

# Type-2 Fuzzy Sets and Systems: How to Learn About Them

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## Abstract

This article is meant to alert readers to type-2 fuzzy sets and systems by focusing on the **already published** tutorial and educational vehicles about them. It lists these vehicles, provides their abstracts and section titles, and makes a few comments about each vehicle. The vehicles are grouped into four categories: starting-out vehicles, technical vehicles unencumbered by derivations, technical vehicles that include detailed derivations, and technical vehicle for a researcher.

## 1. Introduction

During the past decade, type-2 fuzzy sets and systems have become very popular<sup>1</sup>. For the reader who wants to find out, right now, "What are type-2 fuzzy sets and systems?", before they continue with the rest of this article, see item 2.A (Abstract) below.

This article is meant to alert readers to type-2 fuzzy sets and systems by focusing on the **already published** tutorial and educational vehicles about them. By "vehicle" is meant: textbook, published article in a technical transaction, magazine, or newsletter, an article on a website, or, an educational course. Such vehicles will provide the reader with excellent ways to learn about type-2 fuzzy sets and systems.

In this article<sup>2</sup> I list the tutorial and educational vehicles, provide their abstracts and section titles, and make a few comments about each vehicle. I have grouped the vehicles into four categories: starting-out vehicles, technical vehicles unencumbered by derivations, technical vehicles that include detailed derivations, and technical vehicle for a researcher. There is something for everybody!

## 2. Starting-Out Vehicles

These vehicles are easy to read, and include three articles, one each on the Wikipedia website, in a technical newsletter and in a technical magazine. They provide the reader with a **high-level understanding** of type-2 fuzzy sets and systems.

### 2.A "Type-2 Fuzzy Sets and Systems," Wikipedia ([http://en.wikipedia.org/wiki/Type-2\\_Fuzzy\\_Sets\\_and\\_Systems](http://en.wikipedia.org/wiki/Type-2_Fuzzy_Sets_and_Systems))

**Abstract:** Type-2 fuzzy sets and systems generalize (type-1) fuzzy sets and systems so that more uncertainty can be handled. From the very beginning of fuzzy sets, criticism was made about the fact that the membership function of a type-1 fuzzy set has no uncertainty associated with it, something that seems to contradict the word *fuzzy*, since that word has the connotation of lots of uncertainty. So, what does one do when there is uncertainty about the value of the membership function? The answer to this question was provided in 1975 by the inventor of fuzzy sets Prof. Lotfi A. Zadeh, when he proposed more

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<sup>1</sup> On May 24, 2009 when "type-2 fuzzy sets" was entered into Google, results for about 8,000 entries were listed.

<sup>2</sup> This article may appear to be somewhat self-serving, but the fact of the matter is that many of the tutorial and educational vehicles about type-2 fuzzy sets and systems have been written or co-authored by this author.

sophisticated kinds of fuzzy sets, the first of which he called a **type-2 fuzzy set**. A type-2 fuzzy set lets us incorporate uncertainty about the membership function into fuzzy set theory, and is a way to address the above criticism of type-1 fuzzy sets head-on. And, if there is no uncertainty, then a type-2 fuzzy set reduces to a type-1 fuzzy set, which is analogous to probability reducing to determinism when unpredictability vanishes.

**Section Headings:** General type-2 fuzzy sets, Interval type-2 fuzzy sets, Interval type-2 fuzzy logic systems, Computing with words, Further reading, References.

**Comments:** Today, it seems that when one wants to get information about a topic rapidly one goes to Wikipedia. There were already many articles in Wikipedia about fuzzy sets and systems and related topics (see the links to: computational intelligence, expert systems, fuzzy control system, fuzzy logic, fuzzy set, granular computing, rough set, soft computing and vagueness). This relatively new article is a very high-level introduction to type-2 fuzzy sets and systems with 30 references. It is a good place to begin your journey into such topics. Although there is no author listed for this article, it was written by myself with the assistance of Dr. Dongrui Wu, and went on-line in January 2009.

