Stochastic rate-control of interframe video coders for lossy channels

Julián Cabrera*, José I. Ronda*, Antonio Ortega**, and Narciso García*

Abstract

We propose a new algorithm for the real-time control of an interframe video coder operating with a lossy channel such as wireless channels or the Internet. Based on a priori stochastic models of both the source and channel, the proposed formulation leads to an optimal control problem that can be solved off-line. The outcome of this optimization is an off-line control policy that is optimal in the sense of minimizing the average expected coding distortion. The on-line complexity of our approach is only that required to identify the state of the system (source and channel). The state of the channel is obtained based on the error-control mechanism, and the source state is computed as rate-based complexity measurements on each incoming frame. Unlike other optimization-based rate control techniques, which require a search for the optimal operating point, here the operating points for each allowable state of the system have been precalculated.

We consider variable-rate packet-based transmission with Automatic Repeat reQuest (ARQ) error control. While a standard model has been adopted to characterize the channel behavior, a new model based on the concept of coding complexity has been devised in order to characterize the video source taking into account the frame dependency that arises as a consequence of the use of interframe coding.

Simulation results based on this new approach are provided and compared to other proposed rate-control strategies. They show how our model-based optimal policies outperform the other considered approaches keeping a negligible on-line computational cost. This result is very promising as an alternative to traditional costly solutions based on Deterministic Dynamic Programming.

Index Terms

Stochastic rate control, coder optimization, wireless channels, Internet, interframe coding, video source model.

Manuscript received ___

*Grupo de Tratamiento de Imágenes, Universidad Politécnica de Madrid, E-28040 Madrid, Spain. E-mail: julian.cabrera@gti.ssr.upm.es.

**Integrated Media Systems Center, Department of Electrical Engineering, University of Southern California, Los Angeles, California 90089-2564.

This work has been partially supported by the Ministerio de Ciencia y Tecnología of the Spanish Goverment under project TIC2002-03692 (DYMAS). A preliminary version of this work has been published in [4].

LIST OF FIGURES

List of Tables

I. Introduction

In this paper we consider the problem of controlling video transmission over a lossy channel in real-time. The control of a video coder in order to optimize its performance over a given communication channel is a standard problem that has received considerable attention. It is well known that standard coders produce a variable numbers of bits per video unit and so there is a need for a *rate control* mechanism to adapt its output rate to the transmission rate of the communication channel. Moreover, the importance of rate control and its difficulty have recently increased due the use of lossy and variable rate channels as those of Internet or mobile communications. The transmission of information through such channels imposes the need to deal with a randomly time-varying bitrate. This characteristic has important consequences in the problem formulation since the stochastic behavior of the channel endows the whole system with an essentially stochastic nature.

For the rate control problem under consideration, a natural approach is to consider the use of specific tools for the control of stochastic systems. Let us assume first that the output of the source coder (which is variable due to changes in both input video and coding parameters) and the channel can be well modeled using stochastic finite state machines. If this is the case, then it is possible to optimize the coding control to meet pre-specified objectives based on *stochastic dynamic programming* (SDP) techniques [2]. This approach results an alternative to classical deterministic dynamic programming (DDP) with two main advantages: (i) SDP allows to take into account the stochastic nature of the system in the optimization process, and (ii) SDP solutions are computed off-line, thus the on-line computational burden of the control operation is drastically reduced. This is important given the non-linear computational complexity of deterministic optimization algorithms, especially in the case of interframe coding.

The traditional approach in the design of rate control strategies consists in the formulation of an optimization problem. The target function is an objective measure of the quality of the compressed video sequence, which usually depends on the output bitrate of the encoder according to its *rate-distortion (R-D) function*. The problem then consists in selecting for each video unit (frame or part of the frame) the coding parameters (usually quantizer) which will result in the adequate number of bits, in the sense of minimizing the coding distortion under the restrictions imposed by the channel (prevention of buffer overflow). Solutions to these optimization problems provide in general good quality results although they require a considerable computational cost which may be a limitation for real-time scenarios.

These approaches were first used for constant bit-rate (CBR) channels, where the encoder must ensure that its buffer never overflows (buffer constraint). Examples include [14] and [11], which formulate the problem of minimizing, under the buffer constraint, the distortion of a set of frames or macroblocks (those that are available at the moment of making a decision). The solution for that problem is then found using optimization techniques, such as Lagrangian optimization or the Viterbi algorithm. The main drawback of these approaches is their large computational cost since the encoder has to compute the R-D characteristics of the data and then search for the best quantizer assignment. Improvements to speed up these algorithms have been proposed by [11] or [16] (for low-delay scenarios). These works propose the use of R-D models so as to reduce the computational burden required.

Nevertheless, even if suitable models are available to reduce the complexity of computing R-D characteristics, some of these approaches still require a costly search to find the best operating point.

These above mentioned techniques were extended to address variable bit-rate channels such as ATM, and more recently Internet and mobile channels. Examples of these extensions can be found in [6][9] for ATM networks where the authors propose a similar optimization problem to that of [14]. The objective is to minimize the distortion of the decoded sequence taking into account the characteristics of ATM together with a deterministic knowledge of the source. In the case of wireless channels, [1] extends the formulas that rule the control strategy of [16] in order to consider the variable capacity of the channel. Similarly, [8] extends the work in [14] and [11], by incorporating in the problem formulation a stochastic model of the channel. The final solution is then found using both the channel model and the deterministic knowledge of the source data. Regarding Internet, [7] [13] formulate the R-D problem on streaming media. The objective of these approaches is to find an optimal control policy that determines which packets should be transmitted at every transmission opportunity. These control policies are computed on-line taking into account elements such as R-D characteristics, channel behavior and packet interdependencies. In [13] each decision is computed based on minimizing the expected distortion of the packets available at the sender buffer. The optimization problem is solved using an exhaustive search for the best policy, although a suboptimal greedy-search algorithm is proposed to speed-up the policy computation. On the other hand, [7] computes optimal policies for a group of data units so as to minimize the expected distortion subject to rate constraints resulting in an expected Lagrangian formulation.

This paper presents the SDP approach for the case of interframe coding and a lossy channel. SDP has been shown to be an alternative to deterministic optimization techniques, such as Viterbi Algorithm or Lagrangian optimization, in solving rate control problems [17][3]. We formulate an SDP minimization problem where the objective is to minimize the expected distortion of a decoded sequence. As indicated above, this requires the introduction of stochastic finite state machine models for the channel and the coder. Provided these models are available, SDP algorithms compute optimal solutions in the form of *control policies*. A control policy is a mapping that assigns the quantizer choice for each possible state of the system. These optimal policies can be computed a priori, so that during codec operation all that is required is to estimate the state of the system. In our problem formulation the state will be estimated by performing a single coding operation on the current frame, and monitoring the result of the ARQ mechanism.

This type of approach is clearly desirable in that the run-time complexity is minimal. Obviously, the main difficulty in designing such a control system is *defining general enough models for both video coder and channel*. A significant amount of work has been done to model the behavior of various types of channels. Thus, in this paper we will make use of existing channel models and focus our attention on much less developed finite state models for video coders. In particular, following our earlier work [3], which developed one such model for intraframe coding, here we tackle the more challenging problem of designing a model for interframe coding.

Recently, besides our previous work, the most closely related work is that of [7][13], where a policy-based system-state approach is proposed. Nevertheless, the main differences with our approach are: i) their system characterization combines a deterministic source model with a probabilistic expression of the expected distortion, while in our formulation we use a probabilistic model of both source and channel; ii) their control policies have to be computed on-line with the corresponding computational burden, while our policies are precomputed.

The main contributions of this paper are i) the extension of our previous work [3] for the interframe case and the validation of the methodology proposed there, and ii) the development of a novel rate-distortion model of the video source that takes into account the influence of previously coded frames. It is well known that considering interframe dependency requieres exponential increases in computational cost if, as is the case in the deterministic optimization approaches, exact knowledge of R-D operating points is needed. Thus, a general approach to cope with this computational requirement consists in limiting either the possible dependencies among video units and/or the number of video units considered in the problem [15][14][8][7][13]. Using our proposed novel source model, we formalize and solve a minimization problem for complete video sequences (300 frames) where the first frame is intraframe coded and the rest are coded as P frames. Although this model has been developed for the SDP problem formulation, it could also be used in other optimization approaches. Within the SDP approach, this model involves an increase in the off-line computational cost (control policy computation) which is perfectly affordable, while keeps the on-line computational cost similar to that of the intracoding case. Regarding the transmission channel, we use standard models of channel behavior that are appropriate for lossy channels under correlated errors, but these could be replaced by other models if a different channel behavior is observed.

In our experiments, we have considered an H.263 encoder [10] operating in interframe coding mode and using a single quantizer for the whole frame. This configuration of the encoder has allowed us to provide a reasonable framework for the comparison of the performance with other different rate control approaches. We have compared our proposed approach with the frame layer strategy presented in [16], an empirical mechanism not optimized for a variable channel, and with that presented in [1], which customizes the previous strategy for a variable channel similar to the considered in this work. We have simulated in our experiments a wireless channel with a selective-repeat ARQ [12] error control mechanism.

The experimental results show that SDP approach clearly outperforms the other two considered algorithms maintaining a reasonable on-line computational cost. This result is of much interest if we consider the computational cost required by algorithms based on deterministic optimization approaches [8][7]. Due to the interframe dependency, the computational cost associated to these algorithms is a real handicap for their use in real-time [15]. However, by means of SDP techniques the computational requirements of using interframe coding are absorbed in the computation of the optimal policies, a process that is carried out off-line. As a consequence, the on-line computational cost for systems using interframe coding is quite similar to that required by similar approaches for intraframe coding.

The rest of the paper is organized as follows. Section II specifies the operation environment and formalizes the control problem. Section III describes the model employed for the video source and Section IV depicts the communication channel model. Section V shows how these models are employed to obtain the SDP formulation for the control problem. Finally, Section VI presents the result of the control policies in the encoding of different test sequences comparing them with other alternatives, and Section VII presents the main conclusions.

II. Problem formulation

In this section we define the concrete control problem we tackle. The formalization of this problem in the language of SDP with a general theoretical treatment will be left for Section V.

We consider the real-time control of a video coder operating in a typical point-to-point transmission scenario involving a video source, a video coder, a transmission channel, and a video decoder with their respective buffers (see Fig. 1). A control system operates at the transmitter side. A key parameter of this transmission system is the maximum *end-to-end delay* ΔT between the instant at which a frame enters the coder and the instant at which it is presented at the output display. Frames that arrive too late at the decoder are discarded. In this case the decoder must apply some error concealment mechanism, which we assume will consist in replacing the lost frame by the last correctly received frame. Note that this assumption is a worst case bound to the general situation where only parts of the frame are lost.

