Cubic Spline Approximation of Rate and Distortion Functions for MPEG Video

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ABSTRACT

The rate control algorithm plays an important role for improving and stabilizing the playback quality for video coded with the MPEG standard. Several optimal control techniques have been proposed to aim at the best possible quality for a given channel rate and buffer size. Some of these approaches are complex in that they require the rate and distortion characteristics of the input data to be measured. This motivates us to pursue a method for approximating the rate and distortion functions to reduce the computations. Previous work has been based to a large extent on modeling the distortion as negative exponential functions of the rate. This type of model ignores many factors in a real MPEG encoding process and is not general enough for all video sources. In this paper, we use piece-wise polynomials to approximate the frame-level rate and distortion. The frame dependency between the predictive frame and its reference frames is also considered in our model. Compared to other models, our method is relatively more complex but gives more accurate results. We observe low average relative model errors, which indicates that the model is accurate for most of the quantization settings. We use the model within our gradient-based rate control algorithm and show how using the model one can closely approximate the solution obtained using the actual data. Finally, we apply a simplified version of the model to a new fast algorithm derived from the MPEG Test Model 5, and demonstrate that both the quality (in terms of PSNR) and stability of the quality can be improved.

Keywords : MPEG, video coding, rate control, spline interpolation

1 INTRODUCTION

MPEG [1,2] is expected to be a dominating video standard after it is adopted by the Digital Video Disk (DVD) and several digital cable systems. In the standard, only the bitstream syntax and the reference decoder have been defined, while several components in the encoder can vary in each implementation. One such essential component, rate control, is the method for choosing the quantization scales so that the bitstream can be transmitted through a communication channel to a decoder for real-time playback without decoder buffer overflow. The channel can be a constant-bit-rate channel for DVD and some digital cable systems, or a leaky-bucket-constrained variable-bit-rate channel for ATM networks. The rate control scheme plays an important role for improving and stabilizing the decoding and play-back quality.

Many rate control schemes for constant-bit-rate encoding use the buffer occupancy to determine the quantization setting (also known as "direct buffer-state feedback scheme") These approaches do not measure and monitor the distortion in the algorithm. Instead, they are trying to maintain the quality at a similar level for different types (I,P,B), by using some general assumptions on the R-D characteristics. One example is the rate control scheme in the MPEG Test Models [3], in which the two most important assumptions are (i) the product of quantization scales and the code length is a constant (also known as the "frame activity measure"); (ii) the distortion (e.g. mean square error, or MSE) linearly proportional to the quantization scales with different slope for each frame type, and the ratios of slopes for different frame types are constants (also known as k_P and k_B parameter, see [3] for details). The problem is that these assumptions are not always true for all possible video sequence and the parameters k_P and k_B depend on the contents of the video, hence the results are not always good. The advantage of these approaches is their low computation complexity. Recently, there has been a growing interest in rate-distortion (R-D) optimal techniques for both bit allocation and rate control. There are several possible frameworks in which to optimize the performance of the rate control algorithm. A popular approach is to use models of the future frame's rate (and sometimes also distortion) and use control techniques to avoid overflow [4,5]. A second alternative is to *measure* the rate and distortion on the frames themselves, thus increasing the required optimization complexity but eliminating the dependency of the results on the choice of a good model. Examples of this approach can be found in [6–8] where techniques like Lagrangian optimization and dynamic programming have been used. These approaches are complex in that they require the rate and distortion characteristics of the input data to be measured. However they are well suited for environments, as in MPEG, where a discrete set of operating points is available and where it may not be easy to find adequate "continuous" models for the data. In this paper, we are trying to bring the two different schemes closer. In order to achieve this goal, we introduce a model to approximate the rate-distortion characteristics and use the model to (i) reduce the computation complexity of the optimum rate control scheme by avoiding the need to measure the R-D data on all possible settings, and (ii) enhance the quality of direct buffer-state feedback scheme by adding to the algorithm a mechanism for monitoring the distortions.

Previous work on rate and distortion modeling has been based to a large extent on the exponential statistics model. For example, in [9,10], exponential expressions were used to model the relationship between the rate, distortion, and quantization step size in a macroblock. These types of models require low computation overhead since they are obtained based on parameters such as block variance which can be easily obtained from the input frames. However, these are continuous models which tend to be better when a large number of quantizers is used. Thus they may suffer from large errors because of the difficulty in modeling the highly nonlinear quantization and entropy coding process. In addition, these models do not take into account the dependencies that arise in the choice of quantizers for the reference frames and the predicted frames [7]. Even when these models take the dependencies into account, as in [11], they ignore some non-linear effects that are typical in video coding. For example, under the general intra/inter selection rule, there is no dependency if the quality of the reference frame is too low.