**2.B "Type-2 Fuzzy Sets: Some Questions and Answers,"** J. M. Mendel, *IEEE Neural Networks Society Newsletter*, vol. 1, August 2003, pp. 10-13.

**Abstract:** To use type-1 fuzzy sets as models for words is scientifically incorrect. Type-2 fuzzy sets let us model the uncertainties that are inherent in words as well as other uncertainties. This article introduces the reader to type-2 fuzzy sets through a series of questions and answers that will hopefully provide the motivation to learn more about them and to use them.

**Section Headings:** Introduction, Some questions and answers, Conclusions.

**Questions:** The following 16 questions are answered:

1. Can you be more specific about the "paradox" of a type-1 fuzzy set?
2. There are different kinds of uncertainty so which one(s) are you referring to, and where does randomness fit into all of this?
3. What exactly does "both kinds of uncertainty should be accounted for" mean?
4. Where do uncertainties occur in a rule-based fuzzy system?
5. What exactly is a type-2 fuzzy set and how is it different from a type-1 fuzzy set?
6. If all uncertainty disappears, does a type-2 fuzzy set reduce to a type-1 fuzzy set?
7. Why are the pictures in Fig. 1 two-dimensional when the membership function of a type-2 fuzzy set is three-dimensional?
8. Is there new terminology for a type-2 fuzzy set?
9. How does one choose the membership function for a type-2 fuzzy set?
10. Is there an increase in computational complexity using three-dimensional membership functions?
11. What is the earlier mentioned dispersion for a type-2 fuzzy set?
12. Other than modeling words, what are some situations where by using type-2 fuzzy sets we may outperform the use of type-1 fuzzy sets?
13. Why do we believe that by using type-2 fuzzy sets we will outperform the use of type-1 fuzzy sets?
14. Why did it take so long for the concept of a type-2 fuzzy set to emerge?
15. Why didn't type-2 fuzzy sets immediately become popular?
16. How can I learn more about type-2 fuzzy sets?

**Comments:** The Socratic style of this article makes it easy to read. The questions and their answers are as relevant today as they were when I wrote this article in 2003, except for the answer to the last question. Other introductory articles have been written since 2003, and you will find an updated answer to this question at the end of the Wikipedia article in the section entitled "Further reading," as well as in this article.

**2.C “Type-2 fuzzy logic: a historical view,”** R. John and S. Coupland, *IEEE Computational Intelligence Magazine*, Vol. 2, pp. 57-62, February 2007.

**Abstract:** Type-2 fuzzy logic offers an opportunity to model levels of uncertainty with which traditional fuzzy logic (type-1) struggles. This article provides a historical perspective of the development of type-2 fuzzy logic with a retrospective review of important developments in the field. We discuss key questions about the technology such as—when should type-2 systems be used? And why has it taken so long for this technology to emerge? The article concludes by highlighting areas where, we believe, type-2 fuzzy logic will have a significant role to play in the future.

**Section Headings:** Introduction, How has the field developed (Type-2 fuzzy sets appear, Type-2 fuzzy interval sets are promoted, Type-reduction is defined, Type-2 fuzzy logic systems are fully defined, The first textbook on the subject of type-2 fuzzy logic appears, The Representation Theorem is defined, Issues of computational complexity begin to be explored, Computing with words appears, Control applications, Medical applications, Summary), Why did it take so long for the concept of a type-2 fuzzy set to emerge?, Why didn't type-2 fuzzy sets immediately become popular, When should you consider using type-2 fuzzy logic?, Future directions, and Conclusions.

