A MODEL-BASED APPROACH TO CORRELATION ESTIMATION IN WAVELET-BASED DISTRIBUTED SOURCE CODING WITH APPLICATION TO HYPERSPECTRAL IMAGERY

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ABSTRACT

In many practical distributed source coding (DSC) applications correlation information has to be obtained at the encoder in order to determine the encoding rate. Coding efficiency depends strongly on the accuracy of this correlation estimation, which often has to be performed under rate and complexity constraints. In this paper we focus on correlation estimation for wavelet-based DSC. We extend our previously proposed model-based estimation techniques, which provided accurate estimates of bit-plane level correlation under rate constraints, in the simple case where bit-planes are generated from the binary representation of the sources. To extend the model-based approach to wavelet-based DSC, we need to address two issues. Firstly, in order to improve coding efficiency, bit-planes are typically generated by more sophisticated algorithms in wavelet-based DSC (e.g., by deciding on the bitplane scan order based on coefficient "significance"), which makes model-based estimation more challenging. Secondly, certain wavelet subbands may not have enough coefficients for reliable model estimation, so that model-based techniques alone may not be sufficiently accurate. We propose solutions to these problems and, using a DSC-based hyperspectral image system as example, we demonstrate that model-based estimation can lead to efficient system implementation with lower computational and data exchange requirements, and improved parallelism, while incurring only small degradation in coding efficiency.

1. INTRODUCTION

Distributed source coding (DSC) [1] addresses the problem of compression of correlated sources that are not co-located. In practical DSC schemes, correlation information is needed at the encoder to determine the encoding rate. However, in many applications exact correlation information may not be available beforehand, and one would need to estimate it as part of the coding process. In such situation the coding efficiency of DSC depends strongly on the accuracy of correlation estimation performed at the encoder. If exact correlation information is available at the encoder, it is possible to construct capacity-approaching codes, e.g., [2, 3, 4]. In addition, [5] has reported that if the correct motion statistics can be estimated at the encoder, a distributed video coding system can indeed match the performance of traditional predictive codec like MPEG.

Correlation estimation in DSC usually has to be performed under rate and complexity constraints. For example, in wireless sensor networks, where communication costs are significant, it is desirable to limit data exchanges between nodes (such as those required to estimate correlation). In low complexity distributed video encoding [6, 7] and DSC-based hyperspectral image compression [8, 9], correlation estimation at the encoder should be computationally efficient. In fact, since these systems target low complexity encoding, reduced complexity correlation estimation may be desirable, even if it leads to somewhat lower coding efficiency.

Several methods have been proposed for correlation estimation problem in DSC. For video applications, low-complexity schemes to classify macroblocks into different correlation classes have been proposed [6], while other methods use a feedback channel to convey correlation information to the encoder [7]. In our prior work for wavelet-based DSC, correlation estimation was performed by direct bitplane comparison between the source and an approximation of the decoder side-information [8].

In this paper we focus on correlation estimation for wavelet-based DSC. Applications of wavelet-based DSC include hyperspectral image compression [8, 9], data compression in wireless sensor network [10], multi-view video coding [11], etc. The main novelty of this work is an extension of *model-based estimation* to take into account specific characteristics of wavelet-based DSC. We first proposed a general approach for model-based estimation for DSC in [12]: appropriate statistical models are chosen for the data, and analytical expressions are derived to estimate the bit-plane level correlation. Our simulation results have shown that model-based estimation can achieve better accuracy than direct sample exchange under information exchange constraints. Our work [12] focused on bit-plane level correlation estimation, in the simple cases where bit-planes are generated directly from the the binary representation of the sources. Recent work [13] has proposed similar correlation estimation methods for binary and Gray code representations of the sources.