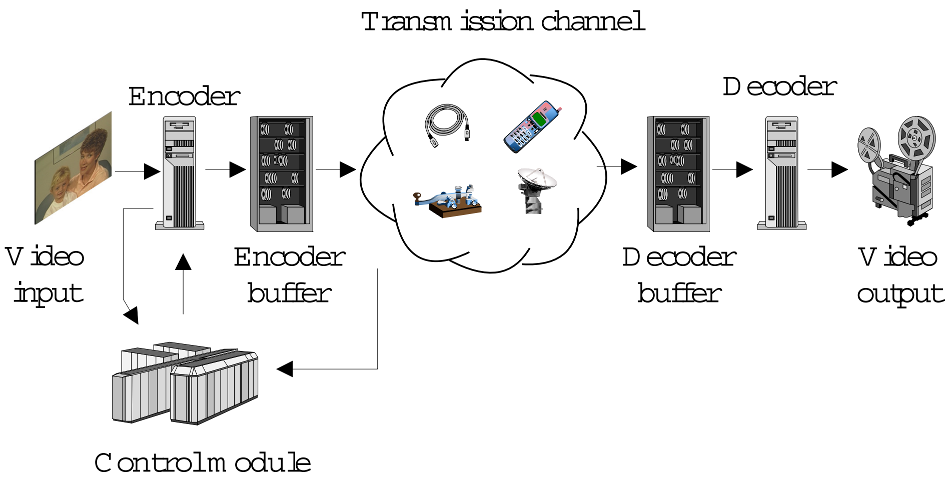


Fig. 1. Block diagram of a generic real-time video system.

We consider that the transmission channel sends FEC-protected packets of a *nominal payload* (C) at a constant *time interval* (T_p). As a consequence of transmission errors, the throughput of the channel varies, so that the effective payload of the packets is either C (when the transmission errors are within the protection capabilities of the FEC) or zero (otherwise). We assume that the probability of undetected errors is negligible. In order to recover from channel errors, a selective-retransmission ARQ mechanism is available, such that each packet is acknowledged by the receiver by sending either a positive or a negative acknowledgement (ACK or NAK respectively). In case of receiving NAKs, the transmitter resends the data. A time-out mechanism is established so that if the ACK/NAK is not received within a specified time, a packet loss is assumed.

We will compute optimal control policies for a simplified scenario in which there are no ACK/NAK losses and these acknowledgements are received with a negligible delay. Note that if the actual ACK/NAK delay is small with respect to the frame period, error propagation due to frame losses can be prevented if the encoder processes each frame only when the previous one has been acknowledged (positive or negatively).

We have also considered the existence of larger ACK/NAK delays by devising two different retransmission mechanisms to avoid the error propagation due to frame losses, described in Section VI-B. Our results will show that, in this way, policies computed using the approximation of a negligible delay operate in a scenario of a larger delay with a limited impact on performance, provided that this delay is short relative to the memory of the channel (average error burst length).

As will be seen, end-to-end delay and finite bandwidth channel limit the total number of bits that can be stored in the encoder/decoder buffers. We assume the physical buffers to be sufficiently large so that the only constraints are those imposed by the end-to-end delay, rather than by the size of the buffer. In our experiments we use an H.263 encoder [10] using interframe coding in the following manner: (i) the first frame is intraframe coded, (ii) the rest of the sequence is coded as P frames using as reference image the last correctly transmitted frame. We focus on the selection of quantizer for a whole frame, although this frame level quantization choice can be also used as a guide for further blockwise bit allocation.

Our purpose will be to define the algorithm for the real-time selection of these quantizers so that the quality of the decoded video signal is optimized. This quality will be the result of two factors: the number of successfully transmitted frames and their quality. We will use objective signal distortion metrics, namely mean square error (MSE). For successfully transmitted frames this error corresponds exclusively to quantization error. In case the frame is lost, the distortion error corresponds to the mean square value of the difference between the original frame and the frame that takes its place at display time, i.e., the one provided by the error concealment algorithm. Note that the assumption of receiving ACKs/NAKs with a negligible delay prevents error propagation, since when the encoder starts the transmission of a new frame, it already knows whether the previous frame will be available at the decoder (then this frame will be used as the reference image for the new frame) or not (in this case, the coder will use the last correctly transmitted frame as reference image). Thus, the final distortion for the whole sequence will be the sum of the distortion incurred by each frame:

$$D_{sequence} = \sum_{k \in \mathcal{T}} d_k + \sum_{m \in \mathcal{L}} d_{lf,m}$$

where d_k indicates the distortion of frame k, $\mathcal{T}$ is the set of indices of the correctly received frames, $d_{lf,m}$ is the distortion incurred by frame m in case it does not arrive to the decoder, and $\mathcal{L}$ is the set of indices corresponding to lost frames.

Our goal is then to choose optimally between (i) employing a finer quantizer, which ensures lower coding distortion but requires more bits (and thus makes it more difficult to retransmit lost data) and (ii) using a coarser quantizer, which produces fewer packets per frame, thus increasing the probability that all these packets will reach the decoder in time (because retransmission is easier). Three types of information about the system will be relevant for our decision:

1) Channel behavior (e.g., likelihood that transmission errors will occur.) Therefore a channel model can be used to estimate loss probabilities based on prior observed behavior.

2) Characteristics of the current frame and, if possible, characteristics of some of the future frames. In this case a probabilistic model can be used to predict future behavior.
3) Maximum number of time slots available for the transmission of the current frame (time left for the current frame). This variable can be computed as a function of the behavior of source and channel.

In order not to increase the total delay, we will not consider any look-ahead to model the video behavior. Thus, we will use a simple precoding of the current frame to estimate the state of the source. Consequently the decision on the current quantizer will be made exclusively on the basis of the coding properties of the current and past frames, past channel information, and the time left for the current frame.

SDP is the standard technique for the optimal control of a stochastic system considering the minimization of a time-additive cost. The video transmission system described fits these two requirements: (i) due to the unpredictable nature of the video source and the channel, the whole system shows a stochastic behavior, (ii) the distortion of the decoded sequence is a time-additive cost. Thus, we propose the use of SDP to solve the rate control problem of the video transmission system described. The formulation of the rate control problem in terms of SDP will be presented in Section V.

Policies obtained by SDP algorithms are optimal for the given models. This means that if the characteristics of either the source or the channel change considerably, a new policy should be computed taking into account these new behaviors. In this work, we consider that the characteristics of the channel and the video source are stationary in the sense that the models can be considered valid for the whole operation of the system. Nevertheless, in a more general scenario where such changes occur, some adaptive mechanism should be provided such as computing several policies for the different models of video sources and/or channels expected. Then, the rate control mechanism should track the characteristics of the source and channel in order to determine the most appropriate policy in each case.

III. Video Source Model

The use of SDP techniques [2] requires the a priori modeling of the system under study as a *controlled Markov chain*, i.e., as a stochastic system in which the state probabilities at time $k+1$ only depend on the state at time k and the control parameter q_k. With this objective, we propose the following model for the video source.

In our prior work [3] we presented a general structure for the video source model based on the definition of a parameter, the *frame complexity*, which characterizes the R-D functions of each frame. This parameter was estimated for every frame by encoding it with a given quantizer (the more bits are produced, the more complex the frame is). Then, we modeled the R-D functions of intraframe coded video using a deterministic model that would, given the complexity of a frame, provide estimates of the rate and distortion for that frame at any quantizer. Moreover, a Markov model was devised to capture the variations over time of frame complexity. In the context of that work, frame complexity completely characterized the current and future behavior of the sequence.

In this work we extend our prior techniques to the case where interframe coding is used in a $IPPP\ldots$ fashion. Thus, the challenge is to capture not only the spatial complexity of the intraframe case, but also the temporal complexity due to dependencies across frames. As before, our final goal is to define a measurement that can model the intrinsic temporal variations of the video signal. We first describe at a high level the assumptions made in our modeling. Then in the follow-up sections we will justify the validity of our assumptions and show how the parameters for our model can be chosen.

In order to develop the source model, we have followed a similar approach to [3]: From the analysis of real R-D measurements for different frames, we aim at deriving functions that characterize the relationships present among these R-D values, and considering the use of interframe coding. In this case, the R-D characteristics for a given frame depend on the quantizer choice for the current frame, and also on choices made for all the previous frames since a coded frame $k-1$ is used as reference image for frame k. Denote q_k a quantization choice for frame k. Then, rate and distortion for frame k will depend on choices made for all frames $0,\ldots,k$, thus, r_k and d_k can be considered as functions of quantizers q_0 to q_k:

$$r_k(q_k, q_{k-1}, \ldots, q_0), \quad d_k(q_k, q_{k-1}, \ldots, q_0)$$

Fig. 2 shows a tree diagram which represents an example of the different coding possibilities that arise as a consequence of the interframe dependency for a simplified encoder with only three different quantizers ($q_k \in \{1,2,3\}$). This example corresponds to a coding scheme where the first frame is coded intraframe and the rest of frames are coded as P frames. For each frame, a circle represents a possible R-D value for a specific choice of quantizers from frame 0 to the frame under consideration. As can be observed, as the frame index k increases the number of possible R-D values increases exponentially, which results in a great difficulty for both, to compute all the possible codings and to capture them in a model [15].

In this scenario, we propose a source model which captures the interframe dependency focusing only on the quantizer choice for the current and previous frame. We assume that most of the R-D variations for frame k depend only on q_k and q_{k-1}. Provided that frames $k-1$ and k are of similar characteristics, this assumption is reasonable since the quality of the reference image for frame k, which depends on q_{k-1}, plays an important role in the R-D values of frame k. Therefore, we consider that R-D values for any frame k are functions of only these two quantizers, that is, $r_k(q_k, q_{k-1})$ and $d_k(q_k, q_{k-1})$. In Fig. 2, it can be observed that this assumption considers that all those encoding combinations that have the same last two branches are considered equivalent. Note that this simplification allows us to establish a tractable modeling problem since the source model has deal with only $n^2(Q)$ R-D values for any frame k instead of $n^k(Q)$ (where $n(Q)$ indicates the number of available quantizers in the encoder). This approach will allow us:

- To establish a reasonable accurate source model which provides estimates of $r_k(q_k, q_{k-1})$ and $d_k(q_k, q_{k-1})$ variables for any pair of quantizers q_k and q_{k-1} in a similar way to the model developed in our previous work [3].
- To capture the R-D behavior of a sequence coded within the video transmission system we are considering.