**Comments:** This is must reading for any reader who has a historical bent. It includes a histogram figure that shows the number of type-2 related publications over time (1975-2006), and a figure that provides a time line depicting the historical development of type-2 fuzzy logic. It also includes a Box entitled “Online resources for type-2 fuzzy logic,” that directs readers to four resources:

- <http://www.type2fuzzylogic.org>: This site hosts a list of people in the field, a community-maintained database of publications related to type-2, an interactive forum and list of links and other useful resources.
- <http://iee-cis.org/technical/standards>: This is the webpage for the Standards Committee of the IEEE Computational Intelligence Society. On this page, under “Benchmark Repository,” one will find an article by J. M. Mendel, H. Hagrass and R. I. John, containing standard background material about interval type-2 fuzzy logic systems that can be used by all authors. The standard is available in pdf and Word formats for both Macintosh and Windows PCs.
- <http://sipi.usc.edu/~mendel/software>: This site has freeware so that you can begin to use type-2 fuzzy logic immediately. The software has been prepared as a collection of M-files for Matlab ©.
- <http://www.cse.dmu.ac.uk/~rij/fstctf.html>: This site is run by a group of people who are charged with promoting extensions to type-1 fuzzy logic (including type-2 fuzzy logic) amongst academia and industry. This group is a Task Force of the Fuzzy Systems Technical Committee of the IEEE Computational Intelligence Society.

### 3. Technical Vehicles Unencumbered By Derivations

These vehicles put meat on the subjects of type-2 fuzzy sets and fuzzy systems and provide the reader with **technical details but do not encumber the reader with their derivations**. They include three technical magazine articles and two educational courses.

**3.A “Type-2 Fuzzy Sets and Systems: an Overview,”** Jerry M. Mendel, *IEEE Computational Intelligence Magazine*, Vol. 2, pp. 20-29, February 2007.

**Abstract:** This paper provides an introduction to and an overview of type-2 fuzzy sets (T2 FS) and systems. It does this by answering the following questions: What is a T2 FS and how is it different from a T1 FS? Is there new terminology for a T2 FS? Are there important representations of a T2 FS and, if so, why are they important? How and why are T2 FSs used in a rule-based system? What are the detailed computations for an interval T2 fuzzy logic system (IT2 FLS) and are they easy to understand? Is it

possible to have an IT2 FLS without type reduction? How do we wrap this up and where can we go to learn more?

**Section Headings:** Introduction, A type-2 fuzzy set and how it is different from a type-1 fuzzy set, New terminology for a type-2 fuzzy set, Important representation of a type-2 fuzzy set, type-2 fuzzy logic systems, Computations in an interval type-2 fuzzy logic system, An interval type-2 fuzzy logic system for real-time computations, Conclusions.

**Comments:** This article is technical in nature, meaning that it has equations. A unique feature of a magazine article is its “Boxes” each of which focuses on a single topic. This article has seven boxes:

- **Box 1:** New terms for type-2 fuzzy sets
- **Box 2:** Two very important representations of a type-2 fuzzy set
- **Box 3:** How type-1 fuzzy set mathematics can be used to derive interval type-2 fuzzy set fired-rule outputs
- **Box 4:** Pictorial descriptions for type-1 and type-2 inferences
- **Box 5:** A comparison of center of sets defuzzification and type-reduction<sup>3</sup>
- **Box 6:** Centroid of an interval type-2 fuzzy set and its computation
- **Box 7:** Uncertainty bounds and related computations

**3.B “Introduction to type-2 fuzzy sets and systems,”** Jerry M. Mendel, *IEEE Expert Now* multi-media educational course, sponsored by the IEEE Computational Intelligence Society, October, 2008, ISBN: 1-4244-1448-2, Run Time: 1:00:00.

