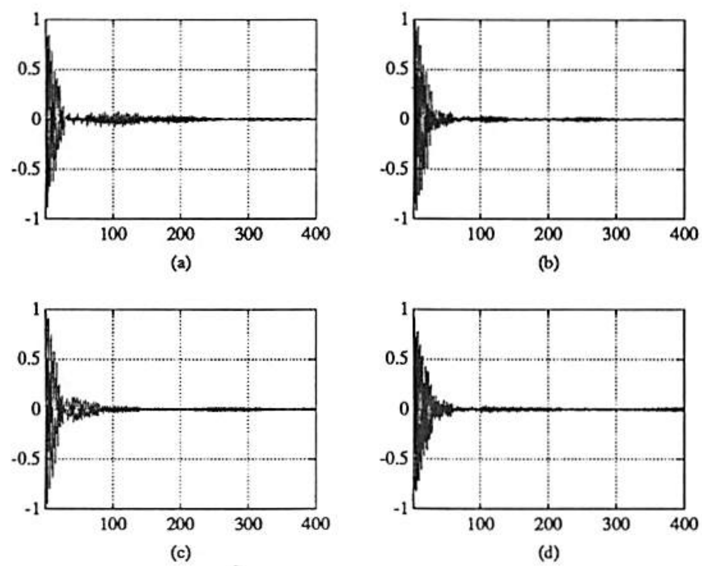


Figure 4: *Experiment 1*: For interference $\{I_2(k)\}$ with $A_2 = 1$, error curves obtained by the ANC–FOS algorithm ($N = 32$) when $\mu_f$ is (a) 0.995 (b) 0.9 (c) 0.85 (d) 0.8.