In this paper we study models that are better suited to rate-distortion optimization in realistic video coding scenarios. We will focus on the two improvements motivated above, namely, we provide models that (i) make relatively few assumptions on the shape of the R-D characteristics and are thus suited when operating with a small number of quantizers and (ii) take into account the dependencies typical of video coding. These models are based on computing a few R-D points and interpolating the remaining points using spline functions. The price to pay for the increased accuracy is a somewhat higher complexity. To demonstrate the performance of the model, we apply it to the gradient-based rate control algorithm [12,13]. The results show that, with the approximation model, similar performance can be achieved with only about 1/10 of computation cost. We then propose a new fast algorithm derived from the MPEG Test Model 5 (TM5) with a simplified version of the approximation model. Simulation results show that both the quality (in terms of PSNR) and the stability of the quality are improved. The paper is organized as follows. In Section 2, we describe the formulation of the spline interpolation functions and inter-frame dependency models, and present the model compliance testing results. In Section 3, we apply the model to the rate control algorithms and present several experimental results. Finally, conclusions and future perspectives on this technique are given in Section 4.

2 APPROXIMATION OF THE RATE AND DISTORTION

In MPEG encoding, the rate-distortion trade-off is controlled by a quantization scale, denoted as q. This parameter is used to compute the step size of the uniform quantizers used for the different DCT coefficients (except for the DC coefficients, see [2] for details). A different q can be assigned to each macroblock. The scheme for adjusting the value of q between macroblocks within an image frame is called "adaptive quantization". There are several schemes for doing the adaptive quantization. For example, in MPEG Test Model 5 (TM5) [3], a nonlinear mapping based on the block variance is used to adapting the q's. Another example is [14], where q's are determined in such a way that a pre-defined cost function (e.g. average MSE) is optimized. In this paper, we do not address the problem of adaptive quantization, and use constant quantization, i.e., keep q constant over an entire frame. Hence, only one q is assigned to each image frame. The rate (the number of bits generated by the coder), $r(q)$, and the distortion (here the MSE is used), $d(q)$, can be measured after the frame has been encoded. The goal of the approximation model is to estimate $r(q)$ and $d(q)$ without calculating them for all the quantization settings. Due to the complex nonlinear properties of the quantization and entropy coding processes, it is difficult to predict the function value accurately enough by using simple mathematical expressions. In this paper, we propose an approach which calls for encoding the data and measuring the R-D functions, but *only on a small set of quantization scales* which we call "control points". Piece-wise polynomials, or splines, are then used to interpolate the function for other q's where the actual data has not been measured.

2.1 Spline Interpolation for R-D Functions

To make the rate and distortion functions more suitable for a gradient-search based optimization algorithm, the first-order derivative of these functions should be well-defined. A good candidate for the interpolation function would be the "interpolating cubic-spline", which possesses the second-order continuous property [15]. The disadvantage of this method is that the interpolation polynomials for any given segment (a segment is defined as a set of points between the two consecutive control points) depend on all the control points, i.e., it will require the coder to encode the source on all the control points even though only a small portion of the function data is required in the rate control algorithm. In this paper, we use another type of spline, which requires smaller computation cost, still possesses first-order continuity, and for which each segment depends only on four nearest control points.

We assume the control points are defined as (x_i, y_i), $i = 0 \ldots M-1$, where M is total number of control points. Fig. 1 shows an example set of control points, where x_i represents the quantization scale (for MPEG, the applicable values are $\{1, 2, \ldots, 31\}$), and y_i represents the actual measured rate or distortion. The function between two consecutive control points, x_i and x_{i+1}, is defined as

$$f_i(x) = a_i \cdot x^3 + b_i \cdot x^2 + c_i \cdot x + d_i \tag{1}$$

where $i = 0 \ldots M-2$. There are $M-1$ polynomials, each corresponding to one segment. For each polynomial, the four parameters, a_i, b_i, c_i, d_i, can be derived from the four control points, (x_{i-1}, y_{i-1}), (x_i, y_i), (x_{i+1}, y_{i+1}), (x_{i+2}, y_{i+2}), by imposing the following two constrains:

1. The interpolated function should take the same values as the original one at the control points, hence:

$$x_i^3 \cdot a_i + x_i^2 \cdot b_i + x_i \cdot c_i + d_i = y_i \tag{2}$$

$$x_{i+1}^3 \cdot a_i + x_{i+1}^2 \cdot b_i + x_{i+1} \cdot c_i + d_i = y_{i+1} \tag{3}$$

2. The first-order derivative should be continuous on the control points. This condition can be achieved by defining the slope at control point x_i as (the first derivative of $f(x)$ is denoted as $f'(x)$):

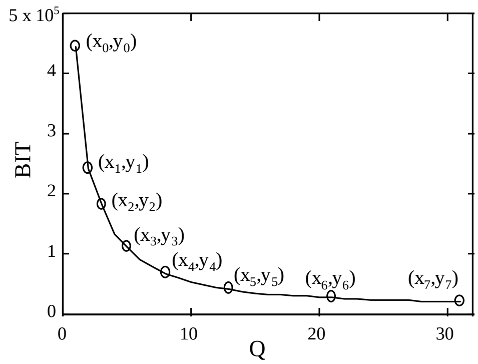
$$f_i'(x_i) = f_{i-1}'(x_i) = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \tag{4}$$

By taking the derivative of (1) and substituting into (4) on the two end points of $f_i(x)$, we get