There are two main challenges in extending model-based estimation to wavelet-based DSC. Firstly, in wavelet-based DSC, bit-planes are usually generated using more sophisticated methods in order to improve coding efficiency. A concrete example of this, which we consider in this paper, is that of bit-planes generated by *set-partitioning* as in SPIHT [14]. This type of bit-plane generation complicates the model-based correlation estimation process, as will be shown. Secondly, some high level wavelet subbands do not have enough coefficients for reliable model parameters estimation, so that model-based techniques alone may not provide sufficient accuracy. In this paper we present techniques to address these two problems. Using the DSC-based hyperspectral imagery system proposed in [8, 9] as an example, we demonstrate model-based estimation can lead to efficient implementation with lower complexity and data exchange requirements, and improved parallelism, while incurring only small degradation in coding efficiency.

This paper is organized as follows. In Section 2 we define the correlation estimation problem. In Section 3 we provide details on how to extend model-based estimation to wavelet-based DSC using hyperspectral image compression as example. Section 4 presents the experimental results and Section 5 concludes the work.

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2. PROBLEM DEFINITION

In this paper we consider encoding a source X with another correlated source Y that is available only at the decoder. Our goal is to design low complexity methods to estimate the correlation between X and Y at the encoder. Assume X and Y are vector sources such that each component can be represented by N bits. A straightforward encoding approach would be to compress the successive bit-planes extracted from the binary representation of X. We have presented correlation estimation tools for this setting in [12]. Here we consider more sophisticated methods to extract bit-planes from the vector sources, in particular those based on set-partitioning algorithms, as those used in SPIHT [14]. Set-partitioning and related methods are typically used when X and Y are wavelet transform coefficients. In these techniques the encoder first signals the *significance* of each of the vector components at a given bit-plane. After a component becomes significant, *sign* information is conveyed and then further *refinement* bits are transmitted. Note that JPEG2000 and SPIHT use different techniques to encode significance, sign and refinement information, but the techniques we propose in the context of SPIHT would also be applicable to JPEG2000. We consider systems where Slepian-Wolf coding is applied to compress the sign/refinement bit-planes, while significance bits are intra-coded. At the decoder, the significance bits of X are used to extract sign/ refinement bit-planes from Y for joint decoding of sign/ refinement bit-planes of X [1]. The focus of this paper is to investigate efficient correlation estimation scheme in this setting.

Denote $b_X(l)$ and $b_Y(l)$ the sign/refinement bit-planes of significance l extracted by set-partitioning from X and Y respectively, and denote $b_X(l,k)$ and $b_Y(l,k)$ the kth binary random variable in $b_X(l)$ and $b_Y(l)$ respectively. We assume $b_X(l,k)$ and $b_Y(l,k)$ are i.i.d. equiprobable. Furthermore, we assume $b_X(l,k)$ and $b_Y(l,k)$ are correlated with *crossover* probability p_l, i.e., $Pr[b_Y(l,k) = 1|b_X(l,k) = 0] = Pr[b_Y(l,k) = 0|b_X(l,k) = 1] = p_l$. In theory $b_X(l,k)$ can be encoded with a rate as low as $H(p_l)$ [4]. Previous work has reported code constructions that can approach this limit, but p_l needs to be known at the encoder to determine the rate. In this setting, the correlation estimation problem becomes estimating the crossover probability p_l of sign and refinement bit-planes.

3. MODEL-BASED APPROACH TO CORRELATION ESTIMATION

In this section we will describe how we extend the model-based approach to estimate the crossover probability of sign/refinement bit-planes, using the DSC-based hyperspectral image system proposed in [8] as example. In what follows we first describe the original system in [8] and our proposed improvements based on model-based estimation. Then we discuss how to derive expressions for crossover probability estimation, with different approaches depending on whether or not subbands contain enough coefficients for reliable model estimation.