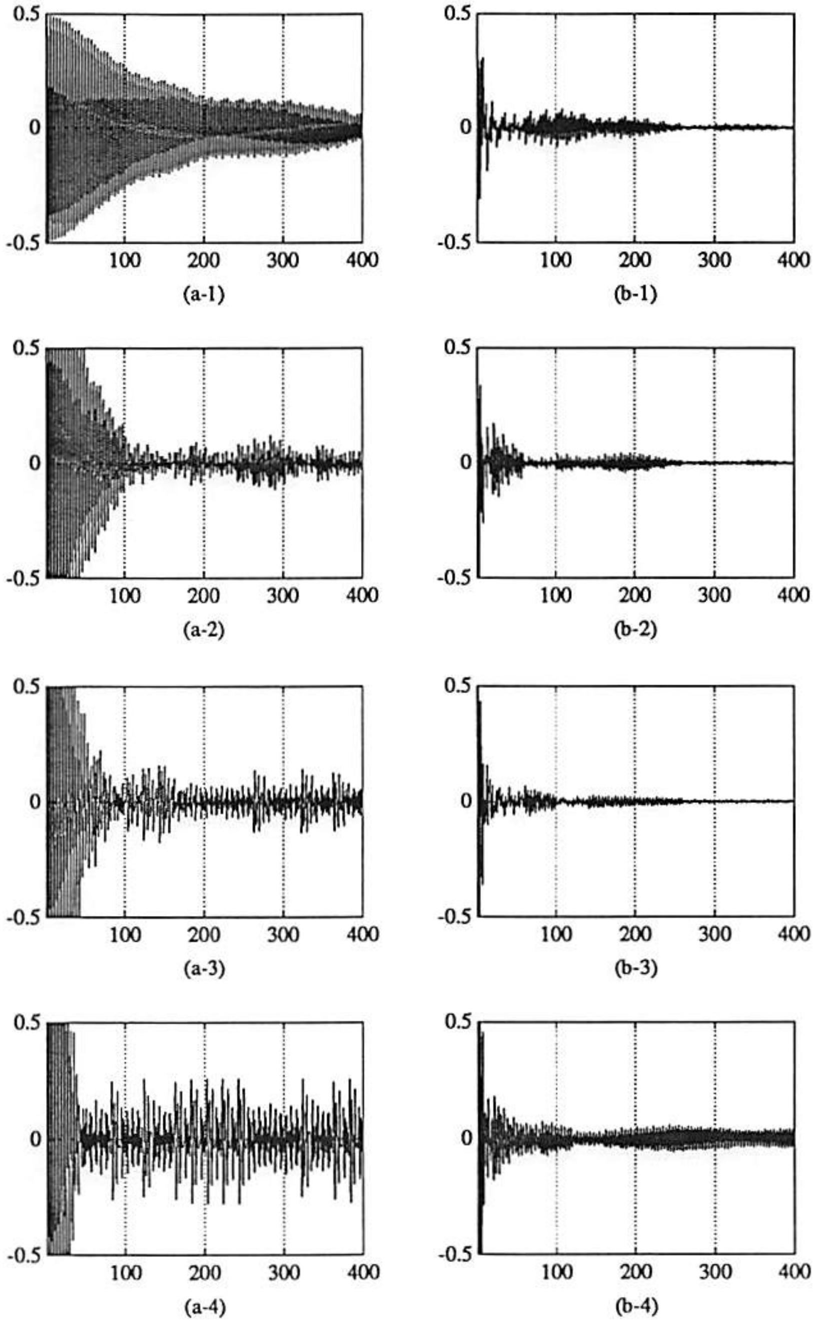


Figure 5: *Experiment 2*: For interference $\{I_2(k)\}$, the first column shows results obtained by the ANC–SOS algorithm ($N = 32$) when $A_2$ is (a–1) 0.5, (a–2) 1, (a–3) $\sqrt{2}$, and (a–4) 2.; the second column shows results obtained by the ANC–FOS algorithm ($N = 8$) when $A_2$ is (b–1) 0.5, (b–2) 1, (b–3) $\sqrt{2}$, and (b–4) 2.

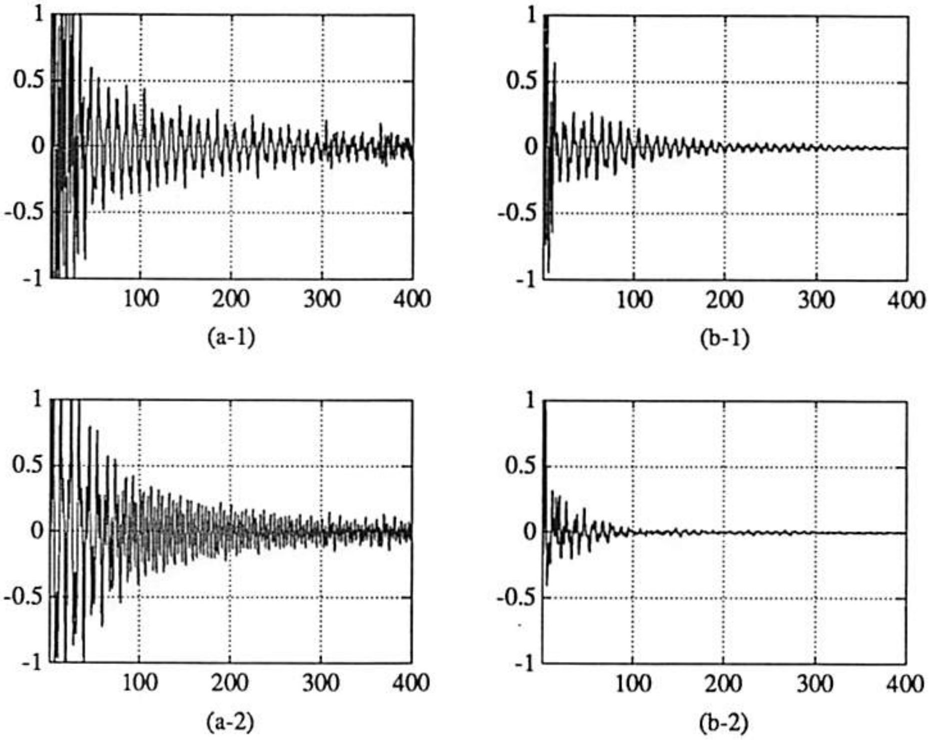


Figure 6: *Experiment 2*: For interference $\{I_1(k) + I_2(k)\}$, the first column shows results obtained by the ANC–SOS algorithm ($N = 32$) when (a–1) $A_1 = \sqrt{2}$ and $A_2 = 0.5$ and (a–2) $A_1 = 0.5$ and $A_2 = 1.$; the second column shows results obtained by the ANC–FOS algorithm ($N = 16$) when (b–1) $A_1 = \sqrt{2}$ and $A_2 = 0.5$ and (b–2) $A_1 = 0.5$ and $A_2 = 1$.