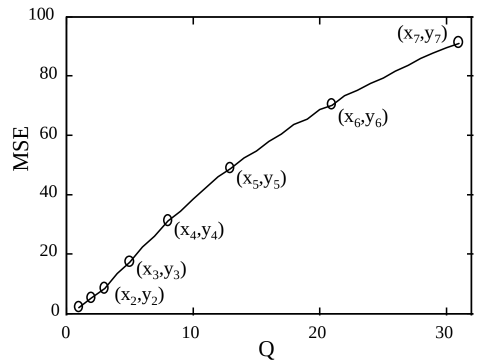
$$3x_i^2 \cdot a_i + 2x_i \cdot b_i + c_i = \frac{y_{i+1} - y_{i-1}}{x_{i+1} - x_{i-1}} \quad (5)$$

$$3x_{i+1}^2 \cdot a_i + 2x_{i+1} \cdot b_i + c_i = \frac{y_{i+2} - y_i}{x_{i+2} - x_i} \quad (6)$$

The four unknowns of $f_i(x)$, a_i, b_i, c_i, d_i, can be readily found from the set of equations (2), (3), (5), and (6).



(a) Bits



(b) MSE

Figure 1: Control points for typical (a) rate and (b) distortion curves. In this figure, a control point (x_i, y_i) represents that if the quantization scale is set to x_i, the measured rate or distortion value is y_i.

In order to capture the exponential-decay property, which is typically observed in R-D data[1], we choose the control points to be with the relation as $x_i = x_{i-1} + x_{i-2}$, so that, in the MPEG case, the set of eight control points becomes $\{1, 2, 3, 5, 8, 13, 21, 31\}$. However, on a typical video sequence at standard rate (e.g. CIF at 1.152 Mbps), the settings for $q = 1, 2$, or even $q = 3, 4$, are rarely used, hence, only 5 to 6 control points are required in most cases.

2.2 Formulation of Inter-Frame Dependency

In MPEG, each image frame can be encoded in one of the three frame types, namely I (intra), P (predictive), and B (bi-directional interpolated) [2]. For I frames, the R-D characteristics are independent to any other frames, so the one dimensional spline functions in Section 2.1 can be directly applied. However, for P and B frames, the R-D characteristics depend on the quality of their reference frames and we have thus to deal with multi-dimensional functions. We will now introduce methods that are less complex than full-blown multi-dimensional models while still capturing the inter-frame dependencies. In order to reduce the computation cost, we make the motion estimation refer to the original reference frame so that the motion vectors are not affected by the quantization settings, and do not have to be recomputed when the P-B frames are repeatedly encoded to sample their respective R-D functions. Note that the ideas presented in this section are introduced in an MPEG framework, but are applicable to more general video coding environments.

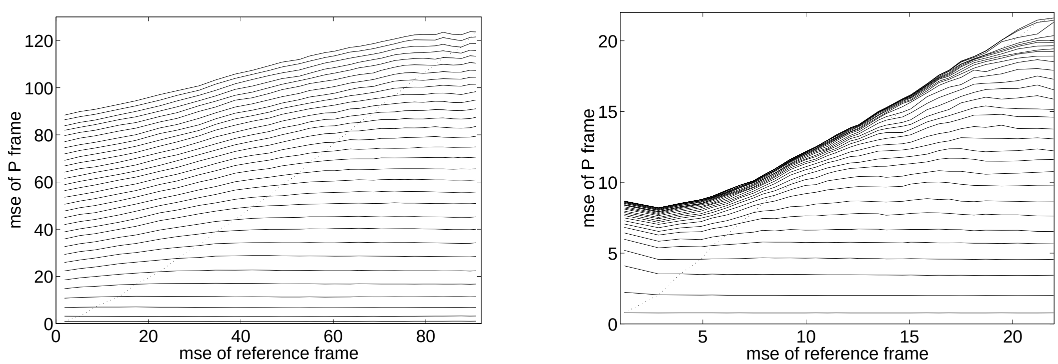
We consider the first P frame in a group-of-pictures (GOP), and its reference I frame[2]. Because of the dependency, the rate and distortion functions become two-dimensional, i.e., they have the form $d(q_I, q_P)$ and $r(q_I, q_P)$, where q_I and q_P are the quantization scales for the I and P frame respectively. Now, the data has to

[1] Note that while approximately exponential characteristics are typical, the error incurred with our approach will normally be smaller, because we have more degrees of freedom, and the characteristics are not exactly exponential.

[2] Note that the model still can be applied when the reference is another P frame.

be sampled in the two-dimensional space. One straightforward extension is to sample the data at the same 6 control points for each dimension (total 36 control points), but this requires many more computations. This is because, in order to compute the data for each additional control points along the q_I axis, the I frame has to be re-compressed and reconstructed again (involving DCT, quantization, de-quantization, and IDCT), and the P frame has to be re-encoded (involving prediction, DCT, quantization, and encoding). This complexity is much higher than the one for computing the data along the q_P axis (only involving quantization and encoding for the P frame). In this section, we introduce a model for inter-frame dependency which only requires two control points along the q_I axis.