In our previous work for the intraframe case, we developed a deterministic model which, for each frame of a sequence, is able to estimate from just one single rate measurement any of the rest of R-D values. As was pointed

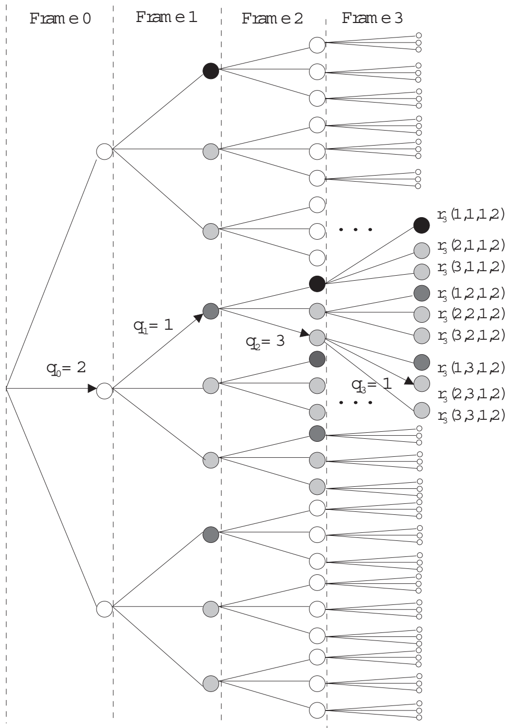


Fig. 2. Tree diagram which represents an example of the different coding possibilities for a simplified encoder with only three different quantizers ($q_k \in \{1, 2, 3\}$). This example corresponds to a coding scheme where the first frame is coded intraframe and the rest of frames are coded as P frames.

out before, this rate measurement was adopted as the frame complexity measurement. This model was based on exploiting the following R-D relationships:

1) Rate relationship. Let $r_k(i)$ and $r_k(j)$ be the rate values that result from the encoding all the frames of the sequence with quantizer i and j. There exists a relationship between $r_k(i)$ and $r_k(j)$ variables for all combinations of i and j, which can be captured by a linear model. Therefore, given a rate value for a reference quantizer, Q_r, we are able to predict the rate value at any other quantizer q_k.
2) Distortion relationship. Similarly, let $d_k(i)$ and $d_k(j)$ be the distortion values that result from the encoding all the frames of the sequence with quantizer i and j. There exists a relationship between $d_k(i)$ and $d_k(j)$ variables for all combinations of i and j, which can be captured by a linear model. Therefore, given a distortion value for a reference quantizer, Q_d, we are able to predict the distortion value at any other quantizer q_k.
3) Cross relationship between rate and distortion. This relationship allowed us to establish a connection between the frame complexity (which is a rate measurement) and the distortion values. Provided an appropriate selection of the reference quantizers Q_r and Q_d, there exists a relationship between $r_k(Q_r)$ and $d_k(Q_d)$ which is also captured by a linear model. Thus, given a rate value for a reference quantizer Q_r we are able to predict the distortion value at Q_d, and using $d_k(Q_d)$ we can estimate any other $d_k(q_k)$ using the model for the distortion relationship.

The source model was completed adopting a Markov model for the variations over time of the frame complexity, which allows to estimate the evolution $r_k(Q_r) \rightarrow r_{k+1}(Q_r)$.

For the interframe case, we propose an extension of the intraframe source model with: (i) a new definition of the frame complexity, adapted to our approach to the modeling of the interframe dependency, (ii) new models for the R-D relationships among frames when interframe dependency is present, which will provide for each frame estimates of any R-D value from its complexity measurement.

The frame complexity will be defined as a single rate measurement over the current frame, but imposing also a condition on the quantizer applied to the previous frame. Specifically, it corresponds to $r_k(Q_0^r, Q_{-1}^r)$, where Q_0^r and Q_{-1}^r are fixed reference quantizers for the current and previous frames respectively. Thus, in order to compute such reference measurement for every frame k two on-line codings are required, i.e, encoding frame $k-1$ with quantizer Q_{-1}^r and then, encoding frame k with quantizer Q_0^r using the previously coded frame as reference image.

Regarding the R-D functional relationships among frames, we have extended those of the intraframe case to the R-D values obtained in the interframe case, and captured in models in a similar way (see section III-A). More specifically:

1) Rate relationship. Given a frame k, it allows the model to estimate rate values for any pair of quantizers $r_k(q_k, q_{k-1})$ from a reference rate measurement at a fixed pair of quantizers, $r_k(Q_0^r, Q_{-1}^r)$ on that frame.

$$r_k(q_k, q_{k-1}) = f_r(r_k(Q_0^r, Q_{-1}^r))$$

2) Distortion relationship. Similarly to the rate relationship, given a frame k it allows the model to estimate distortion values at any pair of quantizers $d_k(q_k, q_{k-1})$ from a reference distortion measurement at a fixed pair of quantizers on that frame, $d_k(Q_0^d, Q_{-1}^d)$. Q_0^d and Q_{-1}^d are the fixed reference quantizers for the current and previous frames respectively.

$$d_k(q_k, q_{k-1}) = f_d(d_k(Q_0^d, Q_{-1}^d))$$

3) Cross relationship between rate and distortion. Given a frame k, it allows the model to estimate the reference distortion measurement $d_k(Q_0^d, Q_{-1}^d)$ from $r_k(Q_0^r, Q_{-1}^r)$, with an appropriate selection of the four reference quantizers Q_0^r, Q_{-1}^r, Q_0^d, Q_{-1}^d.

$$d_k(Q_0^d, Q_{-1}^d) = f_{rd}(r_k(Q_0^r, Q_{-1}^r))$$

These functional relationships will be captured by the deterministic part of the proposed source model: the *frame model*.

The modeling work will be completed with the definition of a Markov model to capture the variations over time of the R-D characteristics based on the frame complexity parameter: the *sequence model*. More specifically, the sequence model will capture the evolution $r_k(Q_0^r, Q_{-1}^r) \rightarrow r_{k+1}(Q_0^r, Q_{-1}^r)$.

In the following sections we will describe how the various components of the model are chosen and will provide experimental evidence that they provide sufficient modeling accuracy. We will present suitable approaches for the different relationships and will discuss how to choose the specific quantizer pairs (Q_0^r, Q_{-1}^r) and (Q_0^d, Q_{-1}^d) to be used in the measurements.

We have developed particular instantiations of the model using R-D measurements of an H.263 encoder using MSE as the distortion metric. Nevertheless, the model structure is not restricted to H.263 and could be applied to other encoders with a similar coding scheme (H.261, MPEG1-2-4, H.264 operating in single frame frame mode, etc.). In the particular case we are considering, each of the quantization parameters q_k corresponds to one of the admissible quantizers defined in the H.263 standard.

A. Frame model

As described before, the frame model comprises the deterministic components of the source model. Given a frame k, its final goal is to estimate any R-D value of a given frame at any choice of quantizers from a single rate measurement, i.e., from its complexity measurement. This goal will be accomplished by modeling the functional relationships present among R-D values between different frames.

1) Rate model: Let (i,j) and (i',j') be two different sets of fixed quantizer choices for frames k and $k-1$. Let $r_k(i,j)$ and $r_k(i',j')$ be the rate variables resulting of encoding all the frames of a given sequence with these quantizers. Then, for all the possible values of i, j, i', and j', we have found a relationship between pairs of rate variables, ($r_k(i,j)$, $r_k(i',j')$). This relationship can be captured as a composition of two linear relationships (see Fig. 3.a as an example of such linear relationships):

- Let us consider pairs of variables with the same quantizer for the previous frame, i.e., pairs of the form $r_k(i,j)$ and $r_k(i',j)$. Then for all values of i, j and i', the relationship between ($r_k(i,j)$, $r_k(i',j)$) can be captured via a linear model. With such a model, given the rate value for a reference pair of quantizers (Q_0^r, j) we will be able to estimate the rate value at any other pair of quantizers of the form (q_k, j). Thus, the linear equations for this part of the model are:

$$r_k(q_k,j) = a(q_k,Q_0^r)\, r_k(Q_0^r,j) + b(q_k,Q_0^r). \tag{1}$$

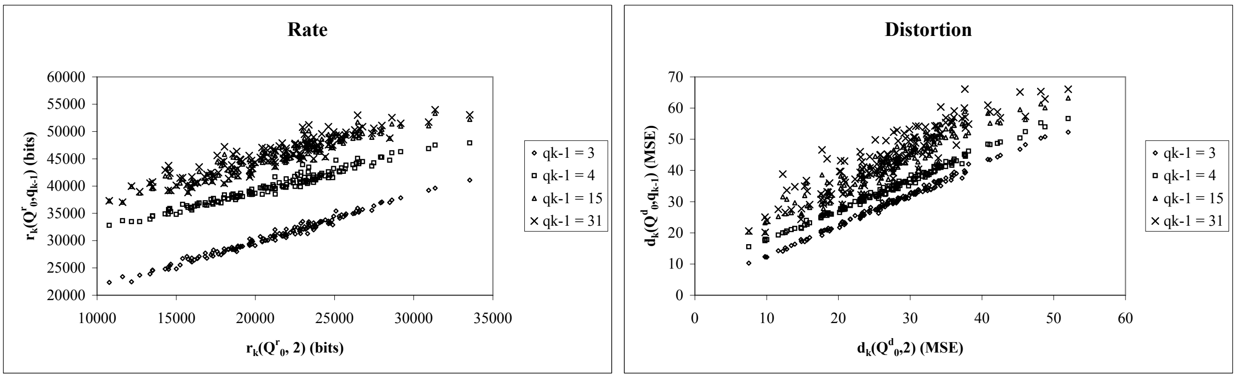
  Note that for any value of q_k different to i there will be a linear equation with coefficients $a(.)$ and $b(.)$.

- Let us now consider pairs of variables with the same quantizer for the current frame, i.e., variables of the form $r_k(i,j)$ and $r_k(i,j')$. Similarly to the previous case, for all values of i,j and j', the relationship between ($r_k(i,j)$, $r_k(i,j')$) variables can be captured via a linear model. With such a model, given the rate value for a reference pair of quantizers (i, Q_{-1}^r) we will be able to estimate the rate value at any other pair of quantizers of the form (i, q_{k-1}). The linear equations for this part of the model are:

$$r_k(i,q_{k-1}) = e(q_{k-1},Q_{-1}^r)\, r_k(i,Q_{-1}^r) + f(q_{k-1},Q_{-1}^r) \tag{2}$$

  Note that for any value of q_{k-1} different to j there will be a linear equation with coefficients $e(.)$ and $f(.)$.