<http://ieeexplore.ieee.org/articleSale/modulesabstract.jsp?mdnumber=EW1081>

**Abstract:** This course provides an introduction to and an overview of type-2 fuzzy sets (T2 FSs) and systems. It locates type-2 fuzzy sets and systems in an educational taxonomy, so that the student will appreciate from the onset the importance of studying such fuzzy sets; explains what a T2 FS is, how it is different from a type-1 FS, and why it is needed; provides careful definitions and pictures of the new terminology of T2 FSs; explains the importance of interval type-2 fuzzy sets over more general T2 FSs; explains important representations for a T2 FS (one is very good for computing, and another is very good for quickly developing the structure of the solution to a new theoretical problem); explains how T2 FSs are used in a rule-based system (a fuzzy logic system-FLS); describes the detailed computations that are used for an interval T2 FLS, relying mostly on graphical pictures; compares those computations with their type-1 counterparts; explains the major obstacle to using a T2 FLS in a real-time application and how that obstacle has been overcome; and wraps up the course with a plug for the applications course and a short reading list.

**Section Headings:** Type-2 fuzzy sets—especially interval type-2 fuzzy sets, Type-1 fuzzy logic systems, Type-2 fuzzy logic systems, and Conclusions.

**Comments:** This educational module has close to 50 slides and costs \$69.95 (The author collects no royalties.) It has voice overlay (not the author’s—a real professional) and uses some multi-media. It is closely connected to and coordinated with Reference 3.A above.

**3.C “Type-2 FLCs: a new generation of fuzzy controllers,”** Hani Hagra, *IEEE Computational Intelligence Magazine*, Vol. 2, pp. 30-43, February 2007.

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<sup>3</sup> Type- reduction is a new kind of computation. It maps the (multiple) fired rules in a type-2 fuzzy logic system into a type-1 fuzzy set, after which that set can be defuzzified into a crisp number.

**Abstract:** Type-1 fuzzy logic controllers (FLCs) have been applied to date with great success to many different applications. However, for dynamic unstructured environments and many real-world applications, there is a need to cope with large amounts of uncertainties. The traditional type-1 FLC using crisp type-1 fuzzy sets cannot directly handle such uncertainties. A type-2 FLC using type-2 fuzzy sets can handle such uncertainties to produce a better performance. Hence, type-2 FLCs will have the potential to overcome the limitations of type-1 FLCs and produce a new generation of fuzzy controllers with improved performance for many applications, which require handling high levels of uncertainty. This paper will introduce briefly the interval type-2 FLC and its benefits. We will also present briefly the type-2 FLC application to three challenging domains: industrial control, mobile robot control and ambient intelligent environmental control.

**Section Headings:** Introduction, Interval type-2 FLC, The application of interval type-2 FLCs to industrial control (The speed control of marine diesel engines, Other interval type-2 FLC applications to industrial control), The application of type-2 FLCs to mobile robots control, The application of type-2 FLCs to ambient intelligent environments control, Conclusions.

**Comments:** Although there are no equations or technical details in this paper, it is included in this section because it will be easier to understand after reading paper 3.A since it uses terms that are defined in that paper. This paper will be especially appealing to readers who develop control systems.

**3. D "Type-2 fuzzy logic controllers: Towards a new approach for handling uncertainties in real world environments,"** Hani Hagra, *IEEE Expert Now* multi-media educational course, sponsored by the IEEE Computational Intelligence Society, October, 2008, ISBN: 1-4244-1451-2, Run Time: 1:00:00.

<http://ieeexplore.ieee.org/articleSale/modulesabstract.jsp?mdnumber=EW1084>

**Abstract:** This course will have a large impact on a large audience as handling uncertainties will be a very important challenge to any real world application that operate in real world changing and dynamic environments. The course will present the theoretical aspects of type-2 FLCs and how to build a type-2 FLC. The course will also present many applications in different areas ranging from Control of Marine Diesel Engines, Autonomous Outdoor mobile Robots as well as Embedded Agents and Ambient Intelligent Environments which deals with how we can embed very efficient computational intelligence and type-2 techniques in small computing and memory platforms. The course will present a very clear description of type-2 Fuzzy Logic Controllers (FLCs), their design and their various application in handling the uncertainties in various real world applications. Different examples will be provided.