Consider the fact that the rate-distortion characteristic of the predicted frame (P or B) depends on the quality of its reference frame(s). When the reference frame has smaller MSE, the prediction residue tends to be smaller, which results in a smaller rate and distortion in the predicted frame. Conversely, if the MSE in the reference is larger, not only the rate and distortion of the predicted frame will become larger, but also more macroblocks will be coded as “intra-block” (given the typical decision rules used in general MPEG encoders, e.g. in [16]), which will decrease the dependency on the reference frame. After some point, the predicted frame will be completely independent of the reference frame (see Fig. 2).



(a) Football sequence (b) Miss America sequence

Figure 2: MSE for the P frames from two video sequences, plot as function of MSE for their reference frames. Each solid line is a MSE curve for a given q in the predictive frame. The dotted line indicates the boundary where q for the predictive and reference frames are equal.

Suppose q_P fixed at a constant C, so that $d(q_I, q_P = C)$ becomes a one-dimensional function with variable q_I. The MSE of the reference frame (I-frame) is denoted as $d_I(q_I)$. Based on the above observation, the frame dependency for the distortion of a P frame is modeled as a linear increasing function with respect to $d_I(q_I)$ for $q_I \leq C$, and becomes a constant function for $q_I > C$, as shown in the following expression:

$$d(q_I, C) = \begin{cases} \alpha - \beta \cdot [d_I(C) - d_I(q_I)] & \text{if } q_I \leq C \\ \alpha & \text{if } q_I > C \end{cases} \tag{7}$$

where q_I is the only variable in the model. The two model parameters, α and β, can be determined by encoding and measuring the distortion at two values of q_I. For example, as shown in Fig. 3(a), if the two values are chosen to be 5 and 13, and the same spline model with 6 control points (as in Section 2.1) is used along q_P axis, the set of 12 control points becomes:

$$\left\{ \begin{array}{cccccc} (5,3) & (5,5) & (5,8) & (5,13) & (5,21) & (5,31) \\ (13,3) & (13,5) & (13,8) & (13,13) & (13,21) & (13,31) \end{array} \right\} \tag{8}$$

To interpolate the function value for any given settings, say $(10, 10)$, the above interframe model is applied 4 times with C set to $\{5, 8, 13, 12\}$, so that the function values are derived at $(10, 5)$, $(10, 8)$, $(10, 13)$ $(10, 21)$. Then a spline interpolation function is used to derive the value at $(10, 10)$ using the 4 derived data.

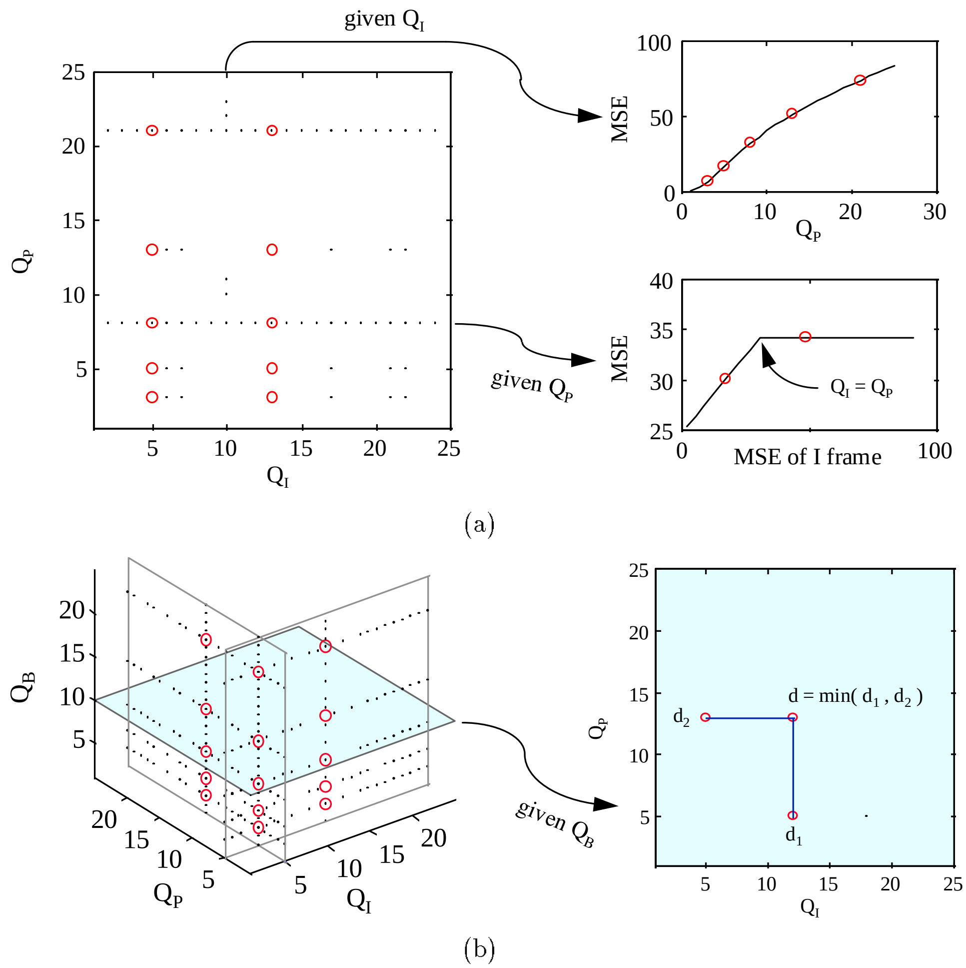


Figure 3: Reconstruction of distortion model for (a) P frame and (b) B frame.