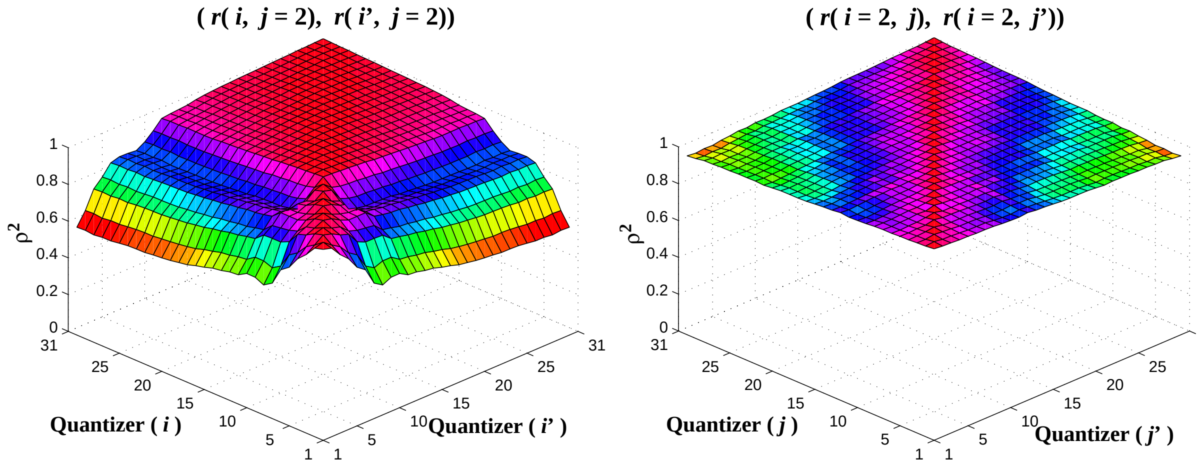
Therefore, given the rate value for a reference pair of quantizers (Q_0^r, Q_{-1}^r) and using eqs. (1)(2) we are able to estimate the rate value at any other pair of quantizers (q_k, q_{k-1}).



(a) (b)

Fig. 3. Example of linear relationship between pairs of variables $(r_k(Q_0^r, 2), r_k(Q_0^r, q_{k-1}))$ and $(d_k(Q_0^d, 2), d_k(Q_0^d, q_{k-1}))$ for several quantizers q_{k-1}. These values correspond to the sequence "silent" using $Q_0^r = 2$ and $Q_0^d = 31$.

In order to assess the accuracy of this approach, we have computed the squared coefficient of correlation (determination coefficient) ρ^2 for several pairs of $(r_k(i,j), r_k(i',j))$ and $(r_k(i,j), r_k(i,j'))$ for different values of i, j, i' and j'. We have used several sequences obtaining similar results in all cases. As a sample, Fig. 4 shows the results for sequence "silent". As can be observed, the value of ρ^2 is significantly large in most of the cases, thus the use of linear models is justified.



(a) (b)

Fig. 4. Determination coefficient ρ^2 for pairs of variables (a) $(r_k(i, j = 2), r_k(i', j = 2))$ and (b) $(r_k(i = 2, j), r_k(i = 2, j'))$. Each graphic corresponds to the coding of the "silent" sequence. Note that the higher correlation level is, the better fitting to the proposed model equations is achieved.

2) Distortion model: In the case of distortion variables, $d_k(i,j)$ and $d_k(i',j')$, an analogous analysis can be applied: for all the possible values of i, j, i', and j', we have found that there exists a relationship between pairs of rate variables, $(d_k(i,j),\ d_k(i',j'))$, which can be captured again as a composition of two linear relationships (see Fig. 3.b as an example of such linear relationships). The equations for the distortion variables are analogous to those of the rate analysis:

$$d_k(q_k, j) = c(q_k, Q_0^d)\, d_k(Q_0^d, j) + d(q_k, Q_0^d), \tag{3}$$

$$d_k(i, q_{k-1}) = g(q_{k-1}, Q_{-1}^d)\, d_k(i, Q_{-1}^d) + h(q_{k-1}, Q_{-1}^d). \tag{4}$$

where Q_0^d and Q_{-1}^d play the same role as Q_0^r and Q_{-1}^r. Therefore, given the distortion value for a reference pair of quantizers (Q_0^d, Q_{-1}^d) and using eqs. (3)(4) we are able to estimate the distortion value at any other pair of quantizers (q_k, q_{k-1}).

Regarding the accuracy of this approach, we have computed the determination coefficient ρ^2 for several pairs of $(d_k(i,j), d_k(i',j))$ and $(d_k(i,j), d_k(i,j')$ for different values of i, j, i' and j', and we have obtained similar levels of ρ^2 to those of the rate variables.

3) Cross rate-distortion model: As pointed out before, in our previous work on intraframe coding [3], we noticed that there exists a cross relationship between rate and distortion variables with an appropriate selection of the fixed quantizers. This relationship allows us to establish a connection between the frame complexity (which is a rate measurement) and the distortion values. In particular, pairs of variables $(r_k(Q_r), d_k(Q_d))$ showed a high level of correlation for low values of Q_r (fine quantizers) and high values of Q_d (coarse quantizers), suitable for the application of a linear model. This indicates that, given two frames k and l, if encoding frame k using a fine quantizer produces more bits than encoding frame l, the distortion due to using a coarse quantizer on frame k will be higher than that due to using the same quantizer on frame l. This seems reasonable since using a coarse quantizer for a frame with more information results in a larger loss, and therefore, in a higher distortion for the coded frame.

In the development of the model for the interframe case, we have verified that there also exists a reasonable level of correlation between a cross pair of R-D variables $(r_k(Q_0^r, Q_{-1}^r),\ d_k(Q_0^d, Q_{-1}^d))$ with an appropriate selection of quantizer choices Q_0^r, Q_{-1}^r, Q_0^d, and Q_{-1}^d. Thus, it can be also captured via a linear model:

$$d_k(Q_0^d, Q_{-1}^d) = m(Q_0^r, Q_{-1}^r, Q_0^d, Q_{-1}^d)\, r_k(Q_0^r, Q_{-1}^r) + n(Q_0^r, Q_{-1}^r, Q_0^d, Q_{-1}^d). \tag{5}$$

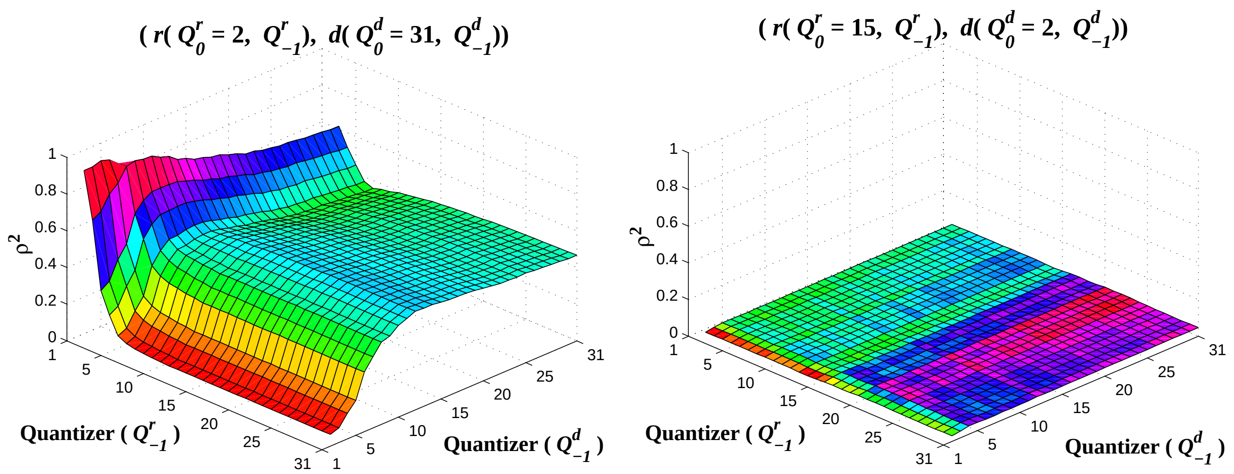
where coefficients $m(.)$ and $n(.)$ depend on the selection of Q_0^r, Q_{-1}^r, Q_0^d, and Q_{-1}^d.

The highest levels of correlation are present for the following selection of quantizers:

- Low values for Q_0^r and high values for Q_0^d. This result is analogous to the one obtained in the intraframe case. The use of a low Q_0^r causes that the resulting number of bits depends on the information contained in the error image: the more information is in the error image, the more bits its encoding produces. In the same way, the use of a coarse quantizer Q_0^d results in a distortion value which depends on the information of the error image: the more information it contains, the more distortion the final coded frame suffers. Fig. 5 shows in (a) a case where this correlation is present, and in (b) how an improper selection of these quantizers may lead to the non existence of correlation between the variables.

- Similar values for Q^r_{-1} and Q^d_{-1}. Provided that Q^r_0 and Q^d_0 have been selected according to the previous discussion, we can observe that higher values of ρ^2 occur when Q^r_{-1} and Q^d_{-1} are similar (see Fig. 5(a)). Note that when Q^r_{-1} and Q^d_{-1} are similar, the reference images for frame k used when computing $r_k(Q^r_0, Q^r_{-1})$ and $d_k(Q^d_0, Q^d_{-1})$ will be also similar, increasing the correlation between those variables. This effect is higher for low values of Q^r_{-1} and Q^d_{-1}.

Therefore, taking into account the above discussion, we have selected as reference quantizers for the computation of source model instantiations $Q^r_0 = 2$, $Q^d_0 = 31$, $Q^r_{-1} = 2$ and $Q^d_{-1} = 2$.



(a) (b)

Fig. 5. Determination coefficient between $r_k(Q^r_0, Q^r_{-1})$ and $d_k(Q^d_0, Q^d_{-1})$ variables for the sequence "silent". Each graphic represents a selection for the quantizers Q^r_0 and Q^d_0, and for all the possible values of Q^r_{-1} and Q^d_{-1}.

In order to assess the accuracy of the model, we have computed the relative error (R_{err}) of the estimates of rate and distortion, defined in both cases as

$$R_{err} = \frac{1}{N} \sum_{i=1}^{N} \frac{|estimated\_value_i - actual\_value_i|}{actual\_value_i}.$$

The coefficients of the model equations have been computed by least squares from the observed R-D values of all the frames of the sequence. To obtain the R-D values ($r_k(q_k, q_{k-1})$ and $d_k(q_k, q_{k-1})$ for all possible pairs of (q_k, q_{k-1})), the sequences have been encoded at 15 frames per second (150 frames in each sequence). For each sequence we have computed several model instantiations using different selections of the reference quantizers. We have obtained similar error results in all sequences. As a sample, Table I shows for the sequence "silent" the minimum and maximum average relative error obtained in the different model instantiatons. We have included the same error measurements provided by the frame model of the intraframe coding case (see [3]) for comparison. These results show that, although the frame model for the interframe case performs worse than its counterpart for the intraframe case, it still provides a reasonable level of accuracy.