**Section Headings:** Sources of uncertainties in real world environments, Type-1 and type-2 FLCs, Overview of the interval type-2 FLC and its various components, Hierarchical type-2 FLCs, Type-2 FLC design, Overcoming the computational overheads in type-2 FLCs, Sample type-2 FLC applications, Future directions in type-2 FLCs, Conclusions

**Comments:** This educational module has close to 50 slides and costs \$69.95 (The author collects no royalties.) It has voice overlay (not the author's—a real professional) and uses some multi-media. It is closely connected to and coordinated with Reference 3.C above.

**3.E "Uncertain fuzzy clustering: insights and recommendations,"** Frank Chung–Hoon Rhee, *IEEE Computational Intelligence Magazine*, Vol. 2, pp. 44-56, February 2007.

**Conclusions:** (No abstract is provided, so instead the Conclusions section is stated, since it reads like an abstract.) In this article, interval type-2 fuzzy sets were used to model the uncertainty that is associated with the various parameters in objective function-based clustering. The purpose was to represent and manage the uncertainty in the cluster memberships by incorporating interval type-2 fuzzy sets. As a result, interval type-2 clustering methods were obtained by modifying the prototype-updating and hard-partitioning procedures in the type-1 fuzzy objective function-based clustering. As a

consequence, the management of uncertainty by an interval type-2 fuzzy approach aids clustering prototypes to converge to a more desirable location than a type-1 fuzzy approach. Several examples illustrate the effectiveness of interval type-2 fuzzy approach methods. Furthermore, the uncertainty associated with the parameters for other existing clustering algorithms can be considered in the development of other interval type-2 clustering algorithms.

**Section Headings:** Introduction, Uncertainties associated with objective function-based fuzzy clustering (Uncertainty in the fuzzifier in the FCM algorithm, Uncertainty in the “bandwidth” in the PCM algorithm, Uncertainty in selection of objective function), Extension to interval type-2 fuzzy sets, Overview of objective function-based interval type-2 fuzzy clustering, Experimental results (Different volume/equal low number “squares” data, Gaussian random generated data set with noise, Image segmentation), Conclusions.

**Comments:** This paper will be especially appealing to readers who are into pattern classification and clustering applications.

## 4. Technical Vehicles That Include Detailed Derivations

These vehicles not only put meat on the subjects of type-2 fuzzy sets and fuzzy systems and provide the reader with **technical details**, but also provide the reader **with their detailed derivations**. They are excellent “learning” vehicles, and include a textbook and two journal articles.

**4.A Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions**, Jerry M. Mendel, Prentice-Hall, Upper Saddle River, NJ, 2001.

**Abstract:** This textbook is organized into four parts. Part 1— *Preliminaries* — contains four chapters that provide background materials about uncertainty, membership functions, and two case studies (forecasting of time-series and knowledge mining using surveys) that are carried throughout the book. Part 2— *Type-1 Fuzzy Logic Systems*—contains two chapters that are included to provide the underlying basis for the new type-2 FLSs, so that the type-2 results for the case studies can be compared with type-1 results. Part 3— *Type-2 Fuzzy Sets*—contains three chapters, each of which focuses on a different aspect of such sets. Part 4— *Type-2 Fuzzy Logic Systems*—which is the heart of the book, contains five chapters, four having to do with different architectures for a FLS and how to handle different kinds of uncertainties within them, and one having to do primarily with four specific applications of type-2 FLSs.

**Chapter Titles:** Ch. 1: Introduction (including a short primer on fuzzy sets and fuzzy logic); Ch. 2: Sources of uncertainty; Ch. 3: Membership functions and uncertainty; Ch. 4: Case studies; Ch. 5: Singleton type-1 fuzzy logic systems: no uncertainties; Ch. 6: Non-singleton type-1 fuzzy logic systems; Ch. 7: Operations and properties of type-2 fuzzy sets; Ch. 8: Type-2 relations and compositions; Ch. 9: Centroid of a type-2 fuzzy set: type-reduction; Ch. 10: Singleton type-2 fuzzy logic systems; Ch. 11: Type-1 non-singleton type-2 fuzzy logic systems; Ch. 12: Type-2 non-singleton type-2 fuzzy logic systems; Ch. 13: TSK fuzzy logic systems; Ch. 14: Epilogue.

