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Waterflood Tomography: Mapping High Contrast Permeability Structures Using Injection/Production Data

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Abstract

The paper presents a novel method to detect the existence and determine the orientation of high permeability channels between injection and production wells in a waterflood. We apply the concept of transmission tomography and model a waterflood reservoir by considering water injection rates as inputs and measured production rates as outputs. We solve the inverse problem, in which the goal is to determine the existence and location of high permeability channels in the field by measuring the lag time in response to the variation in the injection rate. The main advantage over other methods (e.g., tracer testing) is that this technique can be applied without significantly affecting daily operation.

We show that the lag times at the producers can be estimated by monitoring production rates, given that known time-varying injection rates are applied. We propose a mixture model to characterize the lag time for each injector-producer pair, where the system is initialized by assuming that multiple candidate fractures exist. Our algorithm iteratively modifies the length, orientation and location of each high permeability candidate in order to match the measured response time between wells. It is well known that the solution may not be unique if we only have limited measurement data. Thus, in order to choose between possible models, we propose to use the "total length" of high permeability channels as regularization metric. Our method will select the model that fits the measured lag time with minimum total high permeability channel length. It is also possible within our framework to adjust the regularization so that it takes into consideration other type of prior information, e.g., preferred orientation known to exist in a given field.

To validate our approach, we use a commercial simulator to test a synthetic line drive and a 5-spot waterflood. In the first case, we test a five spot with hydraulic fracture located in the central production well. In the second case, we test a single fracture with 45 degree orientation located between rows of producers and injectors. Our results show that our method can provide very accurate estimates of fracture orientation, with decreased uncertainty as the well density in the field increases.

Introduction

Understanding the heterogeneity of the reservoir is very critical in waterflood forecasting. For an ideal waterflooding field, we want the injected water to uniformly "push" fluid towards all producers. But in a tight reservoir, high permeability channels provide a fast pathway and can play a dominant role in explaining the fluid flow for the waterflood. For example, a direct high permeability channel link between injection and production wells will cause water cycling and decrease the sweep efficiency. To achieve improved waterflood optimization, it is desirable to identify the approximate locations of these high permeability channels in the reservoir.

Several characterization methods have been proposed that can be used to estimate reservoir model parameters. Seismic methods can provide very detailed geophysical structure (Sheriff *et al.*[1] 1992) while interference and pulse tests also can map reservoir heterogeneity. Tracer tests measure the interaction of diffusion process between the injection/production wells that provides the information for directional permeability. Vasco[2] *et al*. (1999) combines the dynamic data obtained from tracer tests and that obtained from productions rates to generate high-resolution reservoir representation models.

A common aspect of all these methods is that they require additional equipment and may involve disruption or interruption of daily operations. We note that in a waterflood field injection and production rate data are the most abundant data source. Thus, if we have a model to describe the injection-production relationship it will be possible to estimate an equivalent "lag time" for the fluid using only injection-production rate data. For example, the capacitance model (Yousef *et al*.[3] 2005) has a "time delay" parameter that is roughly proportional to the pressure wave propagation time. We can use this parameter to model the injection-production response and estimate the lag time. Our approach can be extended to any model from which the lag time information can be estimated using injection-production relationship.

Once we determine the response time from the injection/production data, constructing a reservoir model becomes a travel-time tomography problem (Berryman.[4] 1991). We consider a 2D reservoir with measured lag time between the wells. To build a high-resolution model, the main difficulty is that the measured response time data is still too sparse and restricted to the area between the wells. Thus we cannot obtain the response time between any two arbitrary locations in the field. Because we can only measure the lag time between the wells, the spatial resolution is fundamentally limited by the geometric location of wells.

Another important challenge is that lag times are nonlinear functions of the reservoir characteristics, i.e., the pressure wave will not propagate in a straight path between injection/production wells, and instead it will follow the ``least resistance path'' inside the reservoir. Compared to fixed path tomography problems (X-ray, ground penetrating radar) for which paths are known, not knowing the actual pathway significantly increases the difficulty of our problem.

Many reconstruction algorithms have been proposed (Berryman.[4] 1991), but most of them can only successfully recover a low velocity contrast reservoir model. In a real reservoir, it is not unusual for high permeability channels to be present that have pressure wave velocities orders of magnitude greater than those in other areas. Therefore, we cannot apply conventional reconstruction algorithms and need to design a new algorithm that can estimate these high permeability channels. (Lin and Ortega.[5] 2010)

In this paper, we focus on tight reservoirs with few high permeability channels in them. The permeability contrast is very high ($\sim 10^5$) so that the lag time as pressure wave propagates inside the high permeability channels can be considered to be almost negligible. If we estimate the reservoir characteristics by using the conventional iterative least-squares method (Berryman.[4] 1991), this will lead to very poor results in general. One reason is this algorithm uses iterative linearization to solve the nonlinear inverse problem. For the high contrast permeability case, there is a very large velocity gap between different areas and the linearization algorithm usually fails to converge. Another reason is the least-squares method will search for a solution that minimizes the square norm; therefore it will tend to reconstruct a smooth model that cannot correctly capture the high contrast behavior.