	Inter: Avg. Error		Intra: Avg. Error	
	Min	Max	Min	Max
Rate	3.9 %	39.0 %	0.49%	0.68%
Distortion	1.0 %	19.0%	0.74%	1.28%

TABLE I

MINIMUM AND MAXIMUM AVERAGE RELATIVE ERROR OF THE FRAME MODEL FOR THE INTERFRAME AND INTRAFRAME CASE.

B. Sequence Model

Once we have defined models for estimating R-D values within a frame at any possible quantizer, the complete source model still requires the characterization of the variations over time of these values. To cope with this requirement, we have developed the sequence model. Since the frame model is able to estimate R-D values for a given frame from a single rate measurement (its frame complexity), we will focus on the modeling of its evolution along the frames of a sequence. For the sequence model, we will denote the frame complexity as s_k, thus, $s_k = r_k(Q_0^r, Q_{-1}^r)$.

Assuming that sequences we are considering contains frames of similar characteristics, we propose an analogous model to that of the intraframe coding case (see [3]), that is, the evolution of $\{s_k\}$ in k is characterized as a process with independent uniformly distributed samples defined in a certain interval $[s_{min}, s_{max}]$. The limits of that interval have to be estimated a priori based on the characteristics of the sequence being transmitted.

C. Model summary and discussion

The full source model is described by the equations (1)-(5), which have been reordered as they are used to compute estimates from the frame complexity measurement:

$$s_k = r_k(Q_0^r, Q_{-1}^r) \Rightarrow \{s_k\} \in [s_{min}, s_{max}],$$

$$d_k(Q_0^d, Q_{-1}^d) = m(Q_0^r, Q_{-1}^r, Q_0^d, Q_{-1}^d)\, r_k(Q_0^r, Q_{-1}^r) + n(Q_0^r, Q_{-1}^r, Q_0^d, Q_{-1}^d),$$

$$r_k(Q_0^r, q_{k-1}) = e(q_{k-1}, Q_{-1}^r)\, r_k(Q_0^r, Q_{-1}^r) + f(q_{k-1}, Q_{-1}^r),$$

$$d_k(Q_0^d, q_{k-1}) = g(q_{k-1}, Q_{-1}^d)\, d_k(Q_0^d, Q_{-1}^d) + h(q_{k-1}, Q_{-1}^d),$$

$$r_k(q_k, q_{k-1}) = a(q_k, Q_0^r)\, r_k(Q_0^r, q_{k-1}) + b(q_k, Q_0^r),$$

$$d_k(q_k, q_{k-1}) = c(q_k, Q_0^d)\, d_k(Q_0^d, q_{k-1}) + d(q_k, Q_0^d).$$

Note that the model parameters will depend on the particular encoder used, but its structure (its functional dependencies) are valid for any encoder as long as there is a single parameter for each frame, and the R-D values for a frame essentially depend on its control parameter and the one of the previous frame. In the particular case of R-D optimized encoders, in which different quantizers can be applied to the MBs in a frame, the frame control parameter could be defined, for instance, as the Lagrange coefficient, λ.

D. Source model instantiation

As discussed earlier, in general all quantization choices made in frames up to the current one (from the latest intra frame) affect the R-D characteristics of the current frame. However, for simplicity, we only consider all possible combinations of quantization choices for the current and past frame in order to construct our model. This is obviously an approximation and different quantization choices made for the previous frames would result in R-D different values. For example, in Fig. 2, $r_3(1,3,1,2)$ will in general be different from $r_3(1,3,1,1)$. We have verified experimentally that the penalty due to this approximation tends to be modest (see [5]) as long the system operates at rates that are similar to those used in generating the models.

The obtainment of empirical R-D measurements to compute the model parameters, can be done following different procedures. Several of them have been studied and analyzed in [5] under the name of *preprocessing schemes*. In this work, we only present the scheme we have selected because of its good performance. The idea is to simulate the transmission of the sequence following a pre-determined basic rate-control strategy based on buffer occupancy. This simulation uses a model of a transmission system similar to the one considered in this work, except for the channel which is considered reliable. Then, for each frame k we measure $r_k(q_k, q_{k-1})$ and $d_k(q_k, q_{k-1})$ for all possible combinations of q_k, q_{k-1} in the following way: (i) we first encode frame $k-1$ using as reference image the coded frame $k-2$ obtained from the basic rate control employed, and applying every possible quantizer q_{k-1}. Each resulting coded frame is stored for the next step; (ii) we encode frame k using as reference each of the codings of the previous frame, and applying again every possible quantizer q_k. From the obtained R-D values, we have computed the model coefficients using the least squares method. As long as the bitrate of the simulation is close to the effective rate of the channel in the real system, it has been found that the estimates provided by the model are reasonably accurate with respect to the actual operation of the system [5].

The accuracy of the model may depend on the number of frames used to estimate those coefficients. In order to study this effect, we have computed models using several sets of frames of a given sequence. These tests were conducted over several sequences coded at 15 frames per second. The reference quantizers selected, as described before, were $Q_0^r = 2$, $Q_0^d = 31$, $Q_{-1}^r = 2$ and $Q_{-1}^d = 2$. As an example of the results obtained, the R_{err} for sequence "silent" is shown in Fig. 6. Similar results have been obtained for other sequences. As can be observed, the error decreases as the number of frames involved in the computation of the model parameters grows. Nevertheless, it is interesting to note that the model presents low error even when using as few as 20 frames to compute the model parameters for a 150 frames long sequence. Although this number is bigger than that of the intraframe case, it is still possible to develop accurate models for those scenarios where only a small number of representative frames are available.

E. Model of distortion of lost frames

For the definition of our stochastic control problem, we also require a model for the distortion incurred when a lost frame (or skipped frame) is substituted in the decoder by the last received frame. Following the same approach of our previous work for the intraframe case, we propose to characterize the distortion of lost frames through MSE

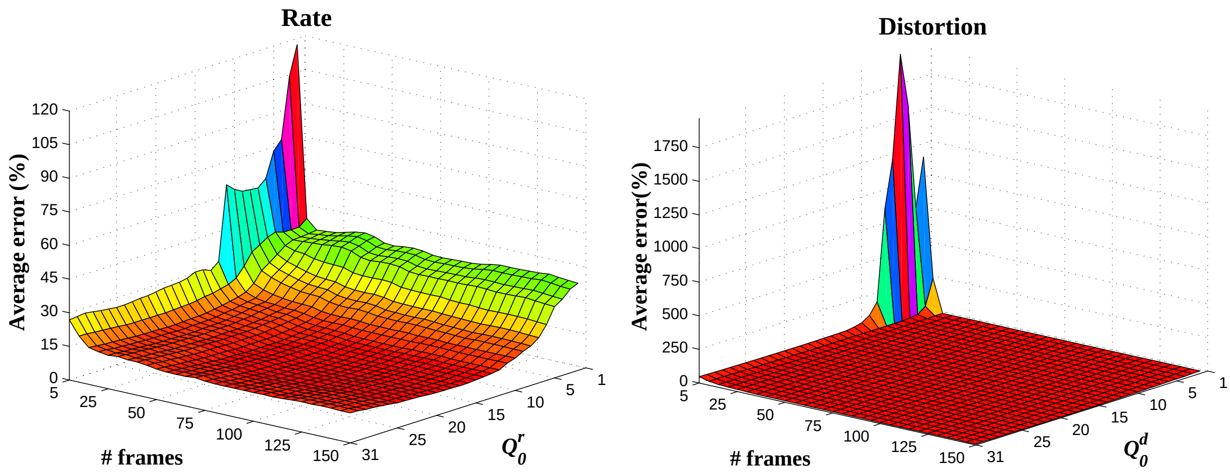


Fig. 6. Average relative error corresponding to rate and distortion, estimates as functions of the number of frames used in the computation of the coefficients of the source model equations. This results corresponds to the sequence "silent".

between the displayed frame and the original one. We model these distortion values, $d_{lf,m}$, as independent identically distributed variables. For the SDP formulation of the problem the only relevant parameter of their distribution is their mean, which we will note as K. For a discussion of the implications of this choice see [3].

IV. Channel Model

We assume a first order Markov model [18] in which the channel switches between states 0 and 1. In state 1 the packet is transmitted correctly while in state 0 the packet is received with errors. The channel behavior is then characterized by the transition probability matrix of this 2-state Markov model. This channel model captures the behavior *after* FEC. Employing parenthesized subindices to represent packet numbers (to avoid confusion with frame numbers), the characterization of the channel is provided by the values of the state transition probabilities

$$p_{ji} = P[c_{(k+1)} = j | c_{(k)} = i]$$

which are usually grouped in the matrix $\mathbf{P} = (p_{ji})_{i=0,1;j=0,1}$.

V. System Dynamics and Optimal Policy

A. System dynamics

Once models have been selected for the video coder and the channel, an analysis of the system dynamics of the whole system (video coder, the channel, and the buffer) will provide us with an adequate definition of the global system state and a procedure to calculate the state transition probabilities. We adopt as the working time unit the duration of a packet time slot T_p (packet slot). Thus, the end-to-end delay measured in packet slots, denoted by ΔN, is:

$$\Delta N = \frac{\Delta T}{T_p}$$

and the frame period, ΔM, also expressed in packet slots can be found as:

$$\Delta M = \frac{T_f}{T_p}.$$

To simplify the development we will assume that ΔN and ΔM are integers.

Every ΔM packet slots a new frame is available at the encoder. Once the rate control module has chosen the appropriate quantizer for the current frame (frame k), it is encoded using as reference image the previous (or the last correctly received at the decoder) coded frame, producing r_k bits. Note that due to our modeling approach of the interframe dependency, r_k will depend on the quantizers q_k and q_{k-1}. The r_k bits will require $n_k = \lceil \frac{r_k}{C} \rceil$ packets to store them. The number of packet slots available to transmit frame k will be termed t_k, and w_k will denote the number of packets actually devoted to the transmission of the frame, either successfully or not. The actual value of w_k for a given frame will be the minimum number of packets employed for which one of these two events occurs:

1) The transmission of the frame is completed, i.e., n_k packets have been successful out of the w_k packets employed.
2) The system realizes the impossibility to transmit the frame, i.e., the number of remaining packet slots has become smaller than the number of packets pending transmission. At this time, the system removes the pending packets from the encoder buffer and starts processing the next frame, if available.