**Comments:** This is a very widely referenced textbook and can be used for a one-semester course or for self-study. To-date it is the only textbook that covers both type-1 and type 2 fuzzy sets and systems in a comprehensive and unified manner. It takes a top-down approach by first presenting results about general type-2 fuzzy sets and systems, and showing how they reduce to those for interval type-2 fuzzy sets and systems. Since its publication its material about interval type-2 fuzzy sets and systems can be presented more easily by using type-1 fuzzy set mathematics, as described in article 4.C.

**4.B “Type-2 fuzzy sets made simple,”** Jerry M. Mendel and Robert I. John, *IEEE Trans. on Fuzzy Systems*, vol. 10, pp. 117-127, April 2002.

**Abstract:** Type-2 fuzzy sets let us model and minimize the effects of uncertainties in rule-based fuzzy logic systems; however, they are difficult to understand for a variety of reasons which we enunciate. In this paper we strive to overcome the difficulties by: (1) establishing a small set of terms that let us easily communicate about type-2 fuzzy sets and also let us define such sets very precisely, (2) presenting a new representation for type-2 fuzzy sets, and (3) using this new representation to derive formulas for union, intersection and complement of type-2 fuzzy sets without having to use the Extension Principle.

**Section Headings:** Introduction, Type-2 fuzzy sets: definitions, A new representation for type-2 fuzzy sets, Applications (Union of type-2 fuzzy sets, Intersection of type-2 fuzzy sets, Complement of type-2 fuzzy sets, Interval type-2 fuzzy sets), Conclusion.

**Comments:** This paper is presently the most widely referenced paper on the subject of type-2. People seem to like a paper with “Made Simple” in its title. The Representation Theorem in this paper is frequently used as a starting place for developing new results both for general and interval type-2 fuzzy sets and systems.

**4.C “Interval Type-2 Fuzzy Logic Systems Made Easy,”** J. M. Mendel, R. I. John and F. Liu, *IEEE Trans. on Fuzzy Systems*, vol. 14, pp. 808-821, Dec. 2006.

**Abstract:** To-date, because of the computational complexity of using a general type-2 fuzzy set (T2 FS) in a T2 fuzzy logic system (FLS), most people only use an interval T2 FS, the result being an interval T2 FLS (IT2 FLS). Unfortunately, there is a heavy educational burden even to using an IT2 FLS. This burden has to do with first having to learn general T2 FS mathematics, and then specializing it to an IT2 FS. In retrospect, we believe that requiring a person to use T2 FS mathematics represents a barrier to the use of an IT2 FLS. In this paper, we demonstrate that it is unnecessary to take the route from general T2 FS to IT2 FS, and that **all of the results that are needed to implement an IT2 FLS can be obtained using T1 FS mathematics**. As such, this paper is a novel tutorial that makes an IT2 FLS much more accessible to all readers of this journal. We can now develop an IT2 FLS in a much more straightforward way.

**Section Headings:** Introduction, Interval type-2 fuzzy sets, Set theoretic operations, Review of type-1 FLS, Interval type-2 FLS (Introduction, Singleton fuzzification and one antecedent, Singleton fuzzification and multiple antecedents, Type-1 non-singleton fuzzification and multiple antecedents, Type-2 non-singleton fuzzification and multiple antecedents, Multiple rules, Output processing, Comments), Conclusions.

**Comments:** If I was teaching a one-semester course on fuzzy sets and systems and wanted to spend one to two weeks on type-2 fuzzy sets and systems, this is the reference I would use, because, as its abstract states, **all of the results that are needed to implement an IT2 FLS can be obtained using T1 FS mathematics**. So, students will have an easy time building upon what they have just learned in their studies about type-1 fuzzy sets and systems.