In order to solve these problems, we develop a new algorithm to estimate the angle and location of high permeability channels directly. Compared to the conventional methods, we do not try to estimate the velocity in each cell. Instead, we assume only two possible velocities can be observed, a constant finite velocity (set to 1 here) for background areas, and infinite for the high permeability channels (equivalently the lag time will be zero through the high permeability channels).

In our approach, the main purpose is to identify the high permeability abnormality in the homogeneous background. In fact, we do not need a precise estimate of background permeability. Under the assumption of background homogeneity, we can use the "ratio" of lag time to the geometrical distance to avoid estimating the background permeability. Any high permeability channels between an injector and a producer will always reduce the lag time, as well as the ratio. The ratio is upper-bounded by a constant, which can be obtained in the case when the wave propagates only through the homogeneous background. The numerical value for this constant is the background permeability. Therefore, we can assume the largest ratio to be the background permeability and only need to compare the relative ratio of each injector-producer pair.

Also, unlike prior work, we model the high permeability channels as line structures in *2D* and approximate the flow path with piecewise linear combinations. Therefore our algorithm estimates directly the location of high permeability channels. This approach significantly reduces the computational complexity required to calculate the flow path and achieves a stable result, as compared to the conventional iterative least squares ray-tracing method.

Our approach can also be extended to multiple discrete permeability values. For example, we can model the fault with zero permeability and try to detect the high permeability channels and faults at the same time. The main difficulty is the computational complexity to calculate the flow path is much higher. When we only consider two possible values for the permeability (one and infinity), it can be shown that the flow path must be a combination of shortest paths between high permeability channels. But this characteristic will no longer be valid if we have multiple or finite permeability values.

It is well known that the reconstructed model may not be unique if we only have a limited number of measurements. In our approach, we define the total length of estimated high permeability channels as the regularization metric. Thus, our chosen reconstructed model will be the one that minimizes a metric that combines goodness of fit for the observation as well as length of resulting estimated fractures. That means that we will tend to choose the "smallest" set of high permeability channels that satisfies the measured response time. Note that our approach could also incorporate other prior information to define an alternative metric. For example, if a geophysical survey provides a rough estimate of orientation, the corresponding estimated angle can be incorporated into the regularization metric so as to encourage high permeability channels to follow that direction.

The rest of paper is structured as follows: in the next section, we define the mathematical model and provide physical intuition for fluid flow in a heterogeneous reservoir. Then the tomography reconstruction concept is introduced and we describe our algorithm. Finally, we provide simulation results and conclusions.

Physical model

In this section we introduce the physical model to describe the response time in a heterogeneous reservoir. Early work in pulse testing (Johnson.[6] 1966) proposed changing the injection rate at a well by alternating between a constant flow rate and a shut-in. Then pressure response is measured in the other wells and the amplitude and lag time is determined between the wells. The lag time t_1 can be related to the reservoir properties by the equation

$$t_1 \approx 20{,}000 \frac{Sr^2}{T\Delta t} \quad (1)$$

Where r is the distance between the wells, and Δt is the pulse length. T and S represent the transmissibility and storage respectively.

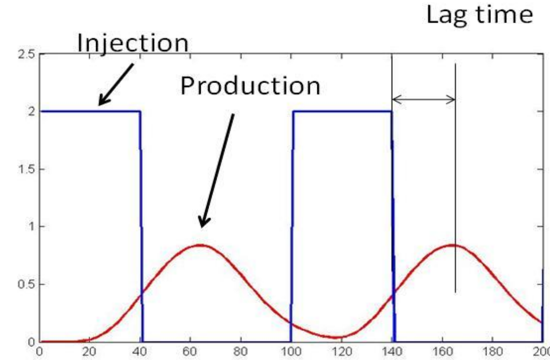


Figure 1 Pulse-Test Terminology (Johnson[6]. 1966)

Recently, the capacitance model proposed by Yousef *et al.*[3] (2005) has generated significant interest in the reservoir modeling community. This work considers the reservoir as an input (injection rate) and output (production rate) system. The mathematical formulation can be written as

$$\tau \frac{dq}{dt} + q(t) = i(t) - \tau * J \frac{dp_{wf}}{dt} \quad (2)$$

Where $i(t)$ is the injection rate and $q(t)$ is the total production rate. P_{wf} represents the flowing bottom hole pressure (BHP) and J stands for the productivity index. The "time constant" τ is defined by the total compressibility and drainage pore volume V_p.

$$\tau = \frac{c_t V_p}{J} \quad (3)$$

Follow the discussion by Yousef *et al.*[3] if we consider a mature reservoir and fix the bottom hole pressure, we can approximate production rates based on only the contribution from injectors.

$$q(t) = \int_{t_0}^{t} \frac{e^{-(t-\varsigma)}}{\tau} i(\varsigma) d\varsigma \quad (4)$$

We use the equation above as a simplified capacitance model to characterize the injection-production relationship. The production response with respect to step injection is shown in Figure 2.