Variables t_k and w_k determine the value of t_{k+1} according to the relationship

$$t_{k+1} = \min\{\Delta N, t_k - w_k + \Delta M\}, \tag{6}$$

which includes the two following possibilities:

1) Frame $k+1$ is not yet available at the encoder at the time the system is ready for its transmission. Then its transmission will start as soon as the frame is ready, and the available period for its transmission will be the end-to-end delay: $t_{k+1} = \Delta N$.
2) When the processing of frame k ends, frame $k+1$ is already available. The transmission of this frame can thus start as soon as the processing of frame k has been completed. As the available number of packet slots for frame k was t_k, the number of slots available for transmitting frame $k+1$ can be computed taking into account that this set of consecutive slots starts w_k slots after the beginning of the corresponding period for frame k, and its due time is delayed ΔM slots with respect to the end of the corresponding period for frame k. So we have $t_{k+1} = t_k - w_k + \Delta M$.

Consequently t_k ranges from ΔM (corresponding to the case in which the previous frame employed all the permitted packet slots, i.e., $w_k = t_k$) to ΔN (when at the time the new frame is available no packets of previous frames are pending of transmission).

We will term l_k the number of the first packet slot devoted to the transmission of frame k. At the time of starting the transmission of this frame, the Markovian property of the channel model implies that the only relevant information on the channel is its last known state value $c_{(l_k-1)}$. It will be represented for short as c_k.

In order to formalize our control problem in SDP terms [2] we now have to identify the state space, calculate the state transition probabilities, and define the cost function whose long-term average will be minimized.

The system state must be defined as a vector x_k including all the *relevant* information *available* from the history of the system, i.e., a set of values known at the moment of making the decision, and which are sufficient to determine probabilistically, together with quantizer q_k, both the system evolution and future cost values. In our case the minimum set of data with these properties can be identified as consisting of

- complexity of the current frame, s_k,
- quantizer used for the previous frame, $a_k = q_{k-1}$, since this variable affects the R-D values of frame k,
- last observed channel state, c_k,
- number of available packet slots before the due time of the current frame, t_k,

so that we have $x_k = (s_k, c_k, t_k, a_k)$. As will be shown next this state vector corresponds to a controlled Markov chain with control parameter q_k which is suitable for the formulation of a SDP problem, because (a) given (s_k, c_k, t_k, a_k) and a choice of q_k, we can find the transition probabilities to all possible $(s_{k+1}, c_{k+1}, t_{k+1}, a_{k+1})$ (see next subsection), and (b) the expected cost per stage is a function of the state (s_k, c_k, t_k, a_k) and the control q_k (see subsection V-C.)

B. State transition probabilities

Since our system comprises two stochastic elements, the source and the channel, its dynamics are captured as their joint transition probabilities for every action. Our goal is thus to obtain the probabilities:

$$P\left[s_{k+1}, c_{k+1}, t_{k+1}, a_{k+1} | s_k, c_k, t_k, a_k, q_k\right]. \tag{7}$$

where the subindex k indicates the stage or frame index and, at that stage, s_k is the complexity of the frame, c_k is the state of the channel, t_k is the time left for the current frame, a_k indicates the previous quantizer (q_{k-1}), and q_k is the selected quantizer. In the remainder of this section, we show the exact computation of the above probabilities. Assuming a Markovian model for s_k and since it is independent of the others variables, we can factor (7) as:

$$\begin{aligned} &P\left[s_{k+1}, c_{k+1}, t_{k+1}, a_{k+1} | s_k, c_k, t_k, a_k, q_k\right] \\ &= P\left[s_{k+1} | s_k\right] P\left[c_{k+1}, t_{k+1}, a_{k+1} | s_k, c_k, t_k, a_k, q_k\right]. \end{aligned} \tag{8}$$

The source model provides the first term of (8), while the second one requires a further elaboration.

As a_{k+1} only depends on q_k we can rewrite it as:

$$\begin{aligned} &P\left[c_{k+1}, t_{k+1}, a_{k+1} | s_k, c_k, t_k, a_k, q_k\right] \\ &= P\left[a_{k+1} | q_k\right] P\left[c_{k+1}, t_{k+1} | s_k, c_k, t_k, a_k, q_k\right], \end{aligned} \tag{9}$$

where $P\left[a_{k+1} | q_k\right]$ is 1 if $a_{k+1} = q_k$ or 0 if $a_{k+1} \neq q_k$.

For the second part of (8) we use again the source model to compute the number of packets n_k of the encoded frame. Using n_k we can rewrite it as follows:

$$P\left[c_{k+1}, t_{k+1} | s_k, c_k, t_k, a_k, q_k\right] = P\left[c_{k+1}, t_{k+1} | n_k, c_k, t_k\right]. \quad (10)$$

Thus, we only need to compute the joint probability $P\left[c_{k+1}, t_{k+1} | n_k, c_k, t_k\right]$. The exact computation of such probability can be found in detail in [3]. Table II shows a summary of the notation and equations.

Notation	
s_k	frame k complexity
c_k	channel state
t_k	time left for frame k
a_k	quantizer for frame $k-1$
q_k	quantizer for frame k
n_k	size of coded frame k in packets
w_k	packets devoted to frame k transmission
l_k	first packet slot for frame k
n_{pp}	no. of padding packets
$c_{(l)}$	channel state at packet slot$l$
$v_{(a,b)}$	no. packets correctly transmitted in [a, b]

Transition Probabilities
$P\left[c_{k+1}, t_{k+1} \| n_k, c_k, t_k\right] = \sum_{w_k} P\left[c_{k+1}, w_k \| n_k, c_k, t_k\right]$
$P[c_{k+1}, w_k \| n_k, c_k, t_k]$ $= \sum_{i=0,1} P[c_{(l_k+w_k+n_{pp})} = c_{k+1} \| c_{(l_k+w_k)} = i]$ $P[c_{(l_k+w_k)} = i, w_k \| n_k, c_{(l_k-1)} = c_k, t_k]$
$P[c_{(l_k+w_k)} = 1, w_k \| n_k, c_k, t_k]$ $= P[v_{(l_k,l_k+w_k)} = n_k, c_{(l_k+w_k)} = 1 \| c_{(l_k-1)} = c_k]$
$P[c_{(l_k+w_k)} = 0, w_k \| n_k, c_k, t_k]$ $= P[v_{(l_k,l_k+w_k)} = w_k - (t_k - n_k + 1), c_{(l_k+w_k)} = 0 \|$ $c_{(l_k-1)} = c_k]$
$P[v_{(l_k,l_k+w_k)} = h, c_{(l_k+w_k)} \| c_{(l_k-1)}]$ $= P[v_{(l_k,l_k+w_k-1)} = h, c_{(l_k+w_k-1)} = 0 \| c_{(l_k-1)}]$ $P[c_{(l_k+w_k)} \| c_{(l_k+w_k-1)} = 0]$ $+P[v_{(l_k,l_k+w_k-1)} = h-1, c_{(l_k+w_k-1)} = 1 \| c_{(l_k-1)}]$ $P[c_{(l_k+w_k)} \| c_{(l_k+w_k-1)} = 1]$

TABLE II

SUMMARY OF NOTATION AND TRANSITION PROBABILITY EQS.

C. Cost function

According to our objectives and modeling assumptions, the cost g_k associated to the decision made for frame k can be defined as the expected resulting distortion, taking into consideration the possibility of not receiving the encoded frame, i.e., as

$$g_k(s_k, t_k, c_k, a_k, q_k) = f_d(s_k, q_k, a_k) \cdot P_{tx}(s_k, t_k, c_k, a_k, q_k) + E[d_{lf,k}] \cdot (1 - P_{tx}(s_k, t_k, c_k, a_k, q_k)) \tag{11}$$

where according to our model $f_d(s_k, q_k, a_k)$ is the distortion due to the quantization process, $E[d_{lf,k}] = K$, and P_{tx} is the probability of receiving the frame in time at the decoder, that can be computed from the results of the last subsection as

$$P_{tx}(s_k, t_k, c_k, a_k, q_k) = \sum_{x_{k+1} \in \mathcal{S}} P[x_{k+1} | x_k, q_k]$$

where $\mathcal{S}$ is the set of states to which the system can move if the frame is correctly transmitted.

Using the equations of the dynamics of the system together with the cost function, standard SDP algorithms [2] find optimal stationary policies (i.e., policies that do not depend on the stage), which indicate the appropriate quantizer for every system state: $q_k = \mu(s_k, c_k, t_k, a_k)$. The computation of the policies is carried out off-line, so that during on-line operation only state identification is necessary.

VI. Experimental Results

To assess the performance of the proposed rate control scheme, optimal policies based on our model have been obtained and used in a simulated burst-error transmission environment. For our tests we have used the QCIF sequences “mother-daughter”, “akiyo” and “silent” (300 frames at 30 frames per second) transmitted at 15 frames per second (T_f=1/15 s). The first frame is intraframe coded and the remaining frames are coded as P frames. The channel simulator transmits 328 bit packets every 5 ms ($C = 328$, $T_p = 5$). We have used a model for the channel with an average burst length of 19 packets whose transition probabilities are shown in Table III.

Parameters	H-ERROR
p_{10}	0,0091
p_{01}	0,0526
ϵ	0,1475
$\overline{L}_{err}$	19

TABLE III

Parameters of the channel model.

In order to get a meaningful representation of the channel behavior in every simulation we transmit a sequence 100 times at different starting points in the channel simulator and average the results. The number of frames transmitted in each test adds up to 15,000 frames, and the number of packets used is about 192,000. We have run the simulations with different values of ΔT: 133, 200 and 267 ms ($2T_f$, $3T_f$, and $4T_f$).

With respect to the source model parameters, we have selected as reference quantizers $Q_0^r = 2$, $Q_0^d = 31$, $Q_{-1}^r = 2$, and $Q_{-1}^d = 2$. The range of definition of the frame complexity of each sequence has been estimated from the $r_k(2,2)$ values of all its frames.

During the sequence transmission, the control module has to calculate the value of the state variables for each frame k. The computation of the frame complexity involves the current frame k and the previous frame $k-1$ coded with Q_0^r and Q_{-1}^r in the following way:

- Frame $k-1$, which is stored in memory, is coded with Q_{-1}^r using as reference image frame $k-2$ coded with the quantizer q_{k-2} selected by the control module.
- Frame k is coded with Q_0^r using as reference image the result of the previous step. The resulting number of bits is its frame complexity.
- The control module determines the quantizer for frame k, q_k, according to the state of the system (frame complexity, channel state, time left, and previous quantizer) through the control policy.
- Frame k is coded with q_k using as reference image frame $k-1$ coded with q_{k-1}. This coded image is transmitted.