## 5. Technical Vehicle for a Researcher

In 2006 I was invited to write a state-of-the-art article for the journal *Information Sciences*. Its abstract that is presented below will make it clear why this section is entitled “Technical Vehicle for a Researcher.”

**5.A “Advances in Type-2 Fuzzy Sets and Systems,”** Jerry M. Mendel, *Information Sciences*, vol. 177, pp. 84-110, 2007.

**Abstract:** In this state-of-the-art paper, important advances that have been made during the past five years for both general and interval type-2 fuzzy sets and systems are described. Interest in type-2 subjects is worldwide and touches on a broad range of applications and many interesting theoretical

topics. The main focus of this paper is on the theoretical topics, with descriptions of what they are, what has been accomplished, and what remains to be done.

**Section Headings:** Introduction, General T2 FSs and FLSs (Representation theorem for a T2 FS, Operations on general T2 FSs, Type-2 FLS, Type-reduction (TR) for general T2 FSs, Similarity of T2 FSs), Interval T2 FSs and FLSs (Representation for an IT2 FS, Interpretations of an IT2 FS, Centroid of an IT2 FS, Interval T2 FLSs, Type-reduction and bypassing it for IT2 FLSs, Hardware realization of an IT2 FLS), The fuzzy weighted average (FWA) and TR, Computing with words, Conclusions.

**Comments:** Interests in both general and interval type-2 fuzzy sets and systems has continued since this paper was written and published. Readers who want to know what has happened since 2006 will have to check out the technical literature, e.g., the *IEEE Transactions on Fuzzy Systems*, *Information Sciences*, *Fuzzy Sets and Systems*, and the proceeding of conferences, e.g., IEEE Conference on Fuzzy Systems (FUZZ-IEEE, 2007, 2008, etc.), NAFIPS, IFSA and EUSFLAT.

## 6. Conclusions

This article has alerted readers to type-2 fuzzy sets and systems by focusing on the **already published** tutorial and educational vehicles about them. It has listed these vehicles, provided their abstracts and section titles, and made a few comments about each vehicle. The vehicles have been grouped into four categories: starting-out vehicles, technical vehicles unencumbered by derivations, technical vehicles that include detailed derivations, and technical vehicle for a researcher.

Start reading!



**Jerry M. Mendel** (S'59–M'61–SM'72–F'78–LF'04) received the Ph.D. degree in electrical engineering from the Polytechnic Institute of Brooklyn, Brooklyn, NY.

Currently he is Professor of Electrical Engineering and Systems Architecting Engineering at the University of Southern California in Los Angeles, where he has been since 1974. He has published over 480 technical papers and is author and/or editor of eight books, including *Uncertain Rule-based Fuzzy Logic Systems: Introduction and New Directions* (Prentice-Hall, 2001). His present research interests include: type-2 fuzzy logic systems and their applications to a wide range of problems, including smart oil field technology and computing with words.

Prof. Mendel is a Distinguished Member of the IEEE Control Systems Society, and a Fellow of the International Fuzzy Systems Association (2009). He was President of the IEEE Control Systems Society in 1986. He is a member of the Administrative Committee of the IEEE Computational Intelligence Society and was Chairman of its Fuzzy Systems Technical Committee. Among his awards are the 1983 Best Transactions Paper Award of the IEEE Geoscience and Remote Sensing Society, the 1992 Signal Processing Society Paper Award, the 2002 *Transactions on Fuzzy Systems* Outstanding Paper Award, a 1984 IEEE Centennial Medal, an IEEE Third Millennium Medal, a Fuzzy Systems Pioneer Award (2008) from the IEEE Computational Intelligence Society, and a Pioneer Award from the IEEE Granular Computing Conference, May 2006, for Outstanding Contributions in Type-2 Fuzzy Systems.