ADAPTIVE DE-NOISING OF IMAGES BY LOCALLY SWITCHING WAVELET TRANSFORMS

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Abstract

A local adaptive image de-noising method based on local selection of the best wavelet, among a finite set, within a sliding window at each level of decomposition is developed. The proposed method suggests certain advantages in terms of de-noising efficiency and detail preservation especially when the image includes different regions which may be efficiently represented by different bases or a priori information on the image is limited. This work concerns the method of using an adaptively varying base and the best wavelet selection rule. The method is implemented and comparative results are submitted.

1 Introduction

Performing a wavelet transform, thresholding wavelet transfor coefficients and inverse transforming the thresholded wavelet transform coefficients are three basic steps of the standard orthogonal wavelet approach to signal and image denoising [1],[2]. The thresholding enables the wavelet transform based denoising to become adaptive to unknown smoothness of the signal and the theory confirms that this approach is able to reach the best asymptotic accuracy for the white Gaussian additive noise. However, the practical efficiency strongly depends on how far the used wavelet transform as well as the thresholding rule are agreed with a local behavior of the signal. This observation is a main motivation of the algorithm developed in this paper.

We perform a set of the wavelet transforms for each local portion of the signal within a sliding window. The particular wavelet decomposition structure which is assumed to be the most efficient one, among the finite set, for the local portion is selected. The shrinkage is performed by using the selected decomposition structure and the estimate of the points in the local window are computed. The selection of best decomposition structure is performed by selecting the best filters for each level of decomposition (Figure 1).

Finally, locally selected decomposition structures are used as estimators for the every location of the sliding window. The final pointwise estimate for each pixel is computed by combining the estimates obtained in the overlapping sliding windows.

The set of the used wavelet transforms, the rules for selecting the best local decomposition structure and rules for threshold selection form design variables of this approach.

2 Selection of the best local transform

Let $x(i,j)$,

$$x(i,j) = y(i,j) + \sigma n(i,j),$$

be the corrupted observation, where $y(i,j)$ is the image and $\sigma n(i,j)$ is the corrupting white Gaussian noise with standard deviation σ. Let us consider a single level of wavelet decomposition:

$$[\mathbf{a}_k, \mathbf{d}_k] = \mathbf{T}_k(\mathbf{x})$$

where $\mathbf{a_k}$ is the approximation coefficients and $\mathbf{d_k}$ is the detail coefficients of the observation $\mathbf{x}$ over the k^{th} wavelet transform, $\mathbf{T_1}, \mathbf{T_2}, ...\mathbf{T_k}, ...\mathbf{T_K}$ are the sparse matrices corresponding to K different wavelet transforms. Let the adaptive threshold t_k for the coefficients of k^{th} transform be as follows

$$\begin{aligned} t_k &= \gamma \cdot \text{median}(\text{abs}(\mathbf{d_k})) \\ \text{abs}(\mathbf{a}) &= \{|a_0|, |a_1|, ..., |a_f|\} \end{aligned}$$

where γ is an empirically found constant. Consider the thresholded detail coefficients $\mathbf{d'_k}$ be:

$$\begin{aligned} d'_k(i,j) &= d_k(i,j) \cdot 1(|d_k(i,j)| \geq t_k), \\ 1(|a| &\geq t) = \begin{cases} 1, & |a| \geq t \\ 0, & |a| < t \end{cases}, \end{aligned}$$

where t is a value of the threshold, $1(|a| \geq t)$ is the indicator-function and $d_k(i,j)$ is the $(i,j)^{th}$ pixel of a matrix $\mathbf{d_k}$.

The detail energy under threshold E_k^t for each transform k is:

$$\begin{aligned} E_k^t &= < (\mathbf{d}_k - \mathbf{d}'_k), (\mathbf{d}_k - \mathbf{d}'_k) > \\ < & \mathbf{a}, \mathbf{b} >= \sum a(i,j) b(i,j). \end{aligned}$$

The transform $\mathbf{T_n}$ for which the detail energy E_n^t is the minimum among the wavelet transform set is assumed to be the best transform for any local portion and decomposition level:

$$\begin{aligned} n &= \arg\min_k (E_1^t, ..., E_k^t, ..., E_K^t)] \\ \mathbf{T_n} &= \text{selected transform.} \end{aligned}$$

The algorithm is based on the observation that the estimate of the noise standard deviation $\overline{\sigma}_k$,

$$\overline{\sigma}_k = \sqrt{2}\text{median}(\text{abs}(\mathbf{d}_k)),$$

approaches to the real noise standard deviation σ if the transform $\mathbf{T_k}$ performs a compact representation of the signal [6], since the significant detail coefficients exist only at transition parts. The estimate $\overline{\sigma}_k$ is assumed to be slightly higher than σ if the compaction efficiency is relatively poor. The detail energy under threshold E_k^t is assumed to increase due to relatively poor compaction.