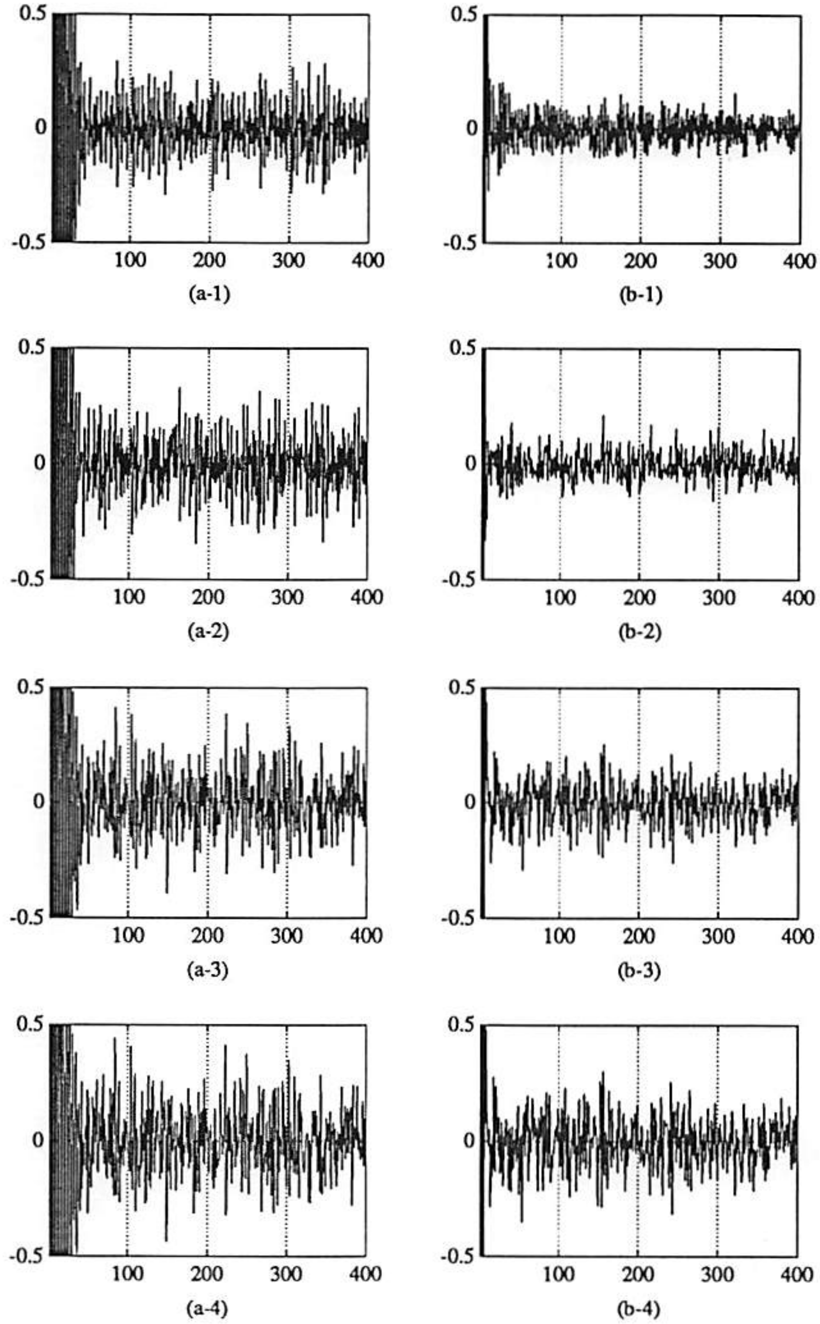


Figure 7: *Experiment 3*: For interference $\{I_2(k)\}$ and additive Gaussian uncorrelated noises, the first column shows results obtained by the ANC–SOS algorithm ($N = 32$) when both noise variances are (a–1) 0.001, (a–2) 0.0025, (a–3) 0.005, and (a–4) 0.0075.; the second column shows results obtained by the ANC–FOS algorithm ($N = 8$) when both noise variances are (b–1) 0.001, (b–2) 0.0025, (b–3) 0.005, and (b–4) 0.0075.