However, due to the difference in properties, a similar model does not work as well for the rate. From several video sequences, we have observed that, for the quantization scales between 3 and 24, the inter-frame dependency for rate is reasonably low. Hence, the following simple piece-wise linear model is used (suppose the two measured points for q_I are x_1 and x_2):

$$r(q_I, C) = \begin{cases} r(x_1, C) & \text{if } q_I \leq x_1 \\ \{r(x_1, C)(d_I(x_2) - d_I(q_I)) + r(x_2, C)(d_I(q_I) - d_I(x_1))\}/\{d_I(x_1) - d_I(x_2)\} & \text{if } x_1 < q_I < x_2 \\ r(x_2, C) & \text{if } q_I \geq x_2 \end{cases} \quad (9)$$

For B frames, the MSE function becomes $d(q_I, q_P, q_B)$, where q_B is the quantization scale for the B frame itself, and q_I and q_P are the quantization scales for the two reference frames. To keep the computation simple, as illustrated in Fig 3(b), we first fix one reference frame by setting $q_I = c$, where c is one of the inter-frame control points, and we evaluate the dependency for the other reference frame by using the same model for P frames and get $d_1(c, q_P, q_B)$. We then fix the other reference frame and derive $d_2(q_I, c, q_B)$. Finally, $d(q_I, q_P, q_B)$ is defined as $\min(d_1(c, q_P, q_B), d_2(q_I, c, q_B))$. This procedure simulates part of the strategy for selecting "forward" or "backward" motion vectors in the MPEG encoder. The same model is also used for the rate function. There are a total of 18 control points to be measured if the same set of control points as in (8) is used.

2.3 Relative Model Errors

We use the MPEG-2 encoder implementation of [16] to test the accuracy the approximation model, by the following steps: We first encode the frame, measure and record the MSE and code length, for every possible quantization settings. Based on the function values at the pre-defined control points ($\{1, 2, 3, 5, 8, 13, 21, 31\}$ for intra-coded frame, $\{5, 13\}$ for inter-frame dependency), we build the model using the procedure described in Section 2.1 and Section 2.2, and calculate the estimated rate and distortion values. The relative error is then calculated by

$$\text{relative\_error} = \left| \frac{\text{estimated\_value} - \text{original\_value}}{\text{original\_value}} \right| \tag{10}$$

For I frames, the average and maximum relative errors are calculated over all the quantization scales. For P and B frames, the average and maximum relative errors are calculated over the typical operating range of quantization scales, which is from 3 to 24. The results are shown in Table 1. The results show relatively small errors for I frame, and also reasonably small for P frames, but somewhat larger for B frames. Note however that the average errors remain small. Moreover, as will be shown in the next section, our model's performance is very close to that achieved using the real data.

	I frames				P frames			
	MSE		BITS		MSE		BITS	
	avgerr	maxerr	avgerr	maxerr	avgerr	maxerr	avgerr	maxerr
Football	0.88%	7.01%	1.04%	6.32%	0.39%	6.60%	0.66%	8.41%
Claire	0.83%	4.37%	0.34%	3.28%	0.88%	12.30%	2.49%	33.02%
Susie	1.08%	6.10%	0.89%	6.11%	1.24%	15.88%	2.92%	15.88%
Miss America	0.95%	3.84%	0.65%	7.18%	0.89%	11.03%	3.27%	45.82%

	Football B frames			
	MSE		BITS	
	avgerr	maxerr	avgerr	maxerr
B1	2.28%	17.56%	2.89%	17.78%
B2	2.29%	14.73%	3.15%	19.65%

Table 1: Relative errors. For I frames, the statistic is over the entire quantization scale range. For P and B frame, it is calculated over the range from 3 to 24. (*avgerr*: average error, *maxerr*: maximum error.)

3 APPLICATION TO THE RATE CONTROL

3.1 Gradient-Based Rate-Control Algorithm

To evaluate the effectiveness of the proposed model, we apply the approximation model to a gradient-based rate control algorithm introduced in [12,13], by substituting the rate and distortion functions with the approximated values. Because of the model error in rate, the original strictly-constant-rate for GOP may be no longer satisfied, but we expect the buffer constraints will still be satisfied most of the time because most of the errors are due to the B frames, which consume the fewest bits. Considering the fact that the model errors in B frames are relatively large, we can further improve the solution by re-allocating bits for the B frames after the solution from the model is obtained. This is done by encoding the I and P frames using the solution from the model, and calculating the total number of bits remaining for the B frames, which we denote R_B. Using this available bit budget, the bit allocation for B frames is then re-optimized. The additional optimization procedure does not cost much in terms of computation, because all the reference frames (I and P) are fixed and all the B frames are independent to each other. Denote the rate and distortion functions of B frames as $r_{Bi}(q_i)$ and $d_{Bi}(q_i)$, where i is the index of B frames. The optimization problem is to determine quantization scales for each B frame ($q_0, q_1, \ldots, q_{M-1}$, where M is the number of B frames), such that the overall distortion is minimized, as below:

$$\text{minimize} \quad \sum_{i=0}^{N-1} d_{Bi}(q_i) \qquad \text{subject to} \quad \sum_{i=0}^{N-1} r_{Bi}(q_i) \leq R_B \tag{11}$$