3 Features of the algorithm

Some features of the algorithm used in the simulations are briefly discussed here.

- A window size of 32×32 was selected. Keeping the window size sufficiently large is necessary for satisfying the considerations about the detail and noise distribution.
- Four one shifted versions of Haar (no shift, horizontal, vertical and diagonal) and Symlet8 wavelets (a total of eight wavelets) were used in the set [1].
- For each location of the sliding window the best decomposition structure was selected.
- The structure was selected for four levels of decomposition.
- The threshold estimate computed for the first level of decomposition were used as the threshold level. An empirically found constant ($\gamma = 3$) was used in a threshold estimation of the local threshold.
- Then, wavelet shrinkage was applied to obtain an estimate for the window contents.
- As a result of the local wavelet shrinkage we obtain the estimate for every pixel (i,j) and neighboring points in the window. As the windows are overlapping we have more than one estimates for every pixel obtained in the different windows. The final estimate for each pixel is obtained by combining the available multiple estimates of the same pixel.

The same approach can be implemented by using different transform and/or threshold selection rules. For example an entropy based [3] or Akaike information criterion [4] based method can be employed for adaptive selection of level of decompositions. The local threshold estimation coefficient γ may also be locally selected [7].

Another alternative for heavily noised images may be one which use adaptive window size selection [5],[7]. However, incorporating local adoption of all parameters to the same filtering algorithm is not realistic in computational sense.

The combination rule for the multiple estimates also affects the performance of the filter. Simple averaging, weighted averaging, ϵ-neigborhood based selective averaging are possible alternatives, ϵ-neigborhood based selective averaging performs slightly better with respect to others.

[1] Discrete Cosine Transform (**DCT**) may also be considered for smaller transform size.

4 Concluding Remarks

The algorithm was implemented in MATLAB and tested with the 'montage' image. The image was corrupted by white Gaussian noise with standard deviation $\sigma = 0.1$. The proposed algorithm attenuated the Root Mean Square Error (RMSE) to 0.0309, while the best basis de-noising by global wavelet transform using Haar wavelet attenuated the RMSE to 0.0585 and Symlet8 to 0.464. Furthermore, the remaining noise can be reduced to 0.303 after post-processing by a median filter of size 3×1. The comparative results are given in Table 1. The original, corrupted and filtered by the proposed method images are plotted in Figures 2,3,4.

Table1: Comparative results.

Used Filter	RMSE	MAE
Proposed Method	0.0309	0.0199
Wiener Filter (5x5)	0.0408	0.0274
Wavelet Package Haar	0.0585	0.0444
Wavelet Package Sym 8	0.0464	0.0310
Wavelet PO Haar (4 level)	0.0567	0.0411
Wavelet PO Sym 8 (4 level)	0.0693	0.0485
Wavelet TI Haar (5 level)	0.0317	0.0205
Wavelet TI Sym 8 (5 level)	0.0365	0.0261

MAE: Mean Absolute Error, PO: Periodic
TI: Translation Invariant

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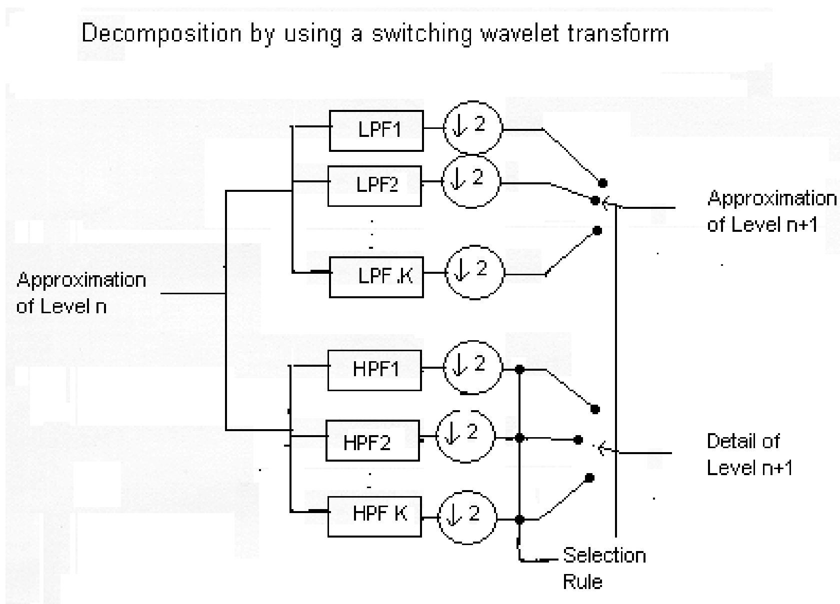

Figure 1: Decomposition by switching the wavelet transforms

Figure 2: Original Image

Figure 3: Corrupted Image

Figure 4: Filtered Image