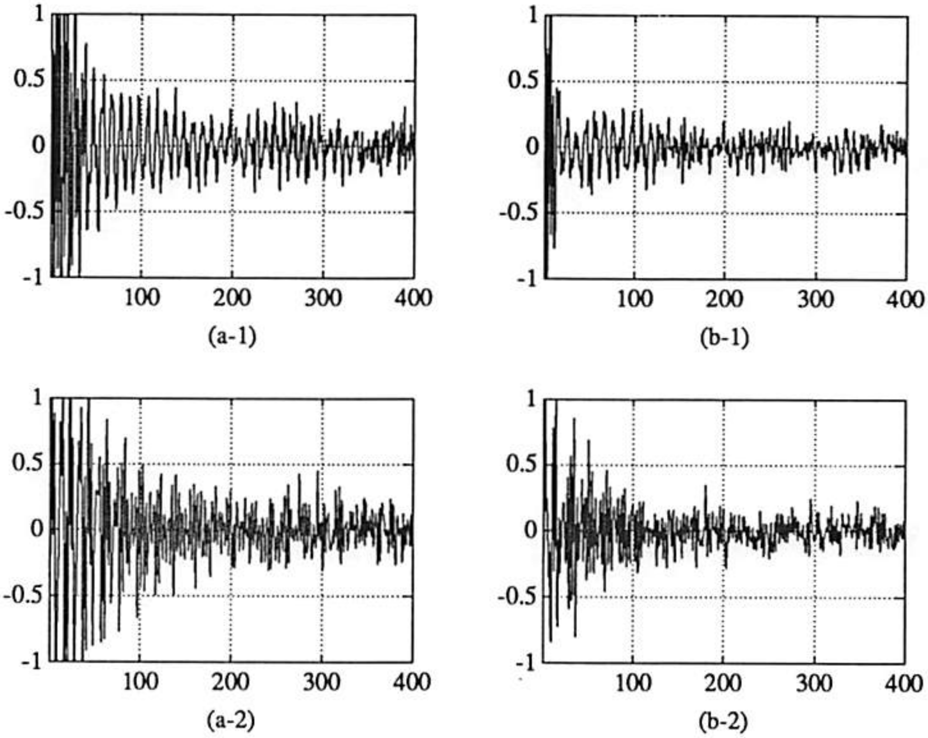


Figure 8: *Experiment 3*: In the presence of additive Gaussian uncorrelated noises with variance 0.005, the first column shows results obtained by the ANC–SOS algorithm ($N = 32$) when the interferences is (a–1) $\{I_1(k)+I_2(k)\}$ with $A_1 = \sqrt{2}$ and $A_2 = 0.5$ and (a–2) $\{I_1(k)+I_2(k)+I_3(k)\}$ with $A_1 = 0.5$, $A_2 = 1$, and $A_3 = 0.75$.; the second column shows results obtained by the ANC–FOS algorithm ($N = 16$) when the interferences is (b–1) $\{I_1(k) + I_2(k)\}$ with $A_1 = \sqrt{2}$ and $A_2 = 0.5$ and (b–2) $\{I_1(k) + I_2(k) + I_3(k)\}$ with $A_1 = 0.5$, $A_2 = 1$, and $A_3 = 0.75$.

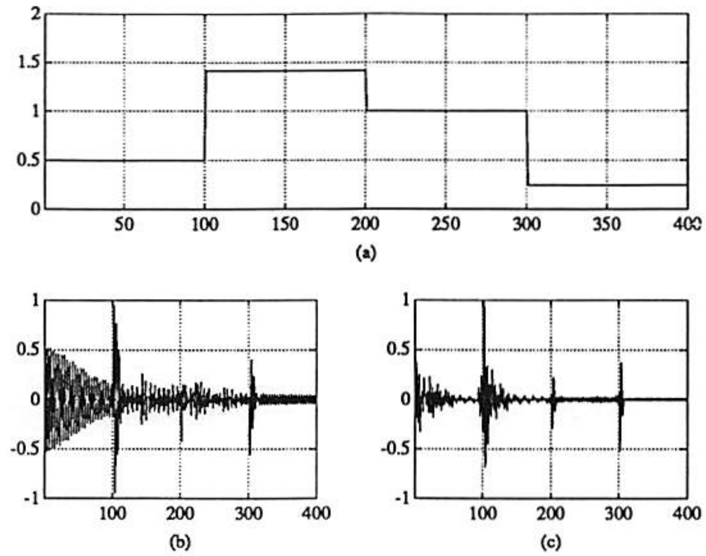


Figure 9: *Experiment 4*: For a wideband interference $\{I_2(k)\}$, (a) amplitude modulation of $A_2$ (b) the error curve obtained by the ANC–SOS algorithm ($N = 16$) and (c) the error curve obtained by the ANC–FOS algorithm ($N = 8$).