The problem can be solved by the method of Lagrange multipliers, by repeatedly solving the following set of unconstrained problems for given λ's,

$$\min_{q_i} \left[d_{Bi}(q_i) + \lambda r_{Bi}(q_i) \right], \quad i = 0 \ldots M-1 \tag{12}$$

and searching for λ, such that the constraint in (11) is satisfied. This can be done efficiently by using the fast search method proposed in [17]. The function values of $r_{Bi}(q_i)$ and $d_{Bi}(q_i)$ are obtained by evaluating the function at control points and interpolating by the intra-frame model. By using this approach, the solution is improved and the strictly-constant-rate for GOP are satisfied again.

To evaluate the effectiveness of the proposed model, we encode two video sequences, football and table tennis, in CIF format at 1.152 Mbps, using three different configurations: (i) gradient-based method with the approximated R-D from the proposed model; (ii) use (i) with additional bit-re-allocation for B frames using Lagrange method; (iii) gradient-based method with the original R-D. The GOP was chosen to be size of 6 (IBBPBB)[3]. The results are shown in the Table 2. An additional result from the Software Simulation Group's MPEG-2 encoder [16], which use an implementation of the TM5 algorithm [3], is also shown only for reference purpose. Note that in this paper, we do not include the adaptive quantization scheme into consideration, while the standard TM5 program we used here includes an adaptive quantization procedure. The problem of adaptive quantization is left for future research. The computation complexities shown in the Table are relative to the TM5 algorithm, and are estimated based on the subroutines in [16], where (i) 13 mults and 29 adds are required for each 8×1 DCT; (ii) two-step search[4] method is used for the motion estimation (takes about 90 percent of overall computations in a single-pass encoding). We assume the memory is large enough to hold all the intermediate data including the motion vectors, reconstructed reference frames, DCT coefficients, etc., so that many of the operations only have to be done once during the evaluation of R-D data on the control points.

	Football		Table Tennis	
	PSNR	Complexity	PSNR	Complexity
Model R-D	33.13	1.68	32.64	1.71
B-Frame Re-Alloc.	33.17	1.70	32.80	1.73
Original R-D	33.17	8.87	32.74	11.35
Test Model 5	32.43	1.00	31.25	1.00

Table 2: Average PSNR and computation complexity with different encoding method. The second second row is based on the model R-D with additional bit-re-allocation for B frames. The computation complexity is relative to the Test Model 5 algorithm.

The results show that, by using the approximated model, *the number of computations is reduced significantly with very little loss in PSNR*. With bit-re-allocating on B frames, we are able to achieve the same PSNR with only a fraction of computation overhead. Note that the relative increase in complexity with respect to TM5 will become larger if fast motion estimation algorithm is used, since the motion estimation is responsible for the bulk of the complexity in the entire encoding process. Another issue is the limitation of GOP size, although its impact is not so high because changing the GOP size from 6 to 15 would result in changing only one frame from P to I for every 10 frames. In the next section, we propose a new method with relatively low complexity and without the limitation of GOP size, by using the R-D predicted from previously coded frames.

[3] We have experienced slow convergence rate of the steepest descent method in higher dimension.

[4] Spiraling outward full search for full-pixel displacement, followed by the search for 8 neighboring half-pixel displacement.

3.2 Rate Control Using Predicted R-D

Consider the fact that, unless there is a scene change, the contents of the image frames are usually similar to each other within a short period of time (e.g. within a GOP). So, when encoding a GOP, it is reasonable to assume the R-D property of an un-coded frame is similar to the most recently coded frame of the same type. Suppose the rate and distortion functions measured from the most recently coded frames are denoted as $r_I(q)$, $d_I(q)$, $r_P(q)$, $d_P(q)$, $r_B(q)$, $d_B(q)$, for the I, P, B frame respectively. Also the number of remaining frames for each frame-type within a GOP are denoted as N_I, N_P, N_B. To simplify the computation, we relax the buffer constraints because the buffer is rarely in overflow when using the standard decoder buffer size recommended by MPEG (e.g. 320 K bit for CIF format)[5]. Based on the specified frame-rate and bit-rate, the average number of bits per frame is calculated and denoted as R. The rate control procedure is listed as below:

1. Initialize the value of N_I, N_P, N_B. Initialize the total bit-budget of a GOP as

   $$T = T_0 + R \cdot (N_I + N_P + N_B)$$

   where T_0 is the number of bits still available from (or if smaller than 0, over-used by) the previous GOP.

2. Evaluate the rate and distortion of the current frame, by repeatedly quantizing and encoding the DCT coefficients of the current image frame for the q's in the set of control points, and then reconstructing the $r(q)$ and $d(q)$ using the spline interpolation functions. According to the current frame type, replace one of the rate functions in $\{(r_I(q), r_P(q), r_B(q)\}$ by the newly derived $r(q)$, and one of the distortion functions in $\{(d_I(q), d_P(q), d_B(q)\}$ by the newly derived $d(q)$.