3.1. DSC based hyperspectral image compression

Figure 1 depicts the original DSC-based hyperspectral image compression [8]. To compress the current spectral band, B_i, its significant, sign and refinement bits are first generated in a similar fashion as in the standard SPIHT algorithm [14]. Then we further compress sign and refinement bit-planes using a Slepian-Wolf code, to be decoded using as side information the sign and refinement bit-planes of same significance extracted from $a\hat{B}_{i-1} + b$, where $\hat{B}_{i-1}$ is the previous adjacent reconstructed band available only at the decoder, and a and b are the linear prediction coefficients. Significance information of B_i is intra-coded. To determine the channel coding rate, we need to estimate the crossover probability between sign/refinement bit-planes and their corresponding side-information. This is accomplished by extracting sign and refinement bit-planes from the *original* previous band B_{i-1} [2] (after linear prediction), and measuring the crossover probabilities by exchanging small subsets of bits (the number is kept small to reduce the information transfer needed, e.g., assuming each band is assigned to a different processor). In order to ensure that the bit-planes are formed with the same wavelet coefficients, we need to apply the significance tree of B_i when extracting bit-planes from B_{i-1}. Note that the extracted sign/refinement bit-planes from B_{i-1} are solely used for correlation estimation. More details can be found in [8, 9].

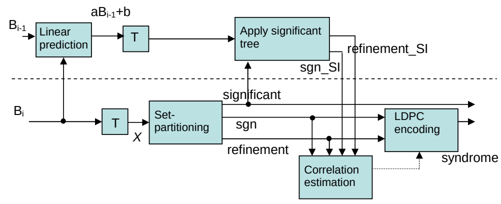


Fig. 1. The original DSC-based hyperspectral image compression.

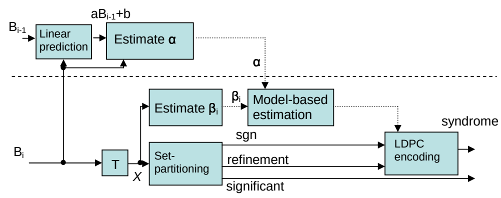


Fig. 2. The proposed system with model-based estimation.

Here, as an alternative, we propose to use the system shown in Figure 2. Significance, sign and refinement bits of B_i are extracted using SPIHT as in the original system. However, the correlation estimation process is improved so as to reduce computational/data exchange requirements, and further enhance the degree of parallelism achievable in the implementation. Specifically, explicit generation of side-information bit-planes approximations at the encoder is no longer required, and instead we use a model-based technique to estimate crossover probability. Denote the wavelet transform coefficients of B_i as X and the wavelet transform coefficients of $\hat{B}_{i-1}$ (after linear prediction) as Y. We assume a system model $Y = X + Z$, where Z is the correlation noise independent of X. The model-based estimation uses the model parameters of X and Z to estimate crossover probabilities through analytical expressions. The main advantage of this approach can be seen by comparing Figures 1 and 2: wavelet transform on $aB_{i-1} + b$ and bit-plane extraction are not required. Also in the original system there was a dependency when applying the significance map of B_i to B_{i-1}, and this hinders parallelism. Instead, the proposed system does not have such dependency.

3.2. Model-based estimation

We first model X and Z and then use this information to estimate crossover probabilities. For X, we use separate models for different

[1]Note that sending significance information in intra-mode allows the decoder to use, in decoding the subset of already significant coefficients X, side information from coefficients in exactly the *same* position in Y.

[2]Note that using the original, i.e., B_{i-1} rather than $\hat{B}_{i-1}$ is acceptable because we are focusing on a high fidelity application. Note also that this way the system is operating in "open-loop", i.e, we do not need to perform decoding at the encoder.

subbands, while for correlation noise Z we use a single model. Separate models are chosen for X to take into account different statistics in different subbands (e.g., variances tend to decrease when going from high level subbands to low level subbands). Since high level subbands may not have enough coefficients to obtain reliable estimate of model parameters, we treat low level and high level subbands differently. Essentially, we use different parametric models for different low level subbands which have enough data to achieve reliable model estimation, while we use a single non-parametric p.m.f. for all the remaining subbands. We will discuss in detail these two cases.