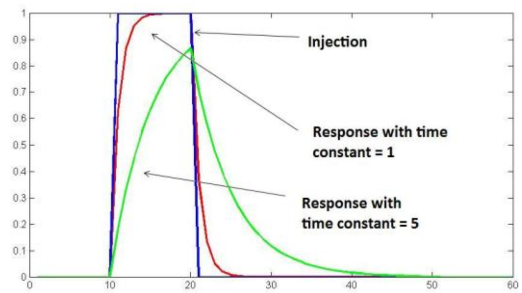


Figure 2 Production response for capacitance model with different time constant

From above, we know that the lag time t_1 or time constant τ are controlled by the storage volume between an injector-producer pair. In this paper we consider a tight reservoir with few high permeability channels so that the fluid will tend to flow through them. The storage volume between the wells is significantly reduced and is approximately propotional to the distance between the high permeability channels (see Fig 3). Therefore, the lag time t_1 or the time constant τ in capacitance model are roughly proportional to the distance the fluid has to travel within the low permeability area. We can choose a model that provides better fitting for the injection-production data, and use corresponding lag time or time constant as the estimated lag time between the wells.

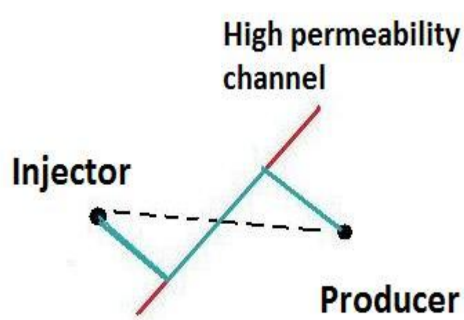


Figure 3 Flow path between injector/producer. Note that the path bends with the high permeability channel.

Tomographic Reconstruction

With measured injection/production data, we can estimate the lag time between wells. We now pose the tomographic reconstruction problem, namely, how to use the lag time information to estimate the reservoir model?

From previous section, we know that the lag time is roughly proportional to the travel distance in a homogeneous low permeability area. In order to reconstruct the reservoir model, we only need to identify these high permeability line structures so that the resulting model fits the lag time. This procedure can be described as a ``forward-backward'' process. In the forward step, we calculate the lag time based on current high permeability structure model. If it does not fit the measured lag time, in backward step we update the high permeability channels according to the difference. This can be summarized as a figure below.

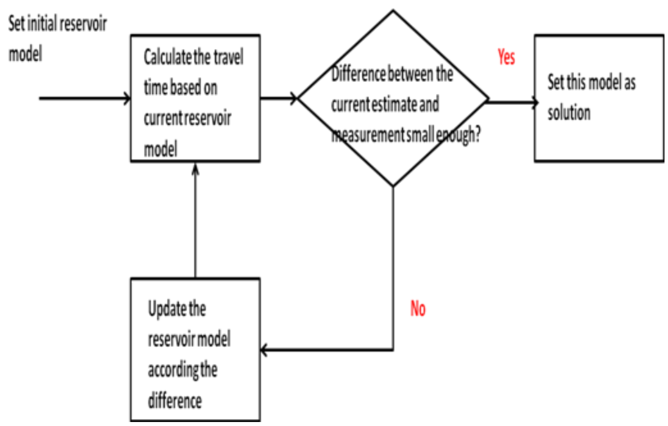


Figure 4 The procedure of estimating the high permeability channels based on the lag time.

First, we introduce the "forward step", that is, calculating the lag time if a high permeability channel is given. With the presence of high permeability channels, the travel path tends to "bend" and uses these channels to achieve fastest path. Since we approximate the lag time inside the high permeability channel as being almost zero, using simple geometry we can calculate the fastest path. For example, if we normalize the permeability of homogeneous background to 1 and consider a single high permeability channel *L*, the path will be either the direct link or the one using high permeability channel, depending on which one is faster (see Figure 3), where the path through the high permeability channel is comprised of the shortest paths to and from the channel and propagation through the channel (which is assumed to take a negligible amount of time as discussed earlier). The response time can be calculated by

$$u(\mathrm{Inj},\mathrm{Pro}) = \mathrm{Min}\begin{cases} d(\mathrm{Inj},\mathrm{Pro}) \\ d(\mathrm{Inj},L) + 0 * d(\mathrm{Inj},L) + d(L,\mathrm{Pro}) \end{cases} \quad (5)$$

$u(\alpha,\beta)$ and $d(\alpha,\beta)$ represent the lag time and the geometrical distance between two points (α,β). By induction this can be extended to n channels: if we already know the fastest path based on $n-1$ channels, adding a new one will decrease the lag time if it causes any "short cut" for the previous flow path.

After deciding the travel path, we can also get the gradient of lag time with respect to any changes in the high permeability channel. Thus, if we increase the length or rotate the high permeability lines, the travel path will follow these changes. In Figure 5 we show that rotating a high permeability channel will change the travel path from $\vec{\alpha}$ into $\vec{\alpha}'$. The gradient of the lag time with respect to the rotation can be calculated by taking the limit of the travel path in homogeneous area divided by the small change in the angle of high permeability channel

$$\frac{\delta u}{\delta \theta} = \lim_{\Delta\theta \to 0} \frac{|\vec{\alpha}'| - |\vec{\alpha}|}{|\Delta\theta|} \quad (6)$$

In this case, the change of travel path $\vec{\beta}$ is equal to the length of high permeability channel multiplied with the small change in angle $l \cdot \Delta\vec{\theta}$. To include all possible changes (increase/decrease length, rotation and shift), we analyze $\vec{\beta}$ as a general case.