3. Find the quantization scales for each frame type, q_I, q_P, and q_B. In this paper, the following two alternative criteria are tested:

   - *Minimum MSE:* The solution of q_I^*, q_P^*, and q_B^* is derived by minimizing

     $$N_I \cdot d_I(q_I) + N_P \cdot d_P(q_P) + N_B \cdot d_B(q_B) \tag{13}$$

     subject to

     $$N_I \cdot r_I(q_I) + N_P \cdot r_P(q_P) + N_B \cdot r_B(q_B) \leq T \tag{14}$$

     $$q_I \leq q_P \leq q_B \tag{15}$$

   - *Smooth MSE:* Based on the current frame type, pick one variable in $\{q_I, q_P, q_B\}$ as a primary variable. For example, suppose the current frame is an I frame, the primary variable is q_I. Given $q_I = x$, the quantization scales for P frames and B frames (denoted as $y^*(x)$ and $z^*(x)$ respectively) are derived by

     $$y^*(x) = \arg\min_y \, [d_P(y) - d_I(x)] \tag{16}$$

     $$z^*(x) = \arg\min_z \, [d_B(z) - d_I(x)] \tag{17}$$

     subject to

     $$d_I \leq d_P \leq d_B \tag{18}$$

     Then, the solution q_I^* is derived by minimizing the following expression over all possible x's:

     $$q_I^* = \arg\min_x \, |[N_I \cdot r_I(x) + N_P \cdot r_P(y^*(x)) + N_B \cdot r_B(z^*(x))] - T| \tag{19}$$

     If the current frame-type is P or B, the solution of q_P^* or q_B^* can be derived by a similar procedure.

[5] Note that this can be a reasonable assumption in some applications (e.g., DVD) but in other cases, especially if buffer size is small due to short delay requirement, the buffer constraint cannot be ignored. Also note that most implementations of the TM5 algorithm also do not control the buffer overflow. They only monitor the buffer occupancy and give warnings when overflow occurs.

Note that the constraints in (15) and (18) are added for better quality of the reference frames, which usually can improve the overall quality.

4. Choose the one quantization scale in $\{q_I^*, q_P^*, q_B^*\}$ corresponding to the current frame-type to encode current frame. Update the value of N_I, N_P, N_B, and T according to the frame-type and the number of bits actually generated by the current frame. If the current frame is the last frame in the GOP, assign T to T_0, advance to the next GOP, and go to step 1. Otherwise, advance to the next frame and go to step 2.

We encode the six test video sequences by the algorithm. The results are shown in Table 3 and Fig. 4. Note that the computation complexity of the new algorithm is similar to the TM5 with only 8 additional quantization and encoding operations. Compared to other operations like motion estimation or DCT, the additional overhead is not signification. It can be sped-up further by using parallel hardware implementation.

	Bicycle				Cheer			
	GOP 6		GOP 15		GOP 6		GOP 15	
	PSNR	Diff	PSNR	Diff	PSNR	Diff	PSNR	Diff
Gradient & Model	27.05	45.86	n/a	n/a	26.59	35.45	n/a	n/a
Prediction & Minimum MSE	26.92	10.80	27.04	10.19	26.31	10.04	26.53	10.66
Prediction & Smooth MSE	26.86	8.83	27.01	7.57	26.19	6.44	26.40	4.96
Test Model 5	26.37	27.07	26.49	27.11	25.86	25.05	26.06	25.86

	Football				Flower			
	GOP 6		GOP 15		GOP 6		GOP 15	
	PSNR	Diff	PSNR	Diff	PSNR	Diff	PSNR	Diff
Gradient & Model	33.17	6.25	n/a	n/a	27.07	26.25	n/a	n/a
Prediction & Minimum MSE	33.14	5.49	33.18	4.10	26.89	15.10	27.62	14.43
Prediction & Smooth MSE	32.98	7.35	33.10	5.19	26.63	12.35	27.24	10.35
Test Model 5	32.43	11.29	32.49	11.01	25.91	14.04	26.70	15.01

	Mobile				Table Tennis			
	GOP 6		GOP 15		GOP 6		GOP 15	
	PSNR	Diff	PSNR	Diff	PSNR	Diff	PSNR	Diff
Gradient & Model	25.25	30.38	n/a	n/a	32.80	7.13	n/a	n/a
Prediction & Minimum MSE	24.70	18.16	25.75	15.19	32.60	7.91	33.43	7.15
Prediction & Smooth MSE	24.75	15.58	25.52	10.74	32.44	6.98	33.14	5.48
Test Model 5	23.96	30.02	25.07	21.44	31.25	8.62	32.13	7.58

Table 3: Average PSNR and first-order difference of MSE for the six test sequences. *Gradient & Model*: gradient method with R-D approximated by the model with additional bit-re-allocation for B frames; *Prediction & Minimum MSE*: Predicted R-D with minimum MSE; *Prediction & Smooth MSE*: Predicted R-D with smooth MSE.