Hence, for each frame k the coder has to perform three coding operations, one over the previous frame with Q_{-1}^r, and two over frame k with Q_0^r and q_k. In our experiments, since $Q_{-1}^r = Q_0^r = 2$ the number of operations can be reduced to two, and using in both encodings of frame k the same motion vectors.

To compare the quality of the results, we have computed:

- PSNR of the decoded sequence, defined as

$$PSNR = 10 \log_{10} \left(\frac{255^2 N}{\sum_{i=0}^{N} MSE_i} \right)$$

  where N is the number of frames of the sequence. The distortion in case of a lost frame corresponds to the difference between the original and the actually displayed (last frame correctly transmitted).
- PSNR of the correctly transmitted frames (PSNR_{tx}).
- Proportion of lost frames (n_{lf}).

The following sections describe the tests conducted and their results.

A. Algorithm Performance

In addition to our proposed algorithm (SDP), the tests have been performed with two other algorithms:

- Algorithm based on the TMN8 [16] rate control (TMN). In this case, we have used the strategy proposed for the frame-layer rate control (prediction of the target rate for the current frame). Then, we use the quantizer that produces the closest encoding rate to the predicted value.

- Algorithm based on the strategy proposed in [1] (TMN-MOD). The authors propose a customization of the TMN frame-layer rate control mechanism for variable bit-rate channels similar to those considered in this work. They include in their algorithm an estimation of the average retransmitted bits which is discounted from the target rate. In our implementation we use the quantizer that produces the closest encoding rate to the predicted value.

Results are shown in Table IV. We can note that the SDP algorithm provides remarkably better results than the other two approaches. In terms of PSNR, the SDP algorithm provides an increase in PSNR with respect to TMN-MOD from 1.18 dB to 2.31 dB, and with respect to TMN from 1.37 to 2.57 dB. Regarding the number of lost frames, the SDP algorithm achieves a significantly lower number of lost frames than TMN-MOD (from 2.4 to 8.3) and TMN (from 3.9 to 12.5). In addition to reducing the number of lost frames SDP provides higher PSNR$_{tx}$ values than the other approaches in most of the cases. These better results are explained by the fact that SDP takes into account for each decision the number of available channel slots for transmiting the frame (which can be considered as a buffer occupancy), the channel state, and the R-D characteristics of the sequence. SDP also estimates the long-term influence of the current coding decisions, leading to long-term optimal solutions. On the other hand, TMN is only based on the buffer occupancy, and TMN-MOD extends TMN considering the expected number of retransmitted bits conditioned to the channel state.

Regarding the on-line computational cost, the three algorithms requires similar resources. In our implementation of TMN and TMN-MOD a variable number of frame coding operations is needed in order to determine the appropriate quantizer. For the SDP algorithm, a maximum of three coding operations for each frame is required: Two operations (or one depending on the selection of the reference quantizers) are required to determine the frame complexity, and the other one corresponds to the selected quantizer.

Therefore, we can conclude that the SDP algorithm provides an increase in quality of the decoded sequence with respect to the other two considered algorithms, maintaining a low on-line computational cost. By means of SDP techniques the complexity requirements of using interframe coding are absorbed in the computation of the optimal policies, process that is carried out off-line. As a consequence, the on-line computational cost required is quite similar to that of the intraframe coding. This result is of particular interest if we consider the computational cost required by algorithms based on Deterministic Dynamic Programming [8].

B. ACK Delay

So far we have considered that there is no delay in receiving the acknowledgements of the transmitted packets (ACK delay), that is, the encoder knows immediately whether a transmission has been successful or not. However, in a real scenario the decoder has to send the ACK back to the decoder, which takes a certain time. Due to this delay, the value of the state variables obtained by the rate control system at a certain time really reflects a situation at a previous instant. Indeed, the following state variables are affected:

- The channel state.
- The value of the time left variable for the current frame.

"mother-daughter"									
	$\Delta T = 2T_f$			$\Delta T = 3T_f$			$\Delta T = 4T_f$		
Algorithm	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}
SDP	34.87	36.49	12.9	35.75	36.98	9.7	36.07	36.97	6.3
TMN-MOD	33.43	36.38	22.5	33.99	35.89	13.5	34.36	35.86	9.8
TMN	33.42	36.46	23.2	33.32	36.46	20.0	33.50	36.40	14.8
"akiyo"									
Algorithm	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}
SDP	37.56	38.74	11.9	37.89	38.94	8.6	38.30	39.05	5.3
TMN-MOD	35.49	36.78	15.8	35.73	36.79	11.0	35.99	36.78	7.9
TMN	35.53	37.24	18.0	35.84	37.25	12.5	36.03	37.19	10.2
"silent"									
Algorithm	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}	PSNR	PSNR$_{tx}$	n_{lf}
SDP	31.26	32.86	13.9	31.87	33.09	9.3	32.46	33.37	6.6
TMN-MOD	30.08	32.38	14.4	30.51	32.36	15.4	30.94	33.34	10.6
TMN	29.89	32.95	26.4	30.05	32.99	21.8	30.13	32.95	19.0

TABLE IV

RESULTS OF THE TEST SEQUENCES.

Moreover, since ACKs/NAKs are delayed, packets have to be stored until their ACK/NAK is received or their due time arrives. Then, when the transmission of a new frame starts there might be some packets whose acknowledgement is still pending. In case of receiving a NAK for a packet before its due time, the packet is retransmitted. In our simulations pending packets are not considered in the computation of the time left for the current frame. Therefore, the actual number of packet slots left for its transmission might be smaller than the one estimated by the rate-control module.

In addition, the existence of ACK delay may cause the "drift" problem between the coder and the decoder in the described system. Note that the coder starts the transmission of a new frame $k+1$ (if it is already available) when one of these events takes place:

1) the coder detects that frame k will not arrive in time at the decoder, thus its pending packets are discarded,
2) the coder has sent all the packets of frame k.

In the first case, the coder knows that frame k will not be at the decoder, hence it will not be used as reference image for frame $k+1$. However, for the second case when the coder starts processing frame $k+1$ it still does not know the final result of the transmission of frame k due to the ACK delay. Therefore, if frame k finally does not arrive at the decoder, error propagation will occur since the decoding of next frames will be erroneous due to frame interdependecy. Note that this possibility cannot occur under the assumption of negligible ACK delay since the transmission result of each packet is known immediately.

Under this new consideration the coder needs to establish a mechanism in order to prevent the drift. In our experiments we have considered two different drift-free procedures:

- Current frame recoding (CFR). If during the transmission of frame k the coder detects that any previous frame is lost at the decoder, it empties the buffer and encodes again frame k using as reference image the last frame received at the decoder. This mechanism avoids sending information of lost frames at the expense of wasting channel resources if a recoding occurs. In this case, the packets that have been sent previously are useless. On the other hand it is also required to send information about the reference frame used.
- Lost frame sending (LFS). Despite detecting that a frame will not arrive in time at the decoder, its pending packets are resent in order to be available at the decoder for correct decoding of future frames. With this strategy channel resources are wasted sending packets of lost frames, but it avoids both the recoding of frames and the specification of the reference frame.

We show in Fig. 7 the results of the transmission of sequence "mother-daughter" in the conditions specified at the beginning of this section, but considering the existence of a constant ACK delay and using the drift-free mechanisms described above. Different ACK delay values have been employed.

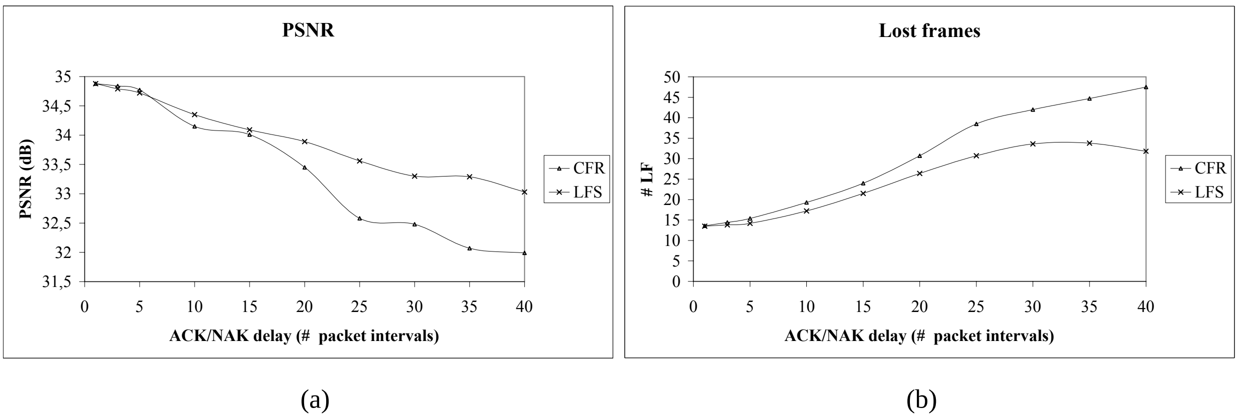


Fig. 7. (a) PSNR and (b) number of lost frames (LF) under different ACK delays and applying CFR and LFS to guarantee the coder-decoder synchronization.

We can observe that for low delay values (ACK delay ≤ 5 packet intervals) the transmission quality is essentially the same as that obtained in a scenario with no ACK delay. This result is reasonable considering that the average burst length is of 19 packets. Thus, a small delay (compared to this value) still allows the rate control module to obtain representative data about the system states. From this point, as the ACK delay increases, the system state identification loses accuracy resulting in a lower transmission quality. Regarding the behavior of the drift-free mechanisms used, it can be observed that when the delay is small (≤ 5 packet intervals) both strategies provide similar results. However, for greater delays LFS outperforms CFR. We can therefore conclude that the proposed problem formulation (which considers a negligible ACK delay) is a valid approach for those scenarios with a small ACK delay with respect to the average burst length.

VII. Conclusions

We have considered the problem of controlling in real-time a video coder over a lossy channel, operating in interframe mode. Our approach is based on R-D optimization premises although we propose an alternative solution to the associated optimization problem by using stochastic dynamic programming. Extending our previous work [3] to the case of interframe coding, we have formulated and solved a minimization problem which involves complete video sequences (300 frames) where the first frame is intraframe coded and the rest are coded as P frames. Using SDP-techniques, we have computed optimal control policies, i.e., the optimal mapping that assigns the quantizer choice for the current frame to each possible observed state of the system. These optimal policies have been computed off-line based on probabilistic models of the source and channel. Therefore, the on-line operation of the system has only required to estimate the state of the system, and to read the optimal quantization choice from a look-up table. According to our problem formulation, estimating the state of the system only involves encoding the current frame using a given quantization choice, and monitoring the result of the ARQ mechanism.