3.2.1. *Estimation in low level subbands*

Our goal is to estimate the crossover probability of refinement and sign bit-planes of significance level l, denoted as $p_{ref}(l)$ and $p_{sgn}(l)$ respectively. Assuming there are enough data, let X_i be the wavelet coefficients in subband i, and assume the system model $Y_i = X_i + Z$, with X_i and Z independent. Notice that the refinement bit-plane of significance level l includes only coefficients that are already significant [14], i.e., $|X_i| >= 2^{l+1}$. Hence the crossover probability of the lth refinement bit-plane for coefficients drawn from subband i, $p_{ref}(l, i)$, is

$$p_{ref}(l, i) = \frac{Pr(R \cap |X_i| >= 2^{l+1})}{Pr(|X_i| >= 2^{l+1})} \qquad (1)$$

where R denotes the event of crossover in refinement bits. We can calculate $Pr(R \cap |X_i| \geq 2^{l+1})$ by integrating the joint p.d.f. of X_i and Y_i, $f_{X_iY_i}$, over the shaded regions in Figure 3(a). In practice, we only need to integrate a few regions where $f_{X_iY_i}$ is non-zero. With the model $Y_i = X_i + Z$, and under the assumption of independence of X_i and Z, $f_{X_iY_i}$ can be factored into

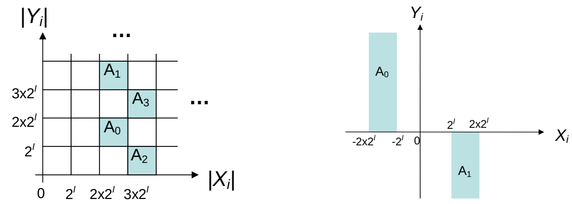
$$f_{X_iY_i}(x, y) = f_{X_i}(x) f_{Y_i|X_i}(y|x) = f_{X_i}(x) f_Z(y - x). \qquad (2)$$

Hence $p_{ref}(l, i)$ can be readily calculated given models for $f_{X_i}(x)$ and $f_Z(z)$. We assume X_i and Z are Laplacian distributed, i.e., $f_{X_i}(x) = \frac{1}{2}\beta_i e^{-\beta_i|x|}$, $f_Z(z) = \frac{1}{2}\alpha e^{-\alpha|z|}$. Model parameters β_i are estimated by maximum likelihood estimation (MLE) using wavelet coefficients from subband i. Model parameter α is estimated by calculating the standard deviation of $B_j - (aB_{j-1} + b)$ in the pixel domain (when compressing jth spectral band), denoted as σ, and using the relationship between standard deviation and model parameter in Laplacian distribution, $\alpha = \sqrt{2}/\sigma$. Note that we calculate the standard deviation in pixel domain in order to avoid computing the wavelet transform of side-information approximation. However, since our filter banks are not orthogonal, the standard deviation of the correlation noise in the pixel domain is not exactly the same as that in transform domain, which introduces some estimation error. Currently we are investigating improved techniques to estimate α in the pixel domain. Note also that the noise model can be estimated using a small percentage of pixels, e.g., 12.5% of pixels are used to calculate the standard deviation in our experiments.

Since refinement bit-planes consist of wavelet coefficients drawn from different subbands, we calculate the proportion of coefficients drawn from subband i (denoted as ρ_i), so that the crossover probability of the *whole* lth refinement bit-plane is calculated as

$$p_{ref}(l) = \sum_{i \in L} \rho_i p_{ref}(l, i) + q_{ref}(l) \sum_{i \in H} \rho_i. \qquad (3)$$

Here L denotes the subset of subbands where there are enough coefficients for reliable estimation of β_i, and H denotes the set of remaining subbands. $p_{ref}(l, i)$ is derived using (1) and (2) if a subband belongs to L. $q_{ref}(l)$ denotes the average probability of bit-plane segments consisting of subbands from H. Estimation of $q_{ref}(l)$ for subbands in H will be discussed in Section 3.2.2. The partition of all subbands into L and H is determined by the number of coefficients in a subband, n_i. Specifically, denote the MLE estimator as $\hat{\beta}_i$, then the *percentage deviation* of MLE estimation, $(\hat{\beta}_i - \beta_i)/\beta_i$, can be shown to be $N(0, 1/n_i)$, i.e., depending on n_i only. So we can determine the threshold on n_i to classify a subband into L or H according to a desired (expected) percentage deviation in the model parameter. Note that for subbands in set L, $p_{ref}(l, i)$ is a function of β_i, α and l, so that estimation can be achieved with low complexity.