4 CONCLUSION AND FUTURE WORK

In this the paper, we have demonstrated that our proposed model provides a good approximation of rate-distortion characteristic. By applying the approximation model to the gradient-based rate control algorithm, the computation complexity can be reduced without degrading much of the quality. We then introduce a new fast algorithm by using the R-D predicted from the previous coded frames, and reduce the computations to a level similar to Test Model 5 while maintaining the higher and smoother PSNR. The following issues are left for future research: (i) apply the inter-frame dependencies model in the new fast algorithm; (ii) address the problem of adaptive quantizations; (iii) study the R-D characteristics and rate control scheme for the interlaced video for MPEG-2 (used in DVD).

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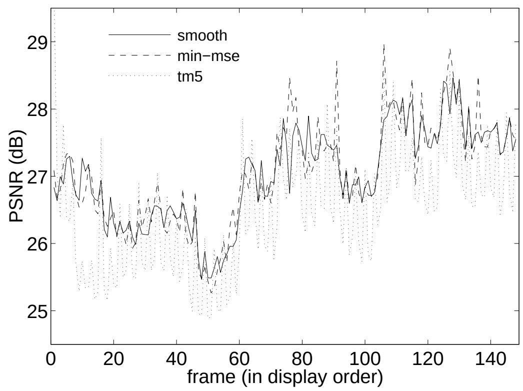
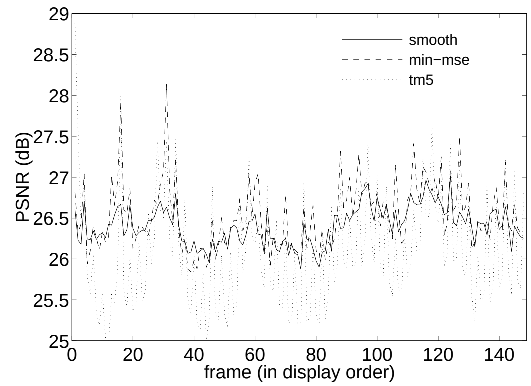
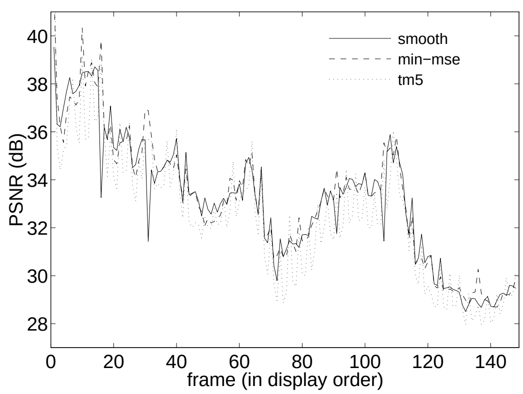


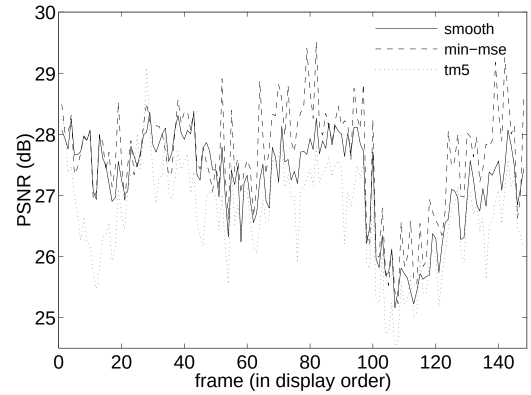
(a) Bicycle



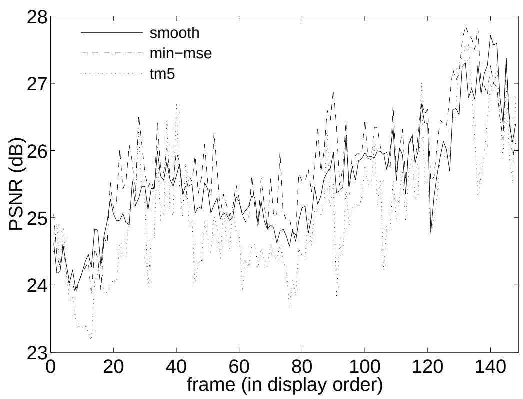
(b) Cheer



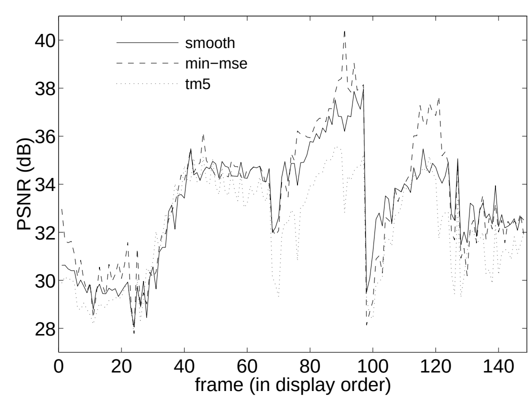
(c) Football



(d) Flower



(e) Mobile



(f) Table Tennis

Figure 4: PSNR of image frames the six video sequences, encoded using GOP size 15. In each figure, *smooth*: optimizing by smooth MSE criterion; *min-mse*: optimizing by minimum MSE criterion; *tm5*: Test Model 5 algorithm.