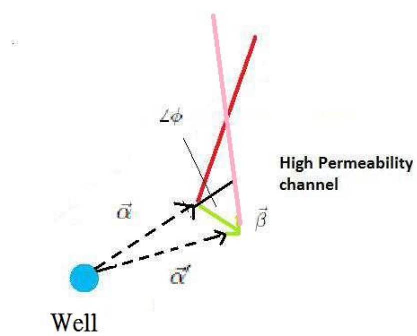


Figure 5 Change in flow path if there is a small rotation in orientation for the high permeability channel.

$$\|\vec{\alpha}'\|^2 = (\vec{\alpha}+\vec{\beta})*(\vec{\alpha}+\vec{\beta}) = \|\vec{\alpha}\|^2 + 2(\vec{\alpha}*\vec{\beta}) + \|\vec{\beta}\|^2 \quad (7)$$

if $\|\vec{\beta}\|$ is very small, then it can be approximated by

$$\approx \|\vec{\alpha}\|^2 + 2\|\vec{\alpha}\|\|\vec{\beta}\|\cos\phi \text{, where } \phi \text{ is the angle between } \vec{\alpha},\vec{\beta}$$

$$= \|\vec{\alpha}\|^2\left(1+\frac{2\|\vec{\beta}\|\cos\phi}{\|\vec{\alpha}\|}\right) \quad (8)$$

$$\|\vec{\alpha}'\| = \|\vec{\alpha}\|\left(1+\frac{2\|\vec{\beta}\|\cos\phi}{\|\vec{\alpha}\|}\right)^{1/2} \quad (9)$$

Using Taylor's expansion on the second term

$$\approx \|\vec{\alpha}\|\left(1+\frac{1}{2}\cdot\frac{2\|\vec{\beta}\|\cos\phi}{\|\vec{\alpha}\|}\right)$$

$$= \|\vec{\alpha}\| + \|\vec{\beta}\|\cos\phi \quad (10)$$

The derivative of travel time is given by

$$\frac{\partial T}{\partial \beta} = \lim_{\delta\beta\to 0}\frac{|\vec{\alpha}'| - |\vec{\alpha}|}{|\vec{\beta}|} = \cos\phi \quad (11)$$

The gradient of lag time is related to the cross angle of travel path $\vec{\alpha}$ and the change of high permeability channel $\vec{\beta}$. If we model high permeability channels as straight lines, we can use the parameters $\{l^{length}{}_i, l^{\angle\theta}{}_i, l^x{}_i, l^y{}_i\}$ to represent an arbitrary high permeability channel. It is easy to relate the $\vec{\beta}$ with small change in $\{l^{length}{}_i, l^{\angle\theta}{}_i, l^x{}_i, l^y{}_i\}$, which correspond to changes length, rotation and shift operations. As we show in Figure 6, if we change the length, then the travel path is modified by $\vec{\beta} = \Delta\vec{l}^{\,length}$. The case of rotation is discussed before $\vec{\beta} = l\cdot\Delta\vec{\theta}$. And the shift is easily understood by $\vec{\beta} = \Delta\vec{x}$ and $\vec{\beta} = \Delta\vec{y}$.

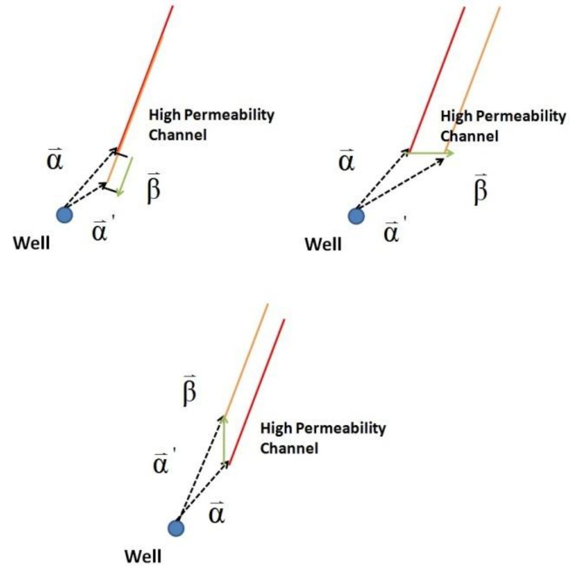


Figure 6 Change in flow path with respect to the increase length, shift for the high permeability channel.