(a) (b)

Fig. 3. Crossover probability estimation. (a) Probability of crossover and "X_i are already significant". E.g., consider $l = 2$ (i.e., the 2nd bit-plane), when X_i takes the value 8 (i.e., already significant), crossover occurs when Y_i takes the value in 4 to 7, or 12 to 15, ..., i.e., when Y_i is in $m \times 2^l$ to $(m + 1) \times 2^l - 1$, where m is an odd number. (b) Probability of sign crossover and "X_i become significant".

The crossover probability of sign bit-planes can be derived in a similar fashion as in refinement bit-planes. The difference here is we need to integrate different regions in the sample space of X_i and Y_i. The lth sign bit-plane includes only coefficients that become significant at significance level l [14], i.e., $2^{l+1} > |X_i| \geq 2^l$. Hence the crossover probability of lth sign bit-plane due to coefficients drawn from subband i, $p_{sgn}(l, i)$, is

$$p_{sgn}(l, i) = \frac{Pr(S \cap 2^{l+1} > |X_i| \geq 2^l)}{Pr(2^{l+1} > |X_i| \geq 2^l)} \qquad (4)$$

where S denotes the event of crossover in sign bits. $Pr(S \cap 2^{l+1} > |X_i| \geq 2^l)$ can be calculated by integrating the joint p.d.f. of X_i and Y_i over the shaded regions in Figure 3(b). We factor the p.d.f. as in (2), and re-use β_i, α and ρ_i from refinement bit-planes estimation, and derive the crossover probability of the whole sign bit-plane $p_{sgn}(l)$ similar to (3).

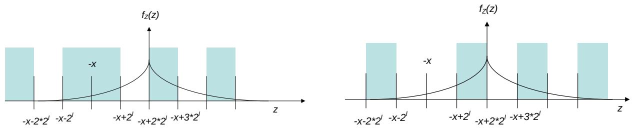
3.2.2. *Estimation in high level subbands*

Subbands in H do not have enough coefficients for reliable β_i estimation. Hence using $f_{X_i}(x)$ to estimate the crossover probability as in (1) and (2) would not provide reliable estimators. Instead, we use the empirical p.m.f. $Pr(X_i = x)$ of all the subbands in H to estimate the crossover probability. This involves more computation than using $f_{X_i}(x)$ as we will discuss later. Specifically, we derive the average crossover probability for the refinement bit-planes segments consisting of only subbands from H by

$$q_{ref}(l) = \sum Pr(R \mid X_i = x) Pr(X_i = x) \qquad (5)$$

where the summation is taken over all the possible values of X_i where $Pr(X_i = x)$ is non-zero. We can determine $Pr(X_i = x)$ empirically during set-partitioning by binning those coefficients drawn from subbands belonging to H. Assuming $Y_i = x + Z$ and using a Laplacian model for Z as before (note that here x is a constant instead of a random variable), $Pr(R \mid X_i = x)$ can be derived by summing the integrals of $f_Z(z)$ over the shaded regions as depicted in Figure 4. In practice we only need to sum over a few regions around $Z = 0$ where the integrals are non-zero. Note that $Pr(R \mid X_i = x)$ is a function of α, l and x, and we need to evaluate the expression for all the x where $Pr(X_i = x)$ is non-zero.

This involves more calculation than computing $p_{ref}(l,i),\ i \in L$. But since we use this method only for high level subbands where the number of coefficients are small, the computational requirement should still be small. After we have calculated $q_{ref}(l)$ we use it in (3) to find the estimate of the whole refinement bit-plane. Similarly,



(a) (b)

Fig. 4. $Pr(R|\ X_i = x)$ when (a) $\lfloor \frac{x}{2^l} \rfloor$ is odd ; (b)$\lfloor \frac{x}{2^l} \rfloor$ is even.

we can derive $q_{sgn}(l)$, the average crossover probability of the sign bit-planes segments consisting of subbands from H, similar to (5), with $Pr(S \mid X_i = x) = \int_{-\infty}^{-|x|} f_Z(z)dz$.