In order to apply the SDP approach, we have modeled the complete system as a discrete-time stochastic dynamical system. We have developed a novel video source model which captures the R-D characteristics of a complete sequence coded in interframe mode. One of the principal features of the model is the characterization of the interframe dependencies, which restricts the combinatorial explosion of frame dependencies, but provides reasonably accurate estimates. Although this model has been developed for the SDP problem formulation, it could also be used in other optimization approaches. Within the SDP approach, this model has allowed to introduce in a simple but effective way the complex R-D characteristics of a video source coded in interframe mode. As a result, the computational requirements of using interframe coding has been derived to the off-line computational cost (control policy computation) which is perfectly affordable, while has kept the on-line computational cost similar to that of the intracoding case.

Experimental results have shown that the SDP approach (i) clearly outperforms the other two considered algorithms and (ii) maintains a reasonable on-line computational cost. This result is of much interest if we consider the computational cost required by algorithms based on deterministic optimization approaches [8][7]. Due to the interframe dependency, the computational cost associated to these algorithms is a real handicap for their use in real-time [15]. However, by means of SDP techniques and the source model proposed, the computational requirements of using interframe coding has been absorbed in the computation of the optimal policies, process that is carried out off-line.

Finally, we have also analyzed the performance of our approach, which considers a negligible ACK delay, in a more realistic scenario where ACK delay is not negligible. We have considered two drift-free mechanisms in the experiments. Results for both approaches have shown that the proposed problem formulation is a valid approach for those scenarios with a small ACK delay with respect to the average burst length.

REFERENCES

[1] S. Aramvith, I-Ming Pao, and Ming-Ting Sun, "A Rate-Control Scheme for Video Transport over Wireless Channels", IEEE Transactions on Circuits and Systems for Video Technology. Vol 11, No 5, pp. 569-580, 2001.

[2] D. P. Bertsekas, "Dynamic Programming and Optimal Control", Athena Scientific, USA, 1995.

[3] J. Cabrera, A. Ortega, and J. I. Ronda, "Stochastic rate-control of video coders for wireless channels", IEEE Transactions on Circuits and Systems for Video Technology, Vol.12, No.6, pp. 496-510, June 2002.

[4] J. Cabrera, J. I. Ronda, A. Ortega, and N. García "Stochastic rate-control of interframe video coders for VBR channels", in Proc. IEEE Signal Processing Society 2003 International Conference on Image Processing (ICIP 2003), Vol.3, pp. 813-816, Barcelona, Spain, September 2003.

[5] J. Cabrera, "Stochastic control of video coders for variable bitrate channels", Ph.D. thesis, Universidad Politécnica de Madrid, December 2002.

[6] J. Chen and D.W. Lin, "Optimal bit allocation for coding of video signals over ATM networks", IEEE Journal on Selected Areas in Communications, Vol.15, No.6, pp. 1002-1015, August 1997.

[7] P. A. Chou and Z. Miao, "Rate-distortion optimized sender-driven streaming over best-effort networks", IEEE Workshop on Multimedia Signal Proc., Cannes, pp. 587-592, Oct. 2001.

[8] C. Hsu, A. Ortega, and M. Khansari "Rate Control for Robust Video Transmission over Burst-Error Wireless Channels", IEEE Journal on Selected Areas in Communications. Special Issue on Multimedia Network Radios, Vol. 17, No. 5, pp. 756-773, May 1999.

[9] C. Hsu, A. Ortega, and A. R. Reibman, "Joint selection of source and channel rate for VBR video transmission under ATM policing constraints", IEEE Journal on Selected Areas in Communications, Vol.15, No.6, August 1997.

[10] ITU-T, "Video coding for low bit-rate communications", ITU-T Recommendation H.263; v.1 Nov.1995, v.2, Jan. 1998.

[11] L. Lin and A. Ortega, "Bit-Rate Control Using Piecewise Approximated Rate-Distortion Characteristics", IEEE Transactions on Circuits and Systems for Video Technology, Vol. 8, No. 4, pp. 446-459, August 1998.

[12] H. Liu, H. Ma, M. El Zarki, and S.Gupta "Error control schemes for networks: An overview", ACM/Baltzer Mobile Networks and Applications Journal, vol. 2, pp. 167-182, 1997.

[13] Z. Miao and A. Ortega, "Optimal scheduling for streaming of scalable media", in Proc. Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 2000.

[14] A. Ortega, K. Ramchandran and M. Vetterli, "Optimal Trellis-based Buffered Compression and Fast Approximations", IEEE Transactions on Image Processing, Vol. 3, No. 1, Jan. 1994.

[15] K. Ramchandran, A. Ortega and M. Vetterli, "Bit Allocation for Dependent Quantization with Applications to Multiresolution and MPEG Video Coders", IEEE Transactions on Image Processing, Vol. 3, No 5, pp 533-545, Sept. 1994.

[16] J. Ribas-Corbera and S. Lei, "Rate Control in DCT Video Coding for Low Delay Communications". IEEE Transactions on Circuits and Systems for Video Technology, Vol.9, No.1, pp. 172-185, Feb. 1999.

[17] J.I Ronda, F. Jaureguizar, and N. García, "Buffer-constrained coding of video sequences with quasiconstant quality", in Proc. IEEE International Conference Image Processing (ICIP 1996), vol. III, pp. 367-370, Lausanne, Switzerland, Sept. 1996.

[18] M. Zorzi, R.R. Rao, and L. Milstein, "On the accuracy of a first-order Markov model for data transmission on fading channels", in Proc. IEEE ICUPC'95, Tokyo, Japan, Nov. 1995.

PLACE PHOTO HERE

Julián Cabrera received the Telecommunication Engineering degree and the Ph.D. degree in Telecommunication, both from the Universidad Politecnica de Madrid (UPM), in 1996 and 2003, respectively.

Since 1996 he is a member of the Image Processing Group of the UPM. He was a Ph.D. scholar of the Information Technology and Telecommunication Programs of the Spanish National Research Plan from 1996 till 2001. Since 2002 he is Assistant Lecturer of Signal Theory and Communications at the Department of Signals, Systems, and Communications at the Escuela Tecnica Superior de Ingenieros de Telecomunicacion of the UPM.

His professional interests include design and development of multimedia communications systems, focusing on viedo transmission over variable rate channels. He has been actively involved in European projects (Acts, Telematics, IST) and national projects.

PLACE PHOTO HERE

J. I. Ronda received the Telecommunication Engineering degree and the Ph.D. degree in Telecommunication, both from the Universidad Politecnica de Madrid (UPM), in 1988 and 1994, respectively. Since 1988 he is a member of the Image Processing Group of the UPM. In 1992 he became a member of the faculty of the UPM, and since 1995 he is Associate Professor of Signal Theory and Communications at the Department of Signals, Systems, and Communications.

PLACE PHOTO HERE

Antonio Ortega was born in Madrid, Spain, in 1965. He received the Telecommunications Engineering degree from the Universidad Politecnica de Madrid (UPM), Madrid, Spain in 1989 and the Ph.D. in Electrical Engineering from Columbia University, New York, NY in 1994. He was a research assistant at the Image Processing Group at UPM (1990). At Columbia he was a graduate research assistant at the Center for Telecommunications Research (1991-94) and was supported by a scholarship from the Fulbright commission and the Ministry of Education of Spain.

In September 1994 he joined the Electrical Engineering-Systems department at the University of Southern California, where he is currently an Associate Professor. At USC he is also a member of the Integrated Media Systems Center (IMSC), an NSF Engineering Research Center, and the Signal and Image Processing Institute. In 1995 he received the NSF Faculty Early Career Development (CAREER) award. He is a senior member of IEEE, and a member of SPIE and ACM. He has been a member of the IEEE Signal Processing Society Multimedia Signal Processing (MMSP) technical committee and is currently a member of the Image and Multidimensional Signal Processing (IMDSP) technical committee. He was the technical program co-chair for the 1998 Workshop on Multimedia Signal Processing and is the technical program co-chair of ICME 2002. He has been an Associate Editor for the IEEE Transactions on Image Processing (1996-2000) and is currently an Associate Editor for the IEEE Signal Processing Letters. In 1997 he was the recipient of the USC School of Engineering Northrop-Grumman Junior Research Award. He received the 1997 IEEE Communications Society Leonard G. Abraham Prize Paper Award for a paper co-authored with C.-Y. Hsu and A. R. Reibman which appeared in the IEEE Journal on Selected Areas in Communications in Aug. 1997. He has also received the IEEE Signal Processing Society 1999 Magazine Award for a review paper co-authored with Kannan Ramchandran that appeared in November 1998.

His research interests are in the areas of image and video compression and communications. They include topics such as adaptive methods for image/video coding, joint source-channel coding for robust video transmission, rate control and video transmission over packet wired or wireless networks. He has over 100 publications on these topics in international conferences and journals.

PLACE
PHOTO
HERE

Narciso García Narciso García was born in Madrid (Spain) in 1954.

He received the Ingeniero de Telecomunicacin degree (five years engineering program) with high honors in 1976 (Spanish National Graduation Award) and the Doctor Ingeniero de Telecomunicacin degree (PhD in Communications) with ”Summa Cum Laude” in 1983 (Doctoral Graduation Award), both from the E.T.S. Ingenieros de Telecomunicacion of the Universidad Politecnica de Madrid.

Since 1973 he is with E.T.S. Ingenieros de Telecomunicacion of the Universidad Politecnica de Madrid, since 1977 he is a member of its faculty (tenure as Associate Professor in 1984), and since 1990 he is Professor of Signal Theory and Communications at the Department of Signals, Systems, and Communications. Since 2002 he is Professor ad-honorem at the Escuela Politecnica Superior of the Universidad Autonoma de Madrid. He leads the Image Processing Group (Grupo de Tratamiento de Imagenes) and has been Vice-Head (1987-1994) and Head (1994-2001) of the Department of Signals, Systems and Communications.

From 1978 till 1988 he was also a Scientific Advisor at the Image Processing Department of the IBM Madrid Scientific Center. He has been Coordinator of the Spanish Evaluation Agency from 1990 till 1992 and auditor and evaluator of European programs since 1990.

His professional interests cover digital image processing, digital television, computer vision, and telecommunication systems. He has been actively involved in European projects (Eureka, Race, Esprit, Cost) and has been one of the cowriters of EBU proposal, base of the ITU standard for digital transmission of TV at 34-45 Mb/s (ITU-T J.81). He is Director of the Spanish delegation at the Management Committee of the Information Society Technologies Programme of the Fifth Framework Programme of the European Union since October 1988.

Professor García is a pioneer of digital image processing activities in Spain and has received the Junior Research Award of the Universidad Politcnica de Madrid in 1987 and the Senior Research Award of the Universidad Politcnica de Madrid in 1994.