Inverse Problem Formulation

The problem we aim to solve is an inverse problem, namely, how to estimate the location, length and angle of high permeability channels given the lag time between the wells. As in many inverse problems, a major difficulty is that the reconstructed reservoir model may not be unique if we only have limited data (see Figure 5). In order to choose between different possible models, we use the total length of estimated high permeability channels as regularization metric. Our algorithm will choose the shortest length model to explain the observed data. This can be stated as a two-step gradient search: in the first step, we refine the structure to satisfy the observed lag time. In the second step, we choose the minimum length structure. We extend our previous reconstruction algorithm (Lin and Ortega,[5] 2010), so that not only the length but also the angle, and centre of high permeability channels are modified in each iteration. The difference in this paper is we only try to reconstruct one single candidate solution. The algorithm can be summarized as follows:

Reconstruction Algorithm

1. Set the initial structure:
Assign the initial structure S with N high permeability channels.
$S = \{l_1, \ldots, l_N\}$, and each l_i has parameter $\{l^{length}{}_i, l^{\angle\theta}{}_i, l^x{}_i, l^y{}_i\}$

2. With measurement, $\vec{t} = (t_1, \ldots, t_k)$
define the cost function as $C(S) = \|\vec{T}(S) - \vec{t}\|^2$

3. Match the measurement t :
Use gradient search to update the structure S.
$S \leftarrow S - \lambda \cdot \nabla C$

4. Minimize the total length of S :
Update the structure S based on same t.
$S \leftarrow \tilde{S}$, if $\sum \tilde{l}_i^{length} \le \sum l_i^{length}$ and $\vec{T}(\tilde{S}) = \vec{T}(S)$

5. Go to step 3 until S converges.

In the first step, we initialize the structure of high permeability channels as lines. We create N lines, where each line is represented by a series of parameters $\{l^{length}{}_i, l^{\angle\theta}{}_i, l^x{}_i, l^y{}_i\}$, which represent its length, angle and center location (x, y). In the second step we define the cost function to quantify the difference of estimated and measured lag time.

In the third step, we calculate the difference between the estimated lag time and the measurement, and use gradient search to refine the model *S*. From above discussion, we know that the gradient of the lag time with respect to $\{l^{length}{}_i, l^{\angle\theta}{}_i, l^x{}_i, l^y{}_i\}$ can be calculated by simple geometry. This will update the length, angle and center position of these line structures.

In the fourth step, we search the models that have same lag time and try to choose one with minimum total length. A major issue for the inverse step is that the reconstructed reservoir model may not be unique if we only have limited data (see Figure 7) Given a pair of channels $\{l_i, l_j\}$, if the gradients $\{\frac{\delta C}{\delta l_i^{length}}, \frac{\delta C}{\delta l_j^{length}}\}$ corresponding to each of the lengths of these channels are not equal, this implies that we can change the lengths of the lines $\{l_i, l_j\}$ while still keeping the same lag time cost C, but reducing the regularization cost.

Assume $\frac{\delta C}{\delta l_i^{length}} > \frac{\delta C}{\delta l_j^{length}}$, then we can choose $\alpha \cdot (\frac{\delta C}{\delta l_i^{length}}) = \beta \cdot (\frac{\delta C}{\delta l_j^{length}})$,with $\alpha < \beta$ Let $l_i^{length} \leftarrow l_i^{length} + \alpha$, $l_j^{length} \leftarrow l_j^{length} - \beta$, the change in cost time function is zero, $\Delta C = 0$ but the total length decreases with $\beta - \alpha$. In this case, we have another structure that has same lag time with lower total length.

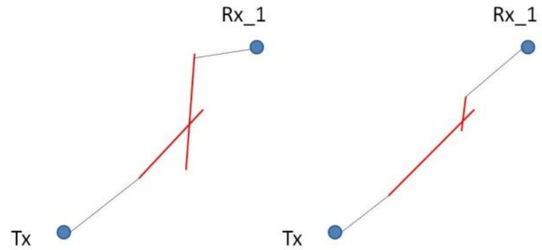


Figure 7 Example of changes in line length between two high permeability lines. Note these two have exactly the same lag time, but that the second one has smaller total length.

Simulation Result

We use a commercial simulator to test our method. The first case is a 5 spot and we want to decide the length and angle for the hydraulic fracture located in the center producer. In this case, we use the capacitance model, CM (Yousef *et al.*[3] 2005) to model the injection-production relationship and retrieve the lag time. But estimating the coefficients in CM is a non-linear optimization problem, for which sometimes one cannot achieve convergence in the search for a solution. In order to avoid this, we use FIR model (Ljung.[7] 1987) as an intermediate model. The reason is we can always get a stable estimate result for FIR coefficients by least-square or linear programming method. First we estimate the FIR model by injection-production data, and then decide the Capacitance model that has best fitting with previous FIR model. We use the PN sequence as injection pattern proposed by Lee *et al.*[8] (2008) and measure the changes in production. Choosing PN sequences as input has been proven to achieve the lowest covariance matrix for estimating the impulse response coefficient in a FIR linear system (Ljung.[7] 1987). We use linear programming to estimate the FIR coefficient, and then we match the FIR with the capacitance model to determine the time-delay constant and use it as the lag time between injector-producer. We use the lag time to reconstruct the reservoir model. The result is shown in Figure 6. The difference between the actual models and the estimated one is within 10 degree.