3.3. Advantages of the proposed scheme

The proposed system has several advantages as compared to the original scheme. First of all, it requires less computation. Instead of explicitly generating the approximations of side-information bit-planes for correlation estimation, which requires wavelet transform and bit-plane extraction on $aB_{i-1} + b$ (Figure 1), our proposed system requires only evaluation of analytical expressions. Estimation of model parameters β_i requires negligible computation by MLE assuming Laplacian model, and estimation of model parameters α requires only calculation of standard deviation [3]. The proposed system also improves parallelism. Assume a parallel implementation where each processor compresses one spectral band. In the proposed system, once we have estimated α in the pixel domain, there is no further dependency between processing units (Figure 5), leading to efficient overall implementation. In addition, the proposed system requires less data traffic compared to the original system, since exchanging of significance tree and sign/refinement bit-planes is no longer required. Accurate estimation of the standard deviation of correlation noise requires only small percentage of pixels.

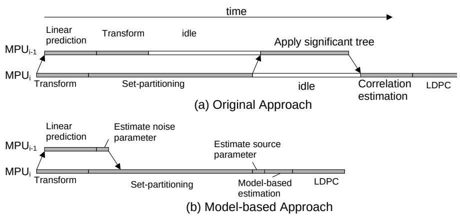
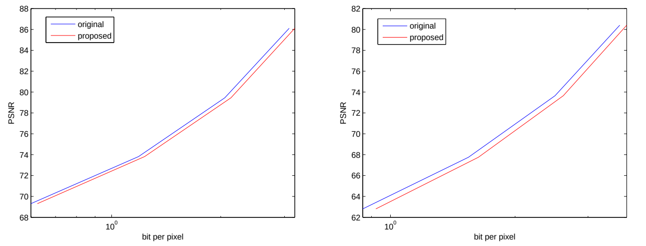


Fig. 5. Time charts of parallel implementation.

4. EXPERIMENTAL RESULTS

We have compared our proposed hyperspectral image system with the original scheme. In the original system we exchange all the sign/refinement bits to determine the *exact* empirical crossover probability, so that optimal estimation accuracy is achieved. In our proposed system, we use 12.5% of pixels to calculate the standard deviation of correlation noise. To prevent decoding error due to underestimating the crossover probability, we allow a larger margin to determine the encoding rate, at the expense of coding efficiency. Figure 6 shows the results of compressing images *Cuprite* spectral band 133 (radiance data) and *Lunar* spectral band 44 (reflectance data). As shown in the figure, our proposed system incurs about 0.5dB degradation in coding efficiency. Note that raw hyperspectral images have 16 bits per pixel, and are usually compressed at high fidelity. So the degradation due to model-based estimation is small. We observe similar results for other hyperspectral image data sets.

[3]Note that we can reduce complexity further by using the model parameters of the previous band for the current band. This may be possible for many bands in a dataset, since the variations in correlation are small in hyperspectral image. Yet some low complexity method is needed to detect large change in correlation.



(a) (b)

Fig. 6. Coding efficiency comparison. (a) Cuprite; (b) Lunar. PSNR $= 10\log_{10}(65535^2/\text{MSE})$, where MSE is the mean squared error between the original and reconstructed bands.

5. CONCLUSIONS

In this paper we have proposed to address correlation estimation in wavelet-based DSC by extending model-based estimation to cases where bit-planes are generated by set-partitioning. We proposed parametric and non-parametric techniques for low level and high level subband coefficients, respectively. Using a DSC-based hyperspectral system as an example, we demonstrated model-based estimation can lead to efficient system implementation while incurring only small coding efficiency degradation. The results suggest that model-based estimation can be a viable low complexity approach to estimate bit-plane level correlation for a variety of DSC applications and correlation structures.

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