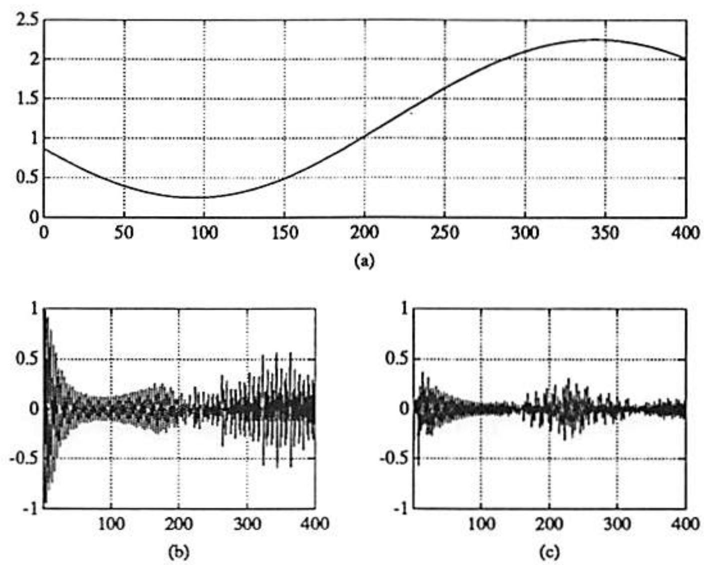


Figure 10: *Experiment 4*: For a wideband interference $\{I_2(k)\}$, (a) amplitude modulation of $A_2$ (b) the error curve obtained by the ANC–SOS algorithm ($N = 16$) and (c) the error curve obtained by the ANC–FOS algorithm($N = 8$).

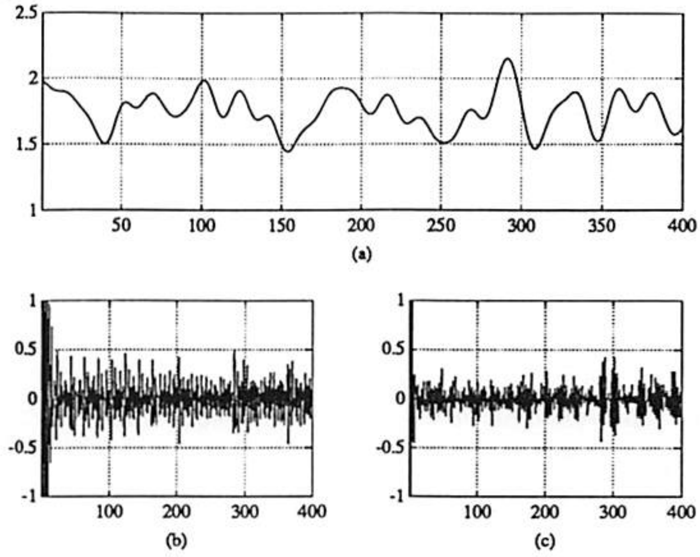


Figure 11: *Experiment 4*: For a wideband interference $\{I_2(k)\}$, (a) amplitude modulation of $A_2$ (b) the error curve obtained by the ANC–SOS algorithm ($N = 16$) and (c) the error curve obtained by the ANC–FOS algorithm ($N = 8$).

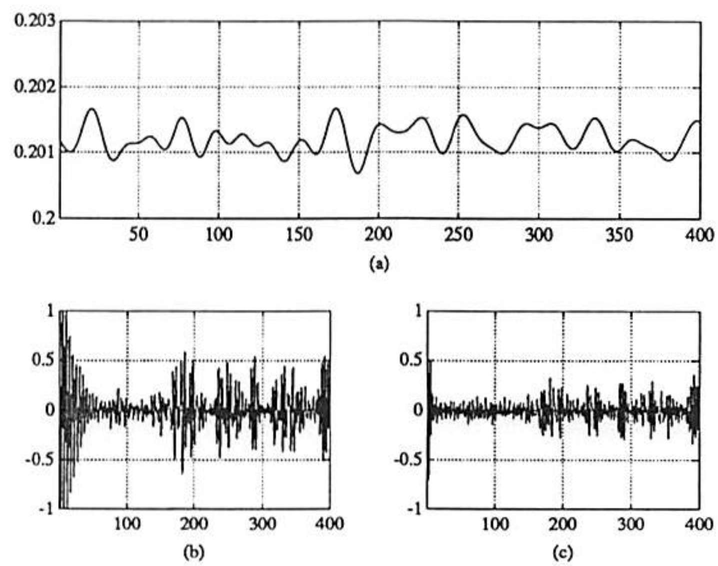


Figure 12: *Experiment 5*: For a wideband interference with constant amplitude, (a) frequency modulation, (b) the error curve obtained by the ANC–SOS algorithm ($N = 16$) and (c) the error curve obtained by the ANC–FOS algorithm ($N = 8$).