The second case is a line drive, with 5-injectors and 5-producers. We do not have any prior information about the location, length or angle about the high permeability channel. We increase the injection rate and determine the lag time from the response of production rate. For the estimated result we can see our method successfully detects the high permeability channels between injector-5 and producer-2.

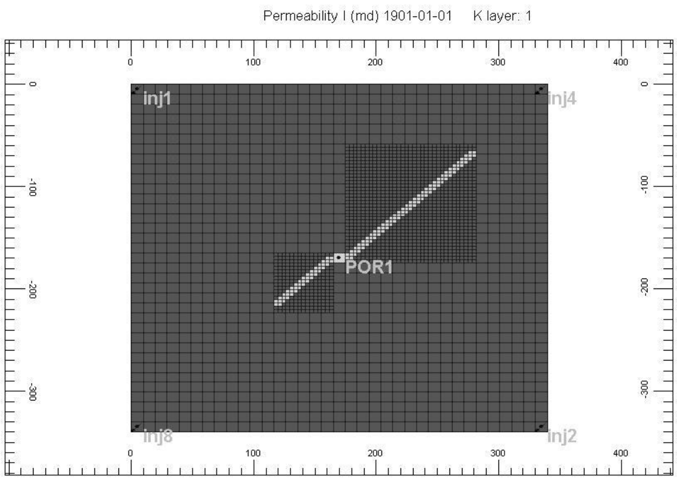
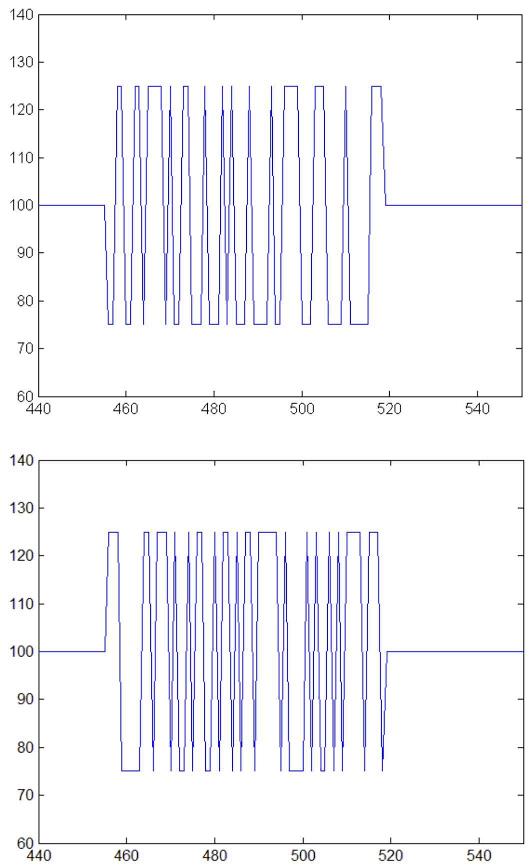


Figure 8 Ground truth in simulator. It is a 5-spot with 4-injector, 1-producer with a 45’ degree high permeability channel. The permeability value is 10 for the background, and 2000 for the high permeability channel.



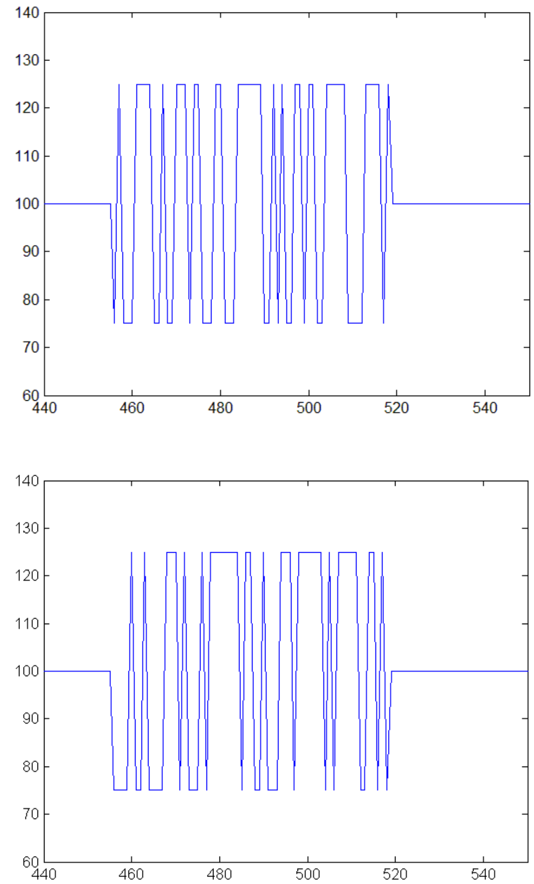


Figure 9 Use PN sequences as the injection pattern. Note all injections have the same average rates.

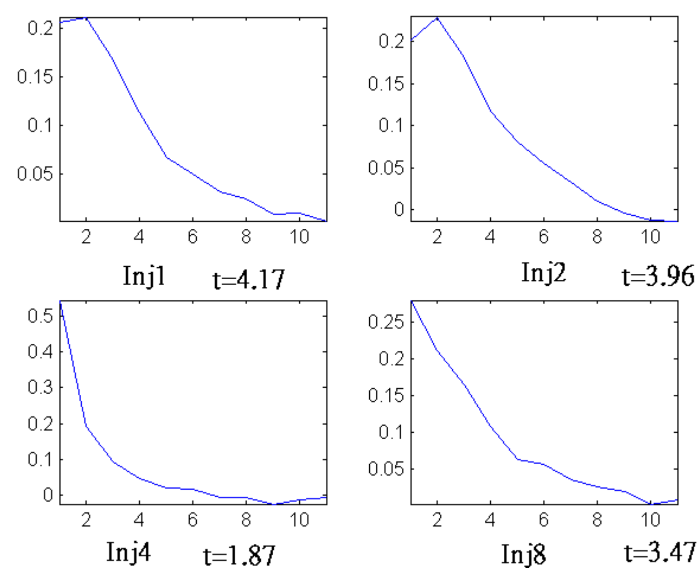


Figure 10 The estimated injection/production response. Note the time constant for Inj4 is much smaller.

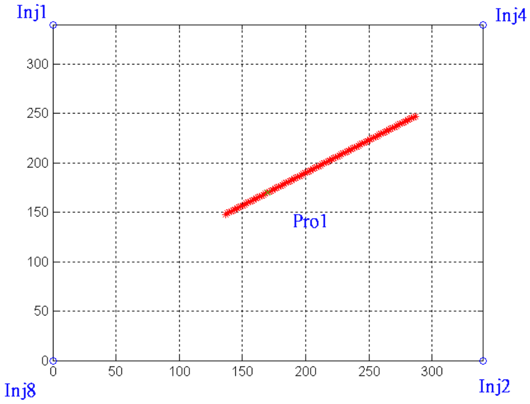


Figure 11 The reconstructed high permeability channel

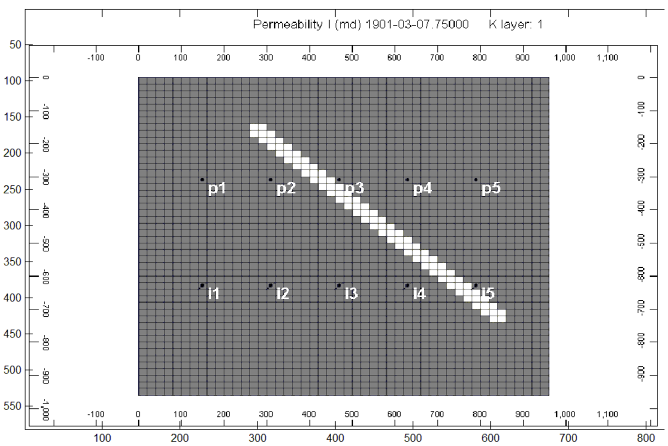


Figure 12 The ground truth in simulator. It's a 5-injector, 5-producer line drive. There is a high permeability channel connect injector-5 and producer-3. The permeability value is 10 for the background, and 2000 for the high permeability channel.

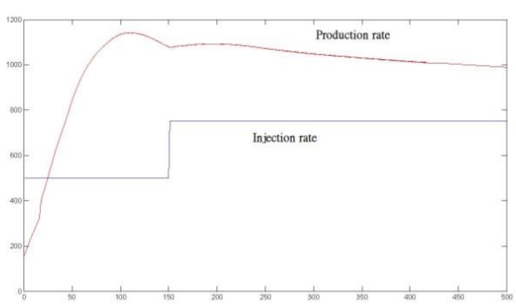


Figure 13 The injection/production rate. We increase the injection rate and measure the response in production to determine the lag time between the wells.

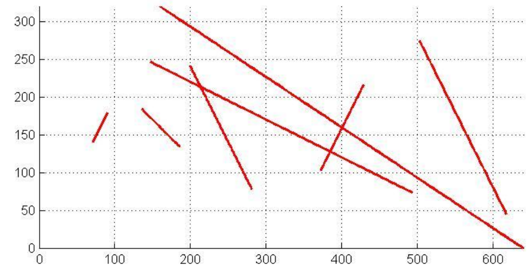


Figure 14(a) The initial structure we start to update.

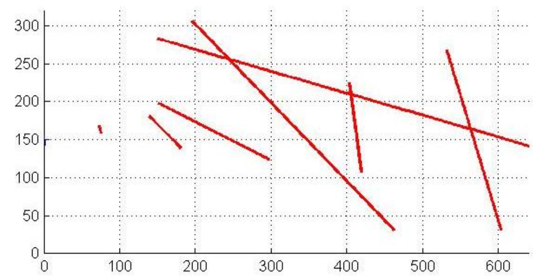


Figure 14(b) The result after 3 iterations.

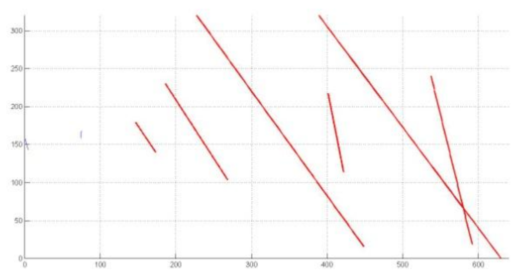


Figure 14(c) The result after 10 iterations.

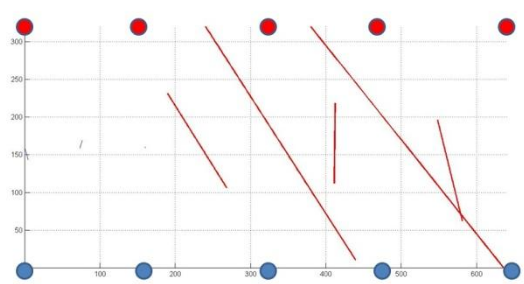


Figure 14(d) The final result after 30 iterations. It can successfully catch the high perm between injector-5 and producer-3. Due the measurement noise, there are some phantoms in other area.

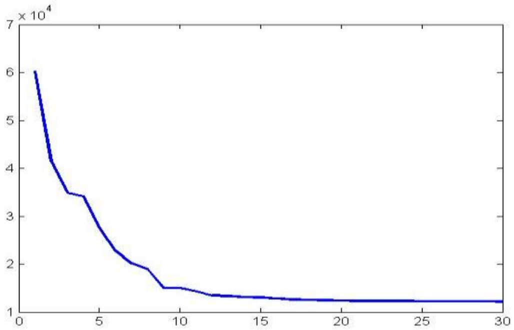


Figure 15 The mismatch cost function. It decrease with iterations, but due to measurement noise the final value is not zero.

Conclusion

In this paper, we propose a new method to use the ``lag time'' to detect high permeability channels. This method only needs the injection-production data and does not require any additional instruments. It can detect the field changes in real-time without altering the average daily production.

In order to apply this to real field data, we note some practical issues. First, the data sampling period and quality are important. Usually we have very reliable daily injection rate data, but production rates are often obtained from bi-weekly well-test data. This reduces the time resolution of estimated lag times. For example, assume we have two pairs of injector-producers and the lag time is 1 day and 10 days. After we increase the injection rates, we cannot tell the difference by looking at the production data because the time-resolution is not high enough (it is only sampled once every 14 days).

The second issue we encounter in practical applications is the distribution of well locations, which is related to spatial resolution. Consider an extreme case where one injector-producer pair is in the horizontal direction. Any high permeability channel exactly in the vertical direction will not affect the lag time. Therefore, it is "invisible" under this situation. In order to detect the high permeability channel in arbitrary direction, we prefer the wells to be uniformly located in the field and to cover all angles, which is not possible in reality. We are currently studying this problem, and plan to address it in future work.

Future work will focus on how to combine the geological information and define a better regularization to choose between possible reservoir models. For example, we may have the seismic survey for the field, but lack of finer details. We can use this kind of information as a prior for our algorithm.

Nomenclature

t	=	*time*
S	=	*storage*
T	=	*transmissibility*
$i(t)$	=	*injection rate*
$q(t)$	=	*total production rate*
Pwf	=	*flowing bottom hole pressure*
J	=	*productivity index*
$l^{length}{}_i$	=	*length parameter for i-th line structure*
$l^{\angle\theta}{}_i$	=	*angle parameter for i-th line structure*
$l^x{}_i, l^y{}_i$	=	*center parameter for i-th line structure*
τ	=	*time delay constant in capacitance model*

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Reference

1. Sheriff, R.E., "Basic petrophysics and geophysics," Investigations in Geophysics, vol. 7, pp. 37–49, 1992.
2. Vasco, D.W., Yoon, S. and Datta-Gupta, A. "Integrating dynamic data into high-resolution reservoir models using streamline-based analytic sensitivity coefficients," SPE Journal, vol. 4, no. 4, 1999.
3. Yousef, A.A, Gentil, P., Jensen, J. and Lake, L. "A capacitance model to infer interwell connectivity from production and injection rate fluctuations," in SPE Annual Technical Conference and Exhibition, 2005.
4. Berryman, J.G., "Lecture notes on Nonlinear Inversion and Tomography: I. Borehole seismic tomography.'' Tech. Rep., UCRL-LR-105358-Rev. 1, Lawrence Livermore National Lab., CA (United States), 1991.
5. Lin, Y.T. and Ortega, A., (2010) "Reconstruction algorithm for high contrast velocity travel time tomography,'' in Proc. ICASSP, Dallas, TX, USA, Mar. 2010.
6. Johnson, C.R., Greenkorn, R.A. and Woods, E.G. "Pulse testing: a new method for describing reservoir flow properties between wells," Journal of Petroleum Technology, vol. 18, no. 12, pp. 1599–1604, 1966.
7. Ljung, L. "System identification: theory for the user. 1987," PTR Prentice Hall, Englewood Cliffs, New Jersey.
8. Lee, K.H., Ortega, A., Ghareloo, A. and Ershaghi, I. "A Method for Characterization of Flow Units between Injection-Production Wells Using Performance Data," in SPE Western Regional and Pacific Section AAPG Joint Meeting